Resonant transparency of materials with negative permittivity

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Abstract

It is shown that the transparency of opaque material with negative permittivity exhibits resonant behavior. The resonance occurs as a result of the excitation of the surface waves at slab boundaries. Dramatic field amplification of the incident evanescent fields at the resonance improves the resolution of the sub-wavelength imaging system (superlens). At the resonance, two evanescent waves have a finite phase shift providing non-zero energy flux through the non-transparent region. It is also shown that the resonant excitation of a surface mode creates a condition for the total transparency of a finite thickness slab to a p-polarized obliquely incident electromagnetic wave for resonant values of the incidence angle and wave frequency.

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1. Introduction

Propagation of the electromagnetic radiation in materials with negative dielectric permittivity (permeability) or the so-called left-handed materials (LHM) has attracted great deal of attention in recent years [1–4]. The increased interest in properties of such media has been driven by their potential applications in various branches of science and technology. One possible application is related to the possibility of creating the so-called superlenses: a subwavelength optical imaging system without the diffraction limit [4–7]. The superlens phenomenon is essentially based on amplification of evanescent waves, facilitated by the excitation of the surface plasmons [6]. Plasma with overcritical density is a simplest example of the negative \( \epsilon \) material; \( \epsilon = 1 - \omega_p^2/\omega^2 < 0 \) for \( \omega < \omega_p \). Phenomena that take place in such plasmas are important in a number of areas, in particular for the inertial confinement fusion (ICF) experiments [1,8–10].

In this work we show that the amplification of the evanescent waves originating from the interaction of two evanescent fields (decaying and growing in space) has a resonant character related to the excitation of surface modes. Such amplification of the evanescent field allows the penetration of the electromagnetic radiation to depths much greater compared to the incident light wave length [2]. The evanescent wave incident on the negative \( \epsilon \) slab is strongly amplified when the resonant conditions for the surface modes are met. For materials with \( \epsilon \neq -1 \) the resonance occurs for finite values of the wave vector \( k_y \) (for \( \epsilon = -1 \), the resonance occurs for \( |k_y| \to \infty \), where \( k_y \) is the in-plane wave vector component (p-polarization is considered). The presence of resonances with finite values of the in-plane wave vector \( k_y \) may significantly improve the overall resolution of the subwavelength imaging system.

Resonant excitation of a surface mode is also the underlying mechanism behind the absolute transparency of a finite thickness slab of material with \( \epsilon < 0 \) to the incident propagating electromagnetic waves \( (|k_y| < \omega/c) \) [2,11]. In this case, the resonant excitation of surface modes by the incident light can be achieved via the presence of a single transition layer with \( 0 < \epsilon < 1 \) on one side of the slab with \( \epsilon < 0 \) [11]. The super-
position of two evanescent waves provides a finite energy flux through the region with \( \epsilon < 0 \), which is equal to that in the incident electromagnetic wave. The radiation is then re-emitted at the other side of the opaque slab.

2. Interference of the evanescent waves and effect of superlensing

Non-propagating (evanescent) modes are basic solutions to the Maxwell’s equations for the electromagnetic fields in materials with negative dielectric permittivity. Often such modes have been neglected assuming that the boundary conditions in an infinite medium preclude the exponentially growing modes while the decaying modes do not contribute to the transmission. It has recently been noted however [6] that in a slab of material while the decaying modes do not contribute to the transmission, an infinite medium preclude the exponentially growing modes with negative permittivity both (growing and decaying) components are present, resulting in amplification of the evanescent modes. As shown by Pendry [6], the effect of amplification of the evanescent spectrum of the incident light (originating from the object) by “negative” materials can be advantageously used for subwavelength imaging applications, potentially leading to optical system without the diffraction limit or superlens.

The imaging problem can be described in terms of the optical transfer function \( \tau(x, k_y, \omega) \) (\( k_y \) designates the in-plane wave vector directed along the surface of the material), defined as the ratio of Fourier components of image field to object field, \( B_{\text{img}}^k(x)/B_{\text{obj}}^k(0) \) (for \( -\infty < k_y < \infty \)) at a given imaging plane \( x \). The transfer function can be found by considering a \( p \)-polarized wave (electric vector in the plane of incidence) incident from vacuum on a thin overcritical density slab of thickness \( d \), dielectric permittivity \( \epsilon < 0 \) and magnetic permeability \( \mu = 1 \) as shown in Fig. 1. The optical properties of this slab are obtained by taking the ratio of the field in the region \( x > a + d \) to that at the object plane (in current calculations the object plane is assumed to be at \( x = 0 \)). The electromagnetic fields in each region of interest are found from solving the well-known wave equation,

\[
\epsilon \frac{d}{dx}\left(\frac{1}{\epsilon} \frac{dB_z}{dx}\right) + \frac{\omega^2}{c^2}(\epsilon - \frac{k_y^2 c^2}{\omega^2})B_z = 0, \tag{1}
\]

with a general solution having the following form,

\[
B_z = (A_1 e^{ik x} + A_2 e^{-ik x}) e^{i(k_y y - \omega t)}, \tag{2}
\]

where \( k = \omega/c \sqrt{(\epsilon - k_y^2 c^2/\omega^2)} \). The electromagnetic fields in vacuum regions \( (x < a \) and \( x > a + d) \) represent a sum of incident and reflected waves \( (x < a) \), and a transmitted wave \( (x > a + d) \). Matching solutions at different boundaries by requiring the continuity of \( B_z \) and \( 1/c dB_z/dx \) across interfaces, one arrives at the expression for the transfer function,

\[
\tau(x, k_y, k_0) = \frac{2k_0 k \epsilon}{\sqrt{k_0^2 - 1}} \frac{k_y^2}{k_0^2 - 1} e^{ik_0 x},
\]

\[
\Xi(d, k_y, k_0) = 2k_0 k \epsilon \sqrt{k_0^2 - 1} \cos(k d),
\]

\[
\Lambda(d, k_y, k_0) = ((1 + k_y^2) k_0^2 - (1 + k_y^2) \epsilon^2) \sin(k d), \tag{3}
\]

where \( x \) is a distance between the source and the image planes and \( k_0 = \omega/c \). Fig. 2 shows the absolute value of the optical transfer function at a distance \( x = 2d \) from the source as a function of the in-plane wave vector \( k_y \). The dielectric constant of the medium and its thickness are \( \epsilon = -1.0292 + 0.00001i \) and \( d = 1.8c/\omega_p \) correspondingly. \( k_0 \) is normalized to classical skin depth \( c/\omega_p \).

Fig. 1. Schematic geometry of the dielectric constant distribution for the case of a single planar medium. Parameters \( a \) and \( d \) define the distance from the source to the slab and the slab thickness correspondingly and \( x \) defines the distance from the source to the imaging plane.

Fig. 2. The absolute value of the optical transfer function at a distance \( x = 2d \) from the source as a function of the in-plane wave vector \( k_y \). The dielectric constant of the medium and its thickness are \( \epsilon = -1.0292 + 0.00001i \) and \( d = 1.8c/\omega_p \) correspondingly. \( k_0 \) is normalized to classical skin depth \( c/\omega_p \).
equation can be reduced to the following form:

$$\tanh(\sqrt{k_y^2 - \epsilon k_0^2} d) = \frac{2\epsilon \sqrt{\epsilon k_0^2 - \epsilon k_y^2 - k_y^2}}{(1 + \epsilon^2)k_y^2 - \epsilon(1 + \epsilon)k_0^2}. \quad (5)$$

In the general case, for a finite value of $d$ there exist two solutions to Eq. (5), as seen from Fig. 2 corresponding to a coupled surface wave running on opposite sides of the slab. As $d$ becomes large so that $\tanh(\sqrt{k_y^2 - \epsilon k_0^2} d) \to 1$, the two solutions degenerate and we obtain,

$$k_y = k_0 \sqrt{\frac{\epsilon}{1 + \epsilon}}, \quad (6)$$

which is the well-known dispersion relation for surface plasmons on an overdense medium-vacuum interface. It is worth noting here that the transfer function given by expression (3) is exact and no approximations were used in its derivation. It includes the resonant contribution of surface modes with a finite in-plane wave vector $k_y$ and in the limit $\epsilon \to -1, |k_y| \gg \omega/c$ it is reduced to that obtained in Ref. [7] where the authors investigated the resolution limit of the slab of “negative” material with dielectric permittivity $\epsilon = -1 + i\epsilon''$ and magnetic permeability $\mu = -1 + i\mu''$. It should also be noted that the location (in $k_y$ wave vector space) of the two resonances not only depends on the absolute value of the dielectric constant $\epsilon$ but also the thickness of the slab $d$. For a very thin slab, the two resonances are separated (smaller slab thicknesses result in greater peak separation in $k_y$ space). As the slab thickness increases, the two resonant points move toward each other, eventually merging together forming a single peak. All of these features in the transfer function have not been noticed in the previous investigations since the authors limited their calculations to the asymptotic case $k_y \gg \omega/c$ and $k_y d > 1$. As we will see below, the presence of such resonances may advantageously be used in the design of superlens for improved image resolution and intensity.

The transfer function $\tau$ can be used to find the reconstructed field in the image plane in the form

$$B_{\text{img}}(x, y, t) = \int S(k_y) \tau(x, k_y, k_0) e^{i(k_y y - \omega t)} dk_y, \quad (7)$$

where $S(k_y)$ is the wave vector spectrum of the source (imaged object). Thus, the ability of the system to image the object is completely determined by the optical transfer function (3), which in itself depends on many system’s physical parameters.

In an ideal case, the transfer function should transfer all spatial harmonics equally or $\tau(x, k_y, k_0) = 1$ for $-\infty < k_y < \infty$. In reality however, the transfer function is a non-monotonous function of the wave vector $k_y$, medium material type defined by $\epsilon$, its thickness $d$, and the position $x$ of the imaging plane relative to the position of the object plane. The superlensing can only be realized for certain set of the above parameters. Fig. 3 shows the optical transfer function for two layers of materials with dielectric permittivities $\epsilon_1 = -1 + 0.001i$ and $\epsilon_2 = -1.0292 + 0.001i$ at the imaging plane $x = 2d$. As one can see the transmission band (in the wave vector space) is significantly broadened in the second material ($\epsilon = -1.0292 + 0.001i$) due to the resonant excitation of a surface plasmon with a finite value of the wave vector $k_y \approx 4$. Such surface plasma wave is not excited in material with $\epsilon = -1$ (assuming the slab thickness $d$ such that the condition $\tanh(\sqrt{k_y^2 - \epsilon k_0^2} d) = 1$ is satisfied), as can be seen from the dispersion relation (6).

Let us estimate how well we can image an object using these two materials. For a sake of simplicity we assume that our object is represented by two slits of a certain width located at a given distance away from each other as shown in Fig. 4. Substituting the Fourier transform of the object together with the transfer function (3) into Eq. (7) we arrive at the reconstructed image shown in Fig. 5 for the case when the incident light wavelength is $\lambda = 350$ nm. As one can see, both materials provided considerable focusing, yielding the sub-wavelength resolution of the object. However, as we expected the image resolution and its intensity for the second layer ($\epsilon = -1.0292$) is superior to that using material with $\epsilon = -1$. This suggests that there is a range of system parameters for which a significant focusing can be achieved [4] necessitating a further parametric study in order to understand the relation between different physical parameters of the system.

### 3. Surface wave induced total transparency of material with negative permittivity

Surface wave induced amplification of incident electromagnetic waves with spectral components of in-plane wave vectors $k_y$ satisfying the condition $|k_y| > \omega/c$ has been considered in the previous section. It was shown that the resonant amplification occurs as a result of the excitation of a surface wave. A slab of negative $\epsilon$ material surrounded by vacuum supports such a surface mode for which its phase velocity is always sub-luminal or $\omega/k_y < c$. As a result, only those modes of the incident light for which $|k_y| > \omega/c$ have been amplified. It is possible however to amplify the propagating modes with $|k_y| < \omega/c$ too, thus creating the conditions for the absolute transparency to the incident propagating wave. This can be done by creating con-
Fig. 4. The field distribution in the object plane. The object comprises two slabs of thickness 15 nm, separated by a distance 80 nm from their centers.

Fig. 5. Field distribution in the image plane for two slabs with $\epsilon_1 = -1.0292 + 0.001i$ (solid line) and $\epsilon_2 = -1 + 0.001i$ (dotted line) and thickness $d = 1.8c/\omega_p$. Magnetic permeability of both materials is assumed to be $\mu = 1$. The wavelength of the incident light is $\lambda = 350$ nm.

Consider $p$-polarized light obliquely incident from vacuum on a two-layer structure having dielectric permittivity distribution shown in Fig. 6. Such a system can be formed by placing an undercritical density plasma layer (with thickness $d$ and electron density corresponding to the plasma frequency $\omega_{p,1}$) to an overcritical density plasma layer (with thickness $a$ and electron density corresponding to the plasma frequency $\omega_{p,2}$). The plasma–plasma interface supports a surface wave with dispersion relation found from the solution to the Maxwell’s equations [1],

$$\alpha_1/\epsilon_1 + \alpha_2/\epsilon_2 = 0 \rightarrow k_y = \sqrt{(\Delta - \omega^2)(\omega^2 - 1)} \overline{1 + \Delta - 2\omega^2},$$  

(8)

where $\alpha_i^2 = k_y^2 - \epsilon_i\omega^2/c^2$, $\epsilon_i = 1 - \omega_{p,i}^2/\omega^2$ ($i = 1, 2$); $\Delta = \omega_{p,1}^2/\omega_{p,2}^2$; $\omega$ is normalized to the plasma frequency in the region where $\epsilon < 0$ and $k_y$ is normalized to the classical skin depth $\delta = c/\omega_{p,2}$. It can be easily seen that the phase velocity of the surface wave on a plasma–plasma interface can be greater than that of light, so that they can couple to radiating electromagnetic fields. This means that the incident $p$-polarized electromagnetic wave may excite a surface mode on a plasma–plasma interface if the resonant condition (external field frequency and its wave vector’s tangential component have to match those that are determined from the dispersion relation for the plasma surface wave) is satisfied. The optical properties of this dual-layer system can be found from matching the fields of the incident/reflected electromagnetic waves with those of the surface wave at the three interfaces. The electromagnetic field in each region $B = (0, 0, B_z)$ is a solution to the wave Eq. (1) with general solution given by expression (2). The electromag-
nematic fields in vacuum regions \((x < -d)\) and reflected wave \((x < -d)\) represent a sum of incident and reflected wave \((x < -d)\), and a transmitted wave \((x > a)\). Matching solutions at different boundaries by requiring continuity of \(B_z\) and \(1/\epsilon dB_z/dx\) across interfaces, we obtain the unknown expansion coefficients with the transmission coefficient having the following form,

\[
T = \left| \frac{4e^{i(k_1d + k_2a - (a+d)\cos\theta)}k_0k_1k_2\epsilon_1\epsilon_2 \cos\theta}{(\Re - \tau)} \right|^2,
\]

\[
\Re = (1 + e^{2ik_1d})k_0k_1k_2\epsilon_1\epsilon_2 \cos\theta
+ e^{2ik_2d}(1 + e^{2ik_1d})k_0k_1k_2\epsilon_1\epsilon_2 \cos\theta,
\]

\[
\tau = 2i e^{ik_1d + k_2a}k_2\epsilon_2\cos(k_2a)(k_2^2 + k_0^2\epsilon_1^2 \cos^2\theta) \sin(k_1d)
+ k_1\epsilon_1 \cos(k_1d)(k_2^2 + k_0^2\epsilon_1^2 \cos^2\theta) \sin(k_2a)
\]

\[
-ik_0(k_2^2\epsilon_1^2 + k_1^2\epsilon_2^2) \cos\theta \sin(k_1d) \sin(k_2a),
\]

where \(\theta = \arcsin(k_d/k_0)\). Fig. 7 shows the transmission coefficient as a function of the incidence angle. As one can see there is a sharp increase in the transmission properties of the system when the angle of incidence matches certain resonant value of \(\theta\) at which point the transmission coefficient reaches unity. The resonant value exactly corresponds to that given by an expression \((8)\) for the dispersion relation of the surface plasma wave.

Thus, the anomalous transmission occurs even for a system consisting of an undercritical density plasma layer adjacent to an overcritical density plasma slab, so that there is no need to form a sandwich-like structure as argued in Refs. [2,12] in which a second surface plasma wave has to be excited on the opposite side of the overcritical density layer to achieve the same effect. In other words, the anomalous light transmission can be achieved through excitation of a single surface plasma wave. The transmission coefficient is also a non-monotonous function of both plasma layer thickness with a single maximum reached at certain correlated values determined from the interference condition between evanescent fields in both plasma slabs.

4. Evanescent wave interference and the energy transport

It is often assumed that the evanescent waves do not carry the energy. Therefore, in the problem of total transparency of a layered structure there occurs a question of how the energy is carried through the non-transparent media where the only solutions are the evanescent modes. One must remember however that the general solution inside the negative \(\epsilon\) medium is a sum of two exponential functions, one that decays with the distance \(\sim e^{-x}\) \((x\) points in the propagation direction\) and the other grows \(\sim e^x\). For the superposition of decaying and growing modes, \(E, B \sim A_1 \exp[-ikx] + A_2 \exp[ikx]\) \((k\) is purely imaginary decay constant\), the \(x\) component of the time averaged Poynting vector \(S_x\)

\[
S_x = \frac{1}{2} \Re[E_x A_1^*] \sim \Re[k (A_1 A_2^* - A_2 A_1^*)]
\]

\[
\sim 2 \Im[A_1 A_2^*],
\]

may become finite when the combination \(A_1 A_2^*\) has a finite imaginary part, which requires a finite phase shift between \(A_1\) and \(A_2\). Therefore, a finite energy flux occurs as a result of the superposition of two evanescent modes with a finite phase shift. The effect of energy transfer through the superposition of two evanescent waves has recently attracted a great deal of attention and is known in literature as the interference of evanescent waves [13,14].

It is easy to show that the required phase shift can be obtained when two evanescent modes inside the negative \(\epsilon\) region are matched to the outgoing (transmitted) wave in vacuum. Matching of the evanescent solutions with the incident vacuum wave (at the other side of the negative \(\epsilon\) region) shows that the total transmission may be obtained only when the transition layer with \(\epsilon > 0\) is included [11]. The condition of the absolute transparency is equivalent to the resonant condition for the excitation of the surface plasma mode. At the resonance the Poynting flux inside the slab becomes equal to that of the incident radiation and the opaque plasma slab becomes absolutely transparent. This can be a possible mechanism of the
anomalously high transparency of overdense plasma recently observed in the experiment [10].

5. Summary

In conclusion, we have shown that the excitation of surface modes leads to the resonant transparency of optically opaque materials. Presence of such resonances may improve the resolution and the signal intensity of the sub wavelength imaging system. It has been also shown that the resonant excitation of surface modes is an underlying mechanism behind the total transparency of an overdense plasma slab to the incident electromagnetic wave. The energy flux through the negative $\epsilon$ region occurs as a result of the interference of the evanescent modes.

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