Hw 1, Math Bootcamp 2016
Due 9 am, August 12. Open book.

1. Solve \( \log_4 x + \log_4(x - 3) = 1 \).

2. Find the domain of 
\[
f(x) = \frac{x}{\sqrt{x^2 - 9}}.
\]

3. Find the derivatives of the following functions
a) \( f(x) = (6x^3 - x)(10 - 20x) \)
b) \( f(x) = \frac{3x+9}{2-x} \)
c) \( f(x) = x^x \).

4. Find the equation of the tangent line to \( x^2 + y^2 = 25 \) at \( (3, -4) \).

5. Use the inverse rule 
\[
\frac{df^{-1}(x)}{dx} = \frac{1}{f'(f^{-1}(x))}
\]
to prove that 
\[
\frac{d \arccos(x)}{dx} = -\frac{1}{\sqrt{1-x^2}}
\]

6. Find the Taylor series of the following functions at \( x = 0 \)
a) \( f(x) = e^x \)
b) \( f(x) = \frac{1}{1-x} \)
c) \( f(x) = \ln(1 + x) \).
7. Use Newton-Raphson to solve for $x$ in $f(x) = 0$, where $f(x) = x^3 - 21x^2 + 135x - 220$. Use initial value $x_0 = 2$.

8. A 15 foot ladder is resting against the wall. The bottom is initially 10 feet away from the wall and is being pushed towards the wall at a rate of .25 ft/sec. How fast is the top of the ladder moving up the wall 12 seconds after we start pushing?

9. Find the following integration
   
   a) $\int_0^1 x^2 e^{-x} \, dx$
   
   b) $\int_4^8 \frac{3x+11}{x^2-x-6} \, dx$.

10. Determine the area of the region enclosed by $y = x^2$ and $y = \sqrt{x}$.