Quiz 6
Math Bootcamp 2012

1. We say $X$ has a binomial distribution, or $X \sim Binomial(n, p)$ if

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}, \quad \text{for } x = 0, 1, \ldots, n.$$ 

a. Prove that $\sum_{x=0}^{n} P(X = x) = 1$.
b. Define the expectation of $X$ as $E(X) = \sum_{x=0}^{n} x P(X = x)$. Prove that

$$E(X) = np.$$ 

2. For $b > a$, we say $X$ is uniformly distributed over $[a, b]$ if it has probability density function

$$f(x) = \begin{cases} 
\frac{1}{b-a} & \text{if } a \leq x \leq b \\
0 & \text{otherwise}
\end{cases}$$ 

We denote it by $X \sim Uniform(a, b)$.
a. Prove that $\int_{a}^{b} f(x)dx = 1$.
b. Define the expectation of $X$ as $E(X) = \int_{a}^{b} x f(x)dx$. Prove that

$$E(X) = (a + b)/2.$$ 

3. Four cards are dealt from the top a well-shuffled deck. What is the probability that they are four aces? The deck has 52 cards.