1. Let $\mathbf{z} = (1, 3, 2)^T$ and $\mathbf{y} = (-2, 1, -1)^T$. Find
   a) the lengths of $\mathbf{z}$ and $\mathbf{y}$
   b) the cosine of the angle between $\mathbf{z}$ and $\mathbf{y}$.

2. Consider $\mathbf{z}_1 = (1, 2, 3)^T$, $\mathbf{z}_2 = (1, 1, 1)^T$ and $\mathbf{z}_3 = (0, 0, 1)$. Are they linearly dependent or independent?

3. Find the rank of the following matrices.
   $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 3 & 0 \\ 1 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{pmatrix}$, and $\mathbf{C} = \begin{pmatrix} 3 & 2 & 0 \\ 2 & 4 & 0 \end{pmatrix}$.

4. Use the definition of inverse matrix. Check that for $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ as long as $\det(\mathbf{A}) \neq 0$.

5. Solve the system of equations \[ \begin{align*}
3x - y &= 7 \\
2x + 3y &= 1
\end{align*} \]
   a) Check that the system of equations becomes $\mathbf{A} \mathbf{z} = \mathbf{c}$, where $\mathbf{c} = (7, 1)^T$.
   b) Check that $\mathbf{z} = \mathbf{c}$ if the solution of the system of equations can be written as $\mathbf{A}^{-1} \mathbf{c}$. 