Directional coupling and switching in multi-core microstructure fibers

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Directional coupling in two-core fibers

\[ \frac{dA_1}{dz} = ik_{12}A_2 + i\delta_a A_1 \]
\[ \frac{dA_2}{dz} = ik_{21}A_1 + i\delta_a A_2 \]

- \( k_{ij} \) – coupling coefficient
- \( \delta_a \) – asymmetry coefficient

For symmetric coupler:
\[ k_{ij} = k_{ji} = k \quad \text{and} \quad \delta_a = 0 \]

Oscillatory solution for initial conditions
\[ P_1(0) = P_0 \quad \text{and} \quad P_2(0) = 0 \]

\[ P_1(z) = P_0 \cos^2(\kappa z) \]
\[ P_2(z) = P_0 \sin^2(\kappa z) \]
The nature of the oscillation

Each of the two supermodes is a stationary solution of the wave equation with its own propagation constant:

\[
\begin{pmatrix}
E_s(x, y) \\
\beta_s
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
E_a(x, y) \\
\beta_a
\end{pmatrix}
\]

light in core 1 only: \( E_1 = \frac{1}{\sqrt{2}} (E_s + E_a) \)

light in core 2 only: \( E_2 = \frac{1}{\sqrt{2}} (E_s - E_a) \)
Microstructured directional coupler

\[ \Lambda = 2.5 \, \mu m, \quad d = 1.0 \, \mu m \]
Modeling light propagation in microstructured fibers

Maxwell’s equations in a dielectric medium:

\[
\nabla \times \vec{E}(\vec{r}, t) = -\frac{1}{c} \frac{\partial \vec{B}(\vec{r}, t)}{\partial t}
\]

\[
\nabla \times \vec{H}(\vec{r}, t) = \frac{1}{c} \frac{\partial \vec{D}(\vec{r}, t)}{\partial t}
\]

\[
\nabla \cdot \vec{H}(\vec{r}, t) = 0 \quad \nabla \cdot \left[ n^2(\vec{r})\vec{E}(\vec{r}, t) \right] = 0
\]

\[
\nabla \cdot \vec{E}(\vec{r}, t) = -\frac{2}{n(\vec{r})} \left[ \nabla \cdot n(\vec{r}) \right] \cdot \vec{E}(\vec{r}, t)
\]

For electromagnetic wave in dielectric medium:

\[
\Delta \vec{E} + \frac{\omega^2 n^2}{c^2} \vec{E} + \nabla \left[ \frac{1}{n} \left( \nabla n \right) \cdot \vec{E} \right] = 0
\]

\[
\Delta \vec{H} + \frac{\omega^2 n^2}{c^2} \vec{H} + \frac{2}{n} \left[ \nabla n \times \left( \nabla \times \vec{H} \right) \right] = 0
\]

It is sufficient to solve only one equation

\[
\vec{H} = \frac{ic}{\omega} \left( \nabla \times \vec{E} \right) \quad \vec{E} = -\frac{ic}{\omega n^2} \left( \nabla \times \vec{H} \right)
\]
The propagation method

When looking for a plane-wave solutions:

\[ E(\vec{r}) = \tilde{E}(\vec{r}) e^{-i k \cdot \vec{r}} \quad H(\vec{r}) = \tilde{H}(\vec{r}) e^{-i k \cdot \vec{r}} \]

propagating along \( z \) we take advantage of the index distribution \( n = n(x, y) \) \( \vec{k} \cdot \vec{\nabla} n = 0 \)

Evolution equations for electromagnetic wave in microstructured medium \( n = n(x, y) \)

\[
\begin{align*}
\frac{\partial \tilde{E}}{\partial z} &= - \frac{i}{2k} \Delta \tilde{E} - \frac{i}{2k} \left( \frac{n^2 \omega^2}{c^2} - k^2 \right) \tilde{E} - \frac{1}{n} \left[ (\vec{\nabla} n) \cdot \tilde{E} \right] \frac{\vec{k}}{k} - i \vec{\nabla} \left[ \frac{1}{n} (\vec{\nabla} n) \cdot \tilde{E} \right] \\
\frac{\partial \tilde{H}}{\partial z} &= - \frac{i}{2k} \Delta \tilde{H} - \frac{i}{2k} \left( \frac{n^2 \omega^2}{c^2} - k^2 \right) \tilde{H} - \frac{1}{n} \left[ (\vec{\nabla} n) \cdot \tilde{H} \right] \frac{\vec{k}}{k} - i \frac{1}{n} \left( \vec{\nabla} n \right) \times \left( \vec{\nabla} \times \tilde{H} \right)
\end{align*}
\]
Propagation method for the magnetic field

\[
\begin{align*}
\frac{\partial H_x}{\partial z} &= -\frac{i}{2k} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) H_x - \frac{i}{2k} \left( \frac{n^2 \omega^2}{c^2} - k^2 \right) H_x - \frac{2}{k} \frac{\partial y n}{n} \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \\
\frac{\partial H_y}{\partial z} &= -\frac{i}{2k} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) H_y - \frac{i}{2k} \left( \frac{n^2 \omega^2}{c^2} - k^2 \right) H_y - \frac{2}{k} \frac{\partial x n}{n} \left( \frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial x} \right) \\
\frac{\partial H_z}{\partial z} &= -\frac{i}{2k} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) H_z - \frac{i}{2k} \left( \frac{n^2 \omega^2}{c^2} - k^2 \right) H_z + \\
& \quad + \frac{2}{k} \frac{\partial_x n}{n} \left( \frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} \right) + \frac{\partial_y n}{n} \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) - \frac{1}{n} \left[ (\partial_x n) H_x + (\partial_y n) H_y \right]
\end{align*}
\]

\[
\begin{align*}
H_x(x, y; z) &\xrightarrow{1} H_x(x, y; z + dz) \\
H_y(x, y; z) &\xrightarrow{2} H_y(x, y; z + dz) \\
H_z(x, y; z) &\xrightarrow{} H_z(x, y, z + dz)
\end{align*}
\]
Λ coupler

Λ = 2.5 μm  d = 1.0 μm
λ = 0.8 μm

Coupling length:
L_c = 0.139 mm
$\sqrt{3} \Lambda$ coupler

$\Lambda = 2.5 \, \mu m$  $d = 1.0 \, \mu m$

$\lambda = 0.8 \, \mu m$

Coupling length:

$L_c = 0.948 \, mm$
$2\Lambda$ coupler

$\Lambda = 2.5 \, \mu m \quad d = 1.0 \, \mu m$

$\lambda = 0.8 \, \mu m$

Coupling length:
$L_c = 2.076 \, mm$
The coupling length $L_c$
Necklace-type six-core $\sqrt{3}\Lambda$ coupler

$E_a$ $E_s$

Channels #1, 3, 5
Channels #2, 4, 6

$L_c^{(6,\text{core})} = 466.4 \mu m = \frac{1}{2} L_c^{(2,\text{core})}$

Power in channel [arb. u.]

Propagation distance $z$, [mm]
Conclusions

• We have developed a full 3D numerical scheme for solving the Maxwell’s equations for modeling light propagation in microstructured media with 2D index of refraction distribution $n(x,y)$.

• The model is used for accurate prediction of the coupling length $L_c$ in various double-core microstructured fiber configurations.

• Microstructured directional fiber couplers are found to behave in a similar way to conventional fiber couplers, the difference being that much shorter coupling lengths (less than a millimeter) can be achieved at no additional production cost.

• Multi-core microstructured fibers are modeled as well, showing promise for parallel switching and mixing.
Coming soon…

Experimental verification of the models

![2 core 4λ coupler](image1)

![6 core 2λ coupler](image2)