Lifetime of Asynchronous Wireless Sensor Networks with Multiple Channels and Power Control

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***Abstract-* We consider wireless sensor networks with data collection traffic that apply asynchronous duty cycling to conserve energy, which is common in many environmental monitoring applications. Under these assumptions, the effect of overhearing dominates the energy consumption of the nodes. We propose multichannel operation with dynamic channel selection and power control for reducing the effect of overhearing in such asynchronous networks. A mathematical model is presented to calculate the maximum network lifetime under these considerations.**

***Keywords:* Wireless sensor networks; power-control; network lifetime; multi-channel routing.**

I. Introduction

The issue of energy conservation and lifetime optimization is critical for reliable long-term operations of wireless sensor networks (WSNs). It is well known that the radio transceiver typically dominates the energy consumption in wireless sensor nodes. The most effective strategy for conserving the energy consumed by the transceiver is duty-cycling between sleep and wake periods, which has been adapted in a large number MAC protocols proposed for WSNs. The key challenge for applying duty-cycling is synchronization of the wake periods between a transmitter and a receiver. If the nodes are time synchronized, then network-wide or local scheduling policies can be applied that can enable nodes to synchronize their wake periods during transmission/reception and go back to sleep at other times. However, challenges in achieving network-wide time synchronization and latency in multi-hop transmissions caused by such synchronized scheduling principles are concerns with this approach. An alternative is asynchronous duty-cycling, where all nodes wake up briefly at periodic intervals of time to check for activity and only remain awake if some activity is detected. Otherwise, the nodes return to their energy-conserving sleep states. Generally, a lengthy preamble is used for each transmitted packet so that the receiving node is able to detect it during its brief wake time. This provides an effective solution for energy conservation in asynchronous WSNs especially under low data rates. Asynchronous duty cycling has been applied to a number of Low Power Listening (LPL) and preamble sampling MAC protocols [1], [2]. One of the key problems with this approach is that it leads to energy wastage from *overhearing*, since unintended neighbors need to receive an entire packet before knowing the destination. Possible solutions to this overhearing problem include mechanisms for providing additional information in the preamble to enable neighbors to interrupt the reception of long preambles when not needed [3], adaptive duty-cycling (EA-ALPL, ASLEEP) [4], [5] and others. Despite these developments, overhearing remains to be a dominating factor in the energy consumption in asynchronous WSNs, especially under high node density and large network sizes.

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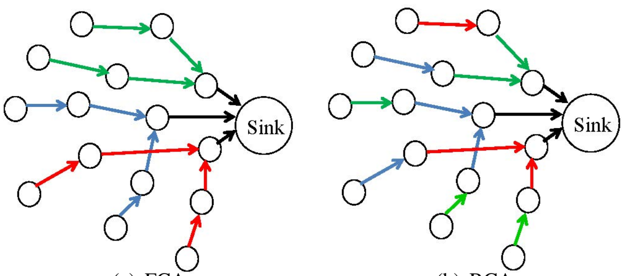
To alleviate the problem of overhearing in such asynchronous WSNs, we propose two approaches. The first is the use of multiple orthogonal channels to reduce the number of co-channel transmissions in a node's neighborhood. Multi-channel operation is supported by typical WSN platforms such as MICAz and Telos, which is typically applied for reducing interference. In our earlier work in [6], [7] we presented the benefits of using a multi-channel MAC protocol by which nodes perform dynamic channel selection to reduce overhearing based on the energy constraints of its neighbors. Secondly, we consider distributed transmit power control [8] that is also an effective mechanism for controlling the effect of overhearing. Our objective is to apply channel selection and power control to adapt the energy consumption in the nodes in order to balance their remaining battery lifetimes, which effectively maximizes the network lifetime.

The main contributions of this paper are as follows. First, we develop a mathematical model to evaluate the network lifetime of single-channel wireless sensor networks under optimal power control. This is derived under a node energy consumption model that assumes asynchronous LPL and a data collection traffic using a link-quality based routing protocol, such as collection tree protocol (CTP) [9]. We first calculate the optimal transmission range of the nodes so that the overall current consumption is minimized. We then apply this result to calculate the network lifetime assuming that all nodes apply transmit power to achieve this optimal transmission range. Our objective is to determine the effect of transmission power control on the lifetime of the network that is primarily affected by energy consumed in transmissions, receiving, and overhearing. Secondly, we extend the network lifetime calculation to consider multi-channel operation where nodes are assumed to dynamically select channels with optimal power control to balance the nodes remaining lifetimes.

II. System Model

We assume a data gathering WSN where all sensor nodes periodically sense some physical parameters and forward them to the sink. Nodes broadcast periodic beacons to exchange various control parameters. We assume that nodes are not time synchronized and they apply the basic LPL principle to conserve energy [10], [11]. We assume that the sender prepends each message with a preamble that is long enough to span the complete length of a sleep-wake cycle to ensure that the receiving node detects it regardless of when it wakes



(a) FCA (b) RCA

Fig. 1. Proposed data collection trees with multiple channels. Each color represents a different channel.

up[1]. Because of this long preamble length (for both beacons and data packets), the effect of overhearing becomes costly. We assume that nodes apply a distributed power control mechanism to reduce the effect of overhearing.

To further reduce the energy consumption and extend the lifetime of the network, we propose two multi-channel transmission schemes. In the first scheme, which we call *flow based channel assignment (FCA)*, we assume that k channels are uniformly distributed over the nodes in the network. Nodes that are on the same channel form a subtree. Thus the scheme partitions the whole network into k vertex-disjoint subtrees as shown in Figure 1(a). Although this channel assignment scheme reduces the average overhearing, it does not allow the nodes to control their energy consumption with respect to their *varying energy resources*, which is our goal for *balancing the remaining lifetimes* of the nodes and thereby maximizing the lifetime of the network. To achieve this, we propose a *receiver based channel assignment (RCA)* that works as follows. We define *receiver channel* as the channel on which a node receives packets. On the other hand *transmit channel* is the channel on which a node transmits, which is the receiver channel on its intended destination. The scheme is shown in Figure 1(b). In RCA, nodes monitor their receiver channels for incoming transmission by default. At the time of transmission, a node temporarily switches to a transmit channel and returns to its receiver channel after transmission. Essentially, RCA allows nodes to choose their transmit channels dynamically to balance the energy consumption of its neighbors so as to balance their residual battery capacities. Details on the implementation of RCA along with experimental results are reported in [6]. Here we develop a mathematical model to analyze the network lifetime.

III. OPTIMAL TRANSMISSION RANGE CALCULATION

We first consider single channel operation, where the estimated current consumption of a node is represented as [12] :

$$\begin{aligned}\mathcal{I} &= \tfrac{I_{Bt}T_{Bt}}{T_B} + M.I_{Dt}T_{Dt} + S.\tfrac{I_{Br}T_{Br}}{T_B} + O.I_{Dr}T_{Dr} \\ &+ F.I_{Dt}T_{Dt} + R.I_{Dr}T_{Dr} + \tfrac{I_sT_s}{T_D} + P.I_pT_p \qquad (1)\end{aligned}$$

where I_x and T_x represent the current drawn and the duration, respectively, of the event x; and T_B represents the beacon interval. Transmission/reception of beacon packets is denoted by B_t/B_r, data transmit/receive is denoted by D_t/D_r and processing and sensing are denoted as p and s, respectively. O and F are the overhearing and forwarding rate respectively and S is the number of neighbors. M is the rate at which a node transmits its own packets and R is the reception rate. P is the number of times a node wakes up and checks the channel. Under these assumptions, we first calculate the optimal transmission range for multi-hop transmission to minimize the overall power consumption.

Let us assume that the current drawn by the receiver electronics in the receiving mode is $I_{Dr} = \alpha_{12}$. In transmit mode, the current drawn is dependent on the transmit power. Assuming that optimal power control is applied, to transmit a packet over a distance d with a path loss exponent of n, the current drawn is

$$I_{Dt} = \alpha_{11} + \alpha_3 d^n \qquad (2)$$

where α_{11} is the current consumed by the transmitter electronics, α_3 accounts for current dissipation in the transmit op-amp. The duration of a packet transmission and reception is proportional to the packet length. We assume that both the data packets and the beacon packets are of same length, thus $T_{Dt} = T_{Dr} = T_{Bt} = T_{Br} = T_l$. [2] Thus, the current consumed by a relay node that receives a packet and transmits it d meters onward is,

$$I_{relay}(d) = (\alpha_{11} + \alpha_3 d^n + \alpha_{12}) .T_l \qquad (3)$$

With a node density (i.e. number of nodes in an unit area) of ρ is the node density the expected number of nodes that overhear the transmission is given by $\pi.d^2\rho - 2$, where we deduct 2 to remove the transmitter and the receiver from consideration of overhearing. Thus the current consumed for overhearing while transmitting a packet is given by $I_{ov} = \left(\pi.d^2\rho - 2\right).\alpha_{12}.T_l$.

We first calculate the total current consumed in the network to transmit a packet from A to B with $K-1$ relays between them as shown in Figure 2(a). The distance between A and B is D. Thus the total current (sum of currents in all nodes) consumed is given by

$$I_T(D) = \textstyle\sum_{i=1}^{K} \left(I_{relay}(d_i) + I_{ov}(d_i)\right) = \sum_{i=1}^{K} I_R(d_i) \qquad (4)$$

where

$$\begin{aligned}I_R(d_i) &= I_{relay}(d_i) + I_{ov}(d_i) \\ &= \left(\alpha_{11} - \alpha_{12} + \alpha_3 d_i^n + \pi.d_i^2\rho.\alpha_{12}\right).T_l \\ &= \left(\alpha_1 + \alpha_2 d_i^2 + \alpha_3 d_i^n\right).T_l \qquad (5)\end{aligned}$$

Theorem 1: Given D and K, $I_T(D)$ is minimized when all hop-distances are equal to $\frac{D}{K}$.

Proof: The proof is obtained similar to that presented in [13]. Note that $I_R(d)$ is strictly convex as $\frac{d^2 I_R}{dt^2} > 0$. Thus from *Jensen's inequality*, we can write

$$\begin{aligned}I_R\left(\tfrac{\sum_{i=1}^{K} d_i}{K}\right) &\leq \tfrac{\sum_{i=1}^{K} I_R(d_i)}{K} \\ \Rightarrow K.I_R\left(\tfrac{D}{K}\right) &\leq \textstyle\sum_{i=1}^{K} I_R(d_i) \quad \Rightarrow K.I_R\left(\tfrac{D}{K}\right) \leq I_T(D) \qquad (6)\end{aligned}$$

which completes the proof. ■

[1] In this work, we do not consider mechanisms for nodes to interrupt unintended receptions using special information transmitted within the preamble, for simplicity.

[2] This is based on the assumption that with low-power operation, the packet size is primarily determined by the long preamble.

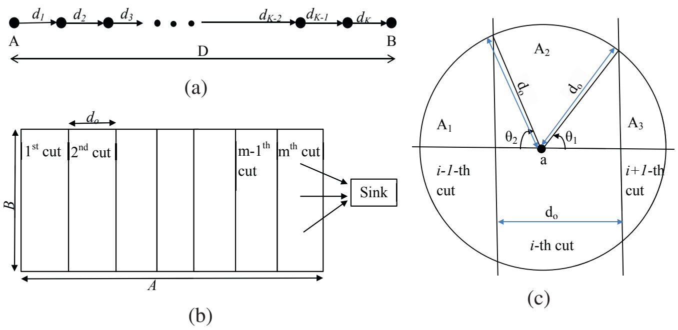


Fig. 2. (a) Introducing $K-1$ relay between A and B. (b) A sensor network with N nodes in a field of A×B, (c) Calculating overhearing at the i-th cut.

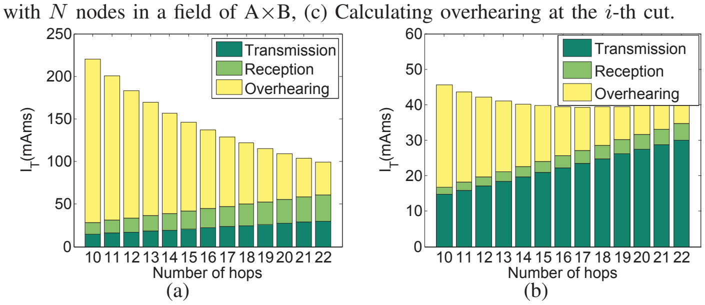


Fig. 3. Variation of I_T with K when D = 200 meters and ρ = 0.0125 nodes/meters^2 for MICA2 motes (a) with typical value of I_r = 10 mA, and (b) with a fictitious value of I_r = 1.5 mA.

Thus, the minimum energy consumption for sending a packet to a distance D using K hops is given by $I_T(D) = \left(\alpha_1 K + \alpha_2 .K.\left(\frac{D}{K}\right)^2 + \alpha_3 .K.\left(\frac{D}{K}\right)^n\right).T_l$. This is minimized when $\alpha_1 - \alpha_2.\left(\frac{D}{K}\right)^2 + (n-1)\alpha_3.\left(\frac{D}{K}\right)^n = 0$. If K_{opt} is the optimal value of K, then the corresponding distance, termed as the *characteristic distance*, is $d_m = \frac{D}{K_{opt}}$. Replacing d_m in the previous equation, we get

$$\alpha_1 - \alpha_2.d_m^2 + (n-1)\alpha_3.d_m^n = 0 \tag{7}$$

By solving this equation 7, we get d_m in terms of α_1, α_2, α_3, n. Note that d_m is independent of D.

Another fact that needs to be considered while calculating the transmission range is network connectivity. In general to ensure connectivity with a high probability, there should be at least $\mathbb{K}$ nodes in the area of $\pi.r^2$, where r is the transmission range[3], i.e.

$$\pi.r^2.\rho \geq \mathbb{K} \Rightarrow r \geq \sqrt{\frac{\mathbb{K}}{\pi.\rho}} \tag{8}$$

Thus the optimal transmission range is $d_o = \max\{d_m, r^{min}\}$ where $r^{min} = \sqrt{\frac{\mathbb{K}}{\pi.\rho}}$. We consider the special case where the nodes from A to B have the same hop-distances (according to Theorem 1), $\mathbb{K}$ = 3.

Theorem 2: If the maximum current drawn by a radio to transmit at its maximum transmit power is I_t^{max} and the current drawn in the receive mode is I_r, then $d_o = r^{min}$ as long as the $I_t^{max} < (\mathbb{K}+1).I_r$.

Proof: Let us assume that I^h and I^{h+1} are the overall current consumption when there are h and $h+1$ hops present in between A and B. Also $r_i = \frac{D}{i}$ is the transmission range when there are i hops in between A and B. To preserve the connectivity with some high probability, $\pi.r_i^2.\rho \geq \mathbb{K}\ \forall\ i$. Then,

$$\begin{aligned} I^h &= \left(I_t^h + (\pi.r_h^2.\rho - 2).I_r + I_r\right).h.T_l \\ I^{h+1} &= \left(I_t^{h+1} + (\pi.r_{h+1}^2.\rho - 2).I_r + I_r\right).(h+1).T_l \\ \Delta^I &= I^h - I^{h+1} = \Delta^T + \Delta^R \end{aligned}$$

where

$$\begin{aligned} \Delta^T &= I_t^h.h.T_l - I_t^{h+1}.(h+1).T_l \\ &= \left(I_t^h - I_t^{h+1}\right).h.T_l - I_t^{h+1}.T_l \\ \Delta^R &= \left(\left(\pi.r_h^2.\rho - 1\right).h - \left(\pi.r_{h+1}^2.\rho - 1\right).(h+1)\right).I_r.T_l \\ &= \left(\left(\pi.\left(\frac{D}{h}\right)^2.\rho - 1\right).h - \left(\pi.\left(\frac{D}{h+1}\right)^2.\rho - 1\right).(h+1)\right).I_r.T_l \\ &= \left(\frac{\pi.D^2.\rho}{h.(h+1)} + 1\right).I_r.T_l \\ &> \left(\frac{\pi.D^2.\rho}{(h+1)^2} + 1\right).I_r.T_l = \left(\pi.r_{h+1}^2.\rho + 1\right).I_r.T_l \\ &> (\mathbb{K}+1).I_r.T_l \end{aligned} \tag{9}$$

When $\Delta^T \geq 0$ then $\Delta^I > 0$. When $\Delta^T < 0$ then $\Delta^T = \left(I_t^h - I_t^{h+1}\right).h.T_l - I_t^{h+1}.T_l > -I_t^{h+1}.T_l > -I_t^{max}.T_l$ as $I_t^h > I_t^{h+1}$. Thus $\Delta^I = \Delta^T + \Delta^R > \left(-I_t^{max} + (\mathbb{K}+1).I_r\right)T_l$ which is positive if $I_t^{max} < (\mathbb{K}+1).I_r$. This concludes that if $I_t^{max} < (\mathbb{K}+1).I_r$, $I^h > I^{h+1}$ i.e. increasing the number of hops results in reduced current consumption as long as $r \geq \sqrt{\frac{\mathbb{K}}{\pi.\rho}}$. At $r = \sqrt{\frac{\mathbb{K}}{\pi.\rho}}$, the current consumption is minimized, i.e. $d_o = r^{min}$. ■

For typical radio transceivers used in sensor networks such as CC1000 (used by MICA2 motes) and CC2420 (used by MICAz motes), $I_t^{max} < 4.I_r$ (obtained under the special case of $\mathbb{K}$ = 3). For instance, for CC1000 radios I_t^{max} = 27 mA and I_r = 10 mA whereas for CC2420 radios I_t^{max} = 17.4 mA and I_r = 19.7 mA. Hence, it is always good to use the minimum power that is sufficient to preserve the network connectivity and required quality with these radios. Figure 3(a) shows the variation of I_T with the number of hops, for MICA2 motes. The maximum number of hops occurs when the distance between each node is r_{min}. It is observed that for smaller number of hops, overhearing dominates due to high transmission range. With the increase in number of hops, overhearing starts reducing whereas consumptions due to reception and transmission increase as the number of relays increases.

Note that for transceivers with $I_t^{max} > (\mathbb{K}+1).I_r$, d_0 has to be calculated as $\max\{d_m, r^{min}\}$. An example of the case when $I_t^{max} > (\mathbb{K}+1).I_r$ is shown in Figure 3(b), where a non-realistic low value of $I_r = 1.5mA$ is assumed with all the other parameters considered to be the same as of MICA2 mote.

IV. Network Lifetime Calculation

We now calculate the upper limit of the lifetime of a network of N sensor nodes that are uniformly distributed in an area of $A{\times}B$. Consider that the network area is divided into rectangular areas (called *cuts*) of width d_o as shown in Figure 2(a). A node in any cut forwards its packets to a node that is located in the cut to its immediate right.

[3]Such as to ensure 1-connectivity in a homogenous network of N nodes with a probability of atleast p, $\pi.r^2\rho \geq -\ln\left(1 - p^{\frac{1}{N}}\right)$ [14].

We first consider single channel operation and calculate the energy consumption in each cut under the assumptions that each node generates b packets/seconds and the beacon rate is B beacons/seconds. The total number of cuts is $m = \frac{A}{d_o}$, with cuts numbered in increasing order from left to right. We assume that the nodes in any cut convey the traffic of the nodes in their left cuts. Thus, nodes in the first cut transmits b packets/seconds, the nodes in the second cut on average transmit $2b$ packets/seconds (their own b packets/seconds + packets generated by the first cut). So, the nodes in the i-th cut on average transmit ib packets/seconds. Thus the expected energy consumed for different actions under our assumptions in the i-th cut can be written as:

$$\mathcal{I}^i_{Dt} = ib\left(\alpha_{11} + \alpha_3 d_o^n\right).T_l \qquad \mathcal{I}^i_{Dr} = (i-1)b\alpha_{12}.T_l$$
$$\mathcal{I}^i_{Bt} = B\left(\alpha_{11} + \alpha_3 d_o^n\right).T_l \qquad \mathcal{I}^i_{Br} = \left(\pi.d_o^2\rho - 1\right)B\alpha_{12}.T_l$$

Now let us calculate the expected overhearing in the i-th cut with the help of Figure 2(b). Let us consider a point a and draw a circle with radius d_o. Thus if we place a node at a, that node overhears all traffic that are forwarded by the nodes that are inside this circle. Nodes that are in A_1, A_2 and A_3 transmit at $(i-1)b$, ib and $(i+1)b$ packets/seconds. The areas of A_1, A_3 and A_2 can be written as $d_o^2\left(\theta_2 - \sin\theta_2\cos\theta_2\right)$, $d_o^2\left(\theta_1 - \sin\theta_1\cos\theta_1\right)$ and $\pi.d_o^2 - A1 - A2$ respectively. Thus the expected number of packets that a node at a overhears in a second is given by:

$$\begin{aligned} ov_i &= E[A_1](i-1)b\rho + (E[A_2]\rho - 1)ib \\ &\quad + E[A_3](i+1)b\rho \text{ for } i < m \\ &= E[A_1](i-1)b\rho + (E[A_2]\rho - 1)ib \text{ for } i = m \\ E[A_1] &= \tfrac{2}{\pi}\int_0^{\frac{\pi}{2}} d_o^2\left(\theta_2 - \sin\theta_2\cos\theta_2\right).d\theta_2 \\ E[A_3] &= \tfrac{2}{\pi}\int_0^{\frac{\pi}{2}} d_o^2\left(\theta_1 - \sin\theta_2\cos\theta_1\right).d\theta_1 \\ E[A_2] &= \pi.d_0^2 - E[A_1] - E[A_2] \end{aligned} \tag{10}$$

Then

$$\begin{aligned} \mathcal{I}^i_{ov} &= \frac{2bi\alpha_{12}.T_l\left(\pi.d_0^2.\rho - 1\right)}{\pi} \text{ for } i < m \\ &= \frac{2bm\alpha_{12}.T_l\left(\pi.d_0^2.\rho - 1\right)}{\pi} \\ &\quad - \frac{2b\alpha_{12}.T_l\rho.d_0^2(m+1)\left(\frac{\pi^2}{8} - \frac{1}{2}\right)}{\pi} \text{ for } i = m \end{aligned} \tag{11}$$

Thus the total current consumption for the nodes in the i-th cut is

$$\mathcal{I}^i = \mathcal{I}^i_{Dt} + \mathcal{I}^i_{Dr} + \mathcal{I}^i_{Bt} + \mathcal{I}^i_{Br} + \mathcal{I}^i_{ov} + \mathcal{I}^i_S + \mathcal{I}^i_P \tag{12}$$

A. Expected Lifetime for Identical Battery Capacities

We define the initial battery capacity of each node by e_0 with τ as the cut-off capacity below which the node does not work. Then the expected lifetime of any node in the i-th cut L_i can be written as $L_i = \frac{e_0 - \tau}{\mathcal{I}^i}$. For any $i < m$, it can be shown that $L_i < L_{i-1}$. Now let us compare L_i for $i = m-1$ and $i = m$. Clearly, $\mathcal{I}^m_{Dt} > \mathcal{I}^{m-1}_{Dt}$ and $\mathcal{I}^m_{Dr} > \mathcal{I}^{m-1}_{Dr}$. But $\mathcal{I}^m_{ov}$ can be greater or less than $\mathcal{I}^{m-1}_{ov}$ based on the values of different parameters. This is because, nodes in the $(m-1)$-th cut overhear from transmissions from both $(m-2)$-th cut and m-th cut, whereas nodes in the m-th cut overhear only from transmissions from $(m-1)$-th cut. Thus L_i is minimum when $i = m$ or $i = m-1$.

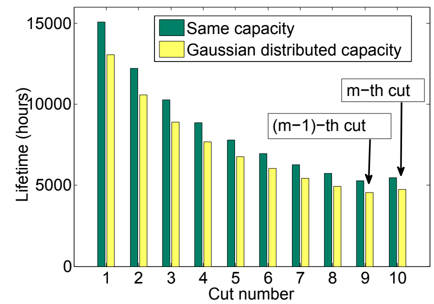


Fig. 4. Expected lifetime in each cut.

B. Expected Lifetime for Different Battery Capacities

In practice, if nothing is done to balance the energy consumption at the nodes, nodes deplete batteries non-uniformly. Consequently, the battery capacities of the nodes at any time are expected to be different. To represent this effect, we model the battery capacity of the nodes at any instant of time to be independent and identically distributed Gaussian random variables with mean μ and standard deviation σ. Define the remaining capacity of any node k in the i-th cut at time t_j to be $e_{ki}(t_j)$. If at a time instance t_0, $e_{ki}(t_0) \sim \mathcal{N}\left(\mu, \sigma^2\right)$, the probability that the remaining capacity of a node in the i-th cut is greater than τ at time $t_j = t_0 + \Delta t$ is

$$\begin{aligned} p_i &= P[e_{ki}(t_j) > \tau] = P[e_{ki}(t_0) - \mathcal{I}^i.\Delta t > \tau] \\ &= P[e_{ki}(t_0) > \tau + \mathcal{I}^i.\Delta t] = Q\left(\tfrac{\tau + \mathcal{I}^i.\Delta t - \mu}{\sigma}\right) \end{aligned} \tag{13}$$

Thus, the expected number of nodes at time t_j in the i-th cut whose capacity is greater then τ is given by $\sum_{x=1}^{\mathbb{N}} x.\binom{\mathbb{N}}{x}.p_i^x.(1-p_i)^{\mathbb{N}-x}$, where $\mathbb{N} = \frac{N}{m}$ is the number of nodes in each cut. If we define the lifetime of a cut to the time until f fraction of the nodes stay alive then

$$\sum_{x=1}^{\mathbb{N}} x.\binom{\mathbb{N}}{x}.p_i^x.(1-p_i)^{\mathbb{N}-x} = f.\mathbb{N} \tag{14}$$

By solving equation (14) we find the expected lifetime of any cut i. To illustrate the results, we take an example with 100 nodes that are uniformly distributed in an area of 200×200 meter^2. The parameters used for the results are listed in Table I. Nodes are assumed to transmit data packets as well as beacons once a minute. We consider two cases: where the initial battery capacities of all nodes are same and equal to 5000 mAHr, and where the battery capacities are normally distributed with mean of 5000 mAHr and standard deviation of 1000 mAHr[4]. For the case of non-uniform battery capacities, the expected lifetime is calculated as the time till the 75% of nodes in a cut survive. The results, depicted in Figure 4 depict that the lifetimes are lower with unequal battery capacities. Also, the $(m-1)$-th cut has the lowest lifetime for both cases.

TABLE I. DIFFERENT PARAMETERS FOR MICA2

Var	Values	Var	Values	Var	Values	Var	Values
I_{Br}	10 mA	T_{Br}	140 ms	I_{Dr}	10 mA	T_{Dr}	140 ms
I_P	10 mA	T_P	3 ms	I_S	7.5 mA	T_S	112 ms

Var	Values	Var	Values
I_{Bt}, I_{Dt}	26.7 mA (10 dBm), 20 mA (8 dBm) 16.8 mA (7 dBm), 14.8 mA (5 dBm) 13.8 mA (4 dBm), 12.8 mA (2 dBm) 11.8 mA (1 dBm), 9.7 mA (-2 dBm)	T_{Bt}, T_{Dt}	140 ms

[4]This is the capacity of batteries that we used in an experimental deployment [12].

C. Expected Lifetime with Multiple Channels and FCA

To analyze this case, we find the average overhearing at a node using the concepts of vertex coloring. First we assume that the network is represented as a *regular graph* of degree δ. A regular graph is a graph where each vertex has the same degree. Now we assign k colors uniformly to the N vertices of this graph. We group all nodes of same color, and let S_i represent the set of nodes colored with the i-th color, where $1 \leq i \leq k$. Clearly, $|S_i| = \frac{N}{k}$ and we assume that N is divisible by k for simplicity. Then

$$\begin{aligned}&\text{Pr(An edge is an overhearing edge)}\\ =\ &\text{Pr(Nodes at the both ends of that edge are from the same color set)}\\ =\ &\frac{k.\binom{|S_i|}{2}}{\binom{N}{2}} \qquad (15)\end{aligned}$$

As the total number of edges in the graph is $\frac{N.\delta}{2}$ then the expected number of overhearing edges is $q = \frac{N.\delta}{2}.\frac{k.\binom{|S_i|}{2}}{\binom{N}{2}} = \frac{N.\delta.(N-k)}{2.k.(N-1)}$. Thus each node has $\frac{2.q}{N} = \frac{\delta.(N-k)}{k.(N-1)}$ overhearers. Also the number of overhearers in unit area is $\rho_c = \frac{\delta.(N-k)}{\pi.d_0^2.k.(N-1)}$. Putting ρ_c in place of ρ in equation (11), we get a modified expression of $\mathcal{I}_{ov}^i$. By changing the expression of $\mathcal{I}_{ov}^i$, we get a new expression of $\mathcal{I}^i$ and L_i for network lifetime with multiple channels. Note that in case of FCA with multiple channels, d_0 is higher than that of single channel. This is because to preserve connectivity with some high probability, there should be atleast $\mathbb{K}$ nodes in the area of $\pi.d_0^2.\rho$ that have the same channel.

D. Expected Lifetime with Multiple Channels and RCA

For simplicity we assume that the forwarding and overhearing rate of a node is proportional to its battery health or remaining capacity at any instance. The residual capacity of node k in the i-th cut at time t_0 (when the network starts) is $e_{ki}(t_0) \sim \mathcal{N}(\mu, \sigma^2)$. Also as the forwarding and overhearing rate is proportional to its remaining capacity, then

$$\mathcal{I}_{Dt}^{ik}(t_j) = b_i(t_j).e_{ki}(t_j) \qquad \mathcal{I}_{ov}^{ik}(t_j) = c_i(t_j).e_{ki}(t_j)$$
$$\mathcal{I}^{ik}(t_j) = b_i(t_j).e_{ki}(t_j) + c_i(t_j).e_{ki}(t_j) + C$$

where $b_i(t_j)$ and $c_i(t_j)$ is the proportionality constant for nodes in the i-th cut at time instance t_j and C is the constant current consumption for other actions such as receptions, sensing etc. The k superscript is used to represent the k-th node. Clearly $b_i(t_j), c_i(t_j) < 1\ \forall\ i$ and j. Then at any instance t_j,

$$\begin{aligned} e_{ki}(t_j) &= e_{ki}(t_{j-1}) - \mathcal{I}^{ik}(t_j)\\ &= e_{ki}(t_{j-1}) - b_i(t_{j-1}).e_{ki}(t_{j-1})\\ &\quad -c_i(t_{j-1}).e_{ki}(t_{j-1}) - C\\ &= (1 - b_i(t_{j-1}) - c_i(t_{j-1}))\, e_{ki}(t_{j-1}) - C\\ &= (1 - b_i(t_{j-2}) - c_i(t_{j-2}))\\ &\quad .(1 - b_i(t_{j-1}) - c_i(t_{j-1}))\, e_{ki}(t_{j-2})\\ &\quad -(1 - b_i(t_{j-1}) - c_i(t_{j-1})).C - C = ...\\ &= \textstyle\prod_{l=0}^{j-1} (1 - b_i(t_l) - c_i(t_l))\, .e_{ki}(t_0) - \mathcal{C} \quad (16)\end{aligned}$$

where $\mathcal{C}$ is a constant. Thus $e_{ki}(t_j)$ is a Gaussian random variable with mean μ and standard deviation of $\sigma_j = \prod_{l=0}^{j-1}(1-b_i(t_l) - c_i(t_l)).\sigma$. As j increases σ_j reduces and gradually approaches zero, i.e. the distribution approaches to a constant figure. This means the residual capacity of all nodes becomes similar and all nodes in a cut die around the same time (i.e. the lifetime of nodes in the i-th cut approaches to $\frac{\mu-\tau}{I_i}$), which increases the worst case network lifetime.

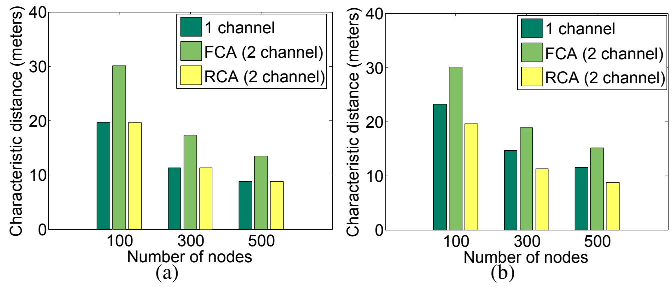


Fig. 5. Characteristic distance with different number of nodes (a) α_r = 10 mA and (b) α_r = 1.5 mA

For both FCA and RCA, we assume that the sink always stays in a designated channel. The nodes that are immediate neighbors to the sink switches to the channel of the sink temporarily while transmitting.

V. Results

We obtain numerical results of the network lifetime under different conditions using network parameters as used in the example described in section IV, unless it is mentioned otherwise. For all the following graphs, we assume $T_l = \frac{1}{P}+15$ ms, thus for P = 8, T_l = 140 ms. Also I_{Br} = I_{Dr} = α_r = T_l, i.e. considering that nodes overhear the whole preamble as well as the data or control packet. The data packets and beacons are transmitted once a minute. For gaussian distributed battery capacities, we assume $\mu = 5000$ mAH with σ = 1000 mAHr. For the following set of graphs we consider two cases. For the first case α_r is assume to be 10 mA ($I_t^{max} < 4.I_r$) and for second case α_r = 1.5 mA ($I_t^{max} > 4.I_r$).

Figure 5 shows the characteristic distance with different number of nodes. It can be observed that the characteristic distance starts reducing with increasing number of nodes because of higher overhearing caused by increasing node density. Also the characteristic distance is same for both single channel and RCA with 2 channels with α_r = 10 mA. This is because when α_r = 10 mA, the characteristic distance is effectively equal to the minimum distance required to maintain connectivity. In case of FCA, to preserve connectivity, there should be atleast $\mathbb{K}$ nodes on the same channel within its characteristic distance, which increases the characteristic distance. For α_r = 1.5 mA, the characteristic distance is lowest in case of RCA with 2 channels. Also due to connectivity considerations, FCA gives higher characteristic distance.

Figure 6 shows the variation of lifetime for the $(m-1)$-th cut with the number of nodes. It can be observed that there is significant improvement in the lifetime with 2 channels in comparison to then single channel case. This is due to lower interference and overhearing. Also we can observe that when α_r = 10 mA, RCA performs better than FCA, based on the assumption that all nodes in a cut die at the same time in RCA. This can be attributed to two reasons. First, to stay connected, d_0 in FCA is higher compared to that in RCA, which results in higher transmit power and higher overhearing.

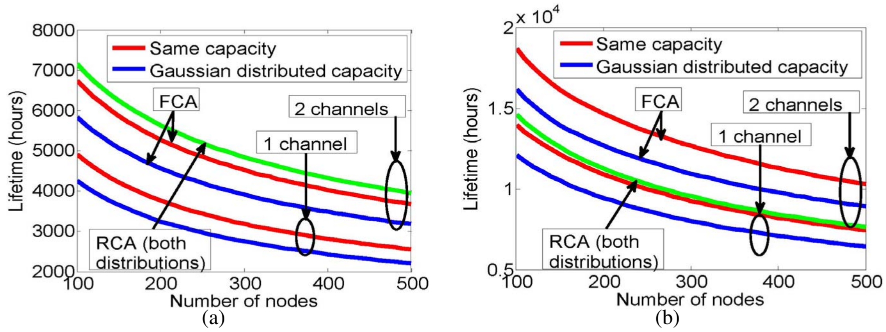


Fig. 6. Expected network lifetime with different number of nodes (a) α_r = 10 mA and (b) α_r = 1.5 mA

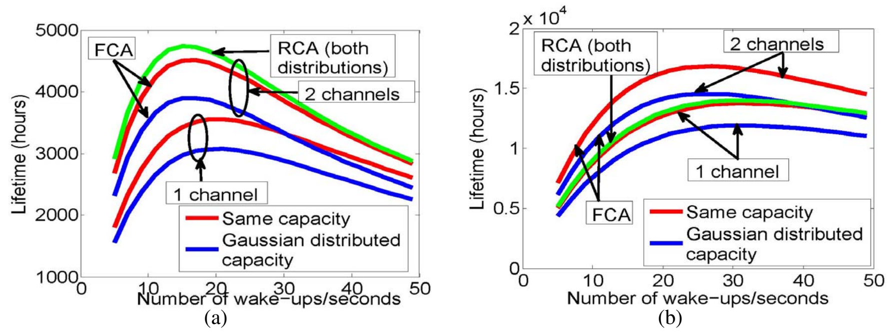


Fig. 7. Expected network lifetime with different wake-up rates (a) α_r = 10 mA and (b) α_r = 1.5 mA

Also RCA balances the overhearing based on a nodes capacity, which results in more overall battery lifetime. Note that when α_r = 1.5 mA, RCA performs poorly. This is because the characteristic distance for RCA decreases which results in more transmissions. In this case, the transmission current is the dominating factor in reducing the network lifetime.

Figure 7 shows the variation of lifetime for the $(m-1)$-th cut with wake-up frequencies. For this set of graphs we kept the number of nodes to be 500. As we can see from these graphs, the lifetime first starts to increase with increasing values of the wake-up frequency, due to a smaller preamble length. But after a certain point the lifetime starts reducing because of higher current consumption due to frequent wake-ups. Similar to the previous set of graphs, for α_r = 1.5 mA, RCA performs poorly with 2 channels because of a higher number of transmission due to smaller d_0.

VI. Conclusion

In this paper, we analyze the battery lifetime of a WSN under data collection traffic and asynchronous duty-cycling. The current consumption in such networks can be optimized by applying transmission power control. This is applied to derive the maximum lifetime of the network. In addition, multi-channel operation with adaptive channel selection is considered as a mechanism to further reduce current consumption as well as to balance the remaining lifetimes of the nodes. Analysis of the network lifetime with multi-channel operation and optimal power control is presented.

Acknowledgement

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