Quiz
August 16, 2010

1. Find the derivatives of the following functions:
   a) \( f(x) = (6x^3 - x)(10 - 20x) \).
   b) \( f(x) = \frac{3x+9}{2-x} \).
   c) \( f(x) = a^x \).

2. Evaluate the following integrations:
   a) \( \int (1 - \frac{1}{x}) \cos(x - \ln x) dx \).
   b) \( \int \sin(1 - x)(2 - \cos(1 - x))^4 dx \).
   c) \( \int \frac{3y}{5y^2 + 4} dy \).
3. Denote the number of $k$-combinations from a given set $\mathcal{S}$ of $n$ elements as

\[
\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k(k-1)\cdots 1} = \frac{n!}{k!(n-k)!}.
\]

Show that
a) For integer $0 < k < n$, \( \binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1} \).

b) \( \sum_{i=0}^{n} \binom{n}{i} = \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n \).

4. Consider choosing 4 numbers from 1, 2, \ldots, 10.

a) The order matters, for example, (1, 2, 3, 4) and (2, 1, 3, 4) are considered to be two different choices. If you can choose with replacement, say (2, 2, 3, 4) is a possible choice, what is the total number of possible choices?

b) The order does not matter, for example, (1, 2, 3, 4) and (2, 1, 3, 4) are considered to be the same choice. If you can choose with replacement, say (2, 2, 3, 4) is a possible choice, what is the total number of possible choices?
5. Find equation of tangent line to $f(x) = 4x - 8\sqrt{x}$ at $x = 16$.

6. Determine $f'(0)$ for $f(x) = |x|$.

7. Show that $f(x) = 2x^3 - 5x^2 - 10x + 5$ has a root in the interval of $[-1, 2]$.

8. Solve $\ln x + \ln(x - 3) = 1$.

9. Graph $x^2 + 2x + y^2 - 8y + 8 = 0$.

10. Determine whether the following statements are right or wrong:

    a) $\lim_{\theta \to 0} \frac{1 - \cos(\theta)}{\theta} = 0$ and $\lim_{\theta \to 0} \frac{\theta}{\sin(\theta)} = 1$.

    b) $\lim_{\theta \to 0} \cos(\frac{\pi}{\theta})$ does not exist.

    c) $\lim_{x \to 0} \frac{1}{x^2} = 1$ and $\lim_{x \to 0} \frac{1}{x}$ does not exist.