Chapter 8

Analog-to-Digital and Digital to Analog Conversion
Chapter 8

Analog-to-Digital and Digital-to-Analog Conversion

- Sampling and Quantization
- Pages 390-391
• Traditional *analog transmission* (AM, FM and PM) are less complex than digital data transmission have been the basis of broadcasting and communication for 100 years.
• **Digital data transmission** (PAM, ASK, PSK, FSK and QAM) is more complex but (perhaps) offers higher performance with control of accuracy and easier storage, simpler signal processing for noise reduction, error detection and correction and encryption.

*S&M Figure 8-1b*
Digital data transmission requires analog-to-digital (ADC) and digital-to-analog (DAC) converters. The ADC process utilizes sampling and quantization of the continuous analog signal.

![Diagram of ADC and DAC processes](image-url)
• ADC sampling occurs at a uniform rate (the *sampling rate*) and has a continuous amplitude.

S&M Figure 8-2a,b
The continuous amplitude sample is then quantized to $n$ bits or resolution for the *full scale* input or $2^n$ levels.
Here \( n = 4 \) and there are \( 2^4 = 16 \) levels for a full scale input of \( 2 \text{ V} \) (\( \pm 1 \text{ V} \)). The step size is \( \frac{2 \text{ V}}{16} = 0.125 \text{ V} \) and the quantized value is the midpoint of the voltage range.

<table>
<thead>
<tr>
<th>Voltage Range of Signal ( x(t) )</th>
<th>Rounded-off (Quantized) Value</th>
<th>Binary Assignment for Quantized Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1.000 \leq x(t) &lt; -0.875 \text{ v})</td>
<td>(-0.9375 \text{ v})</td>
<td>0000</td>
</tr>
<tr>
<td>(-0.875 \leq x(t) &lt; -0.750 \text{ v})</td>
<td>(-0.8125 \text{ v})</td>
<td>0001</td>
</tr>
<tr>
<td>(-0.750 \leq x(t) &lt; -0.625 \text{ v})</td>
<td>(-0.6875 \text{ v})</td>
<td>0010</td>
</tr>
<tr>
<td>(-0.625 \leq x(t) &lt; -0.500 \text{ v})</td>
<td>(-0.5625 \text{ v})</td>
<td>0011</td>
</tr>
<tr>
<td>(-0.500 \leq x(t) &lt; -0.375 \text{ v})</td>
<td>(-0.4375 \text{ v})</td>
<td>0100</td>
</tr>
<tr>
<td>(-0.375 \leq x(t) &lt; -0.250 \text{ v})</td>
<td>(-0.3125 \text{ v})</td>
<td>0101</td>
</tr>
<tr>
<td>(-0.250 \leq x(t) &lt; -0.125 \text{ v})</td>
<td>(-0.1875 \text{ v})</td>
<td>0110</td>
</tr>
<tr>
<td>(-0.125 \leq x(t) &lt; 0.000 \text{ v})</td>
<td>(-0.0625 \text{ v})</td>
<td>0111</td>
</tr>
<tr>
<td>(0.000 \leq x(t) &lt; 0.125 \text{ v})</td>
<td>0.0625 \text{ v}</td>
<td>1000</td>
</tr>
<tr>
<td>(0.125 \leq x(t) &lt; 0.250 \text{ v})</td>
<td>0.1875 \text{ v}</td>
<td>1001</td>
</tr>
<tr>
<td>(0.250 \leq x(t) &lt; 0.375 \text{ v})</td>
<td>0.3125 \text{ v}</td>
<td>1010</td>
</tr>
<tr>
<td>(0.375 \leq x(t) &lt; 0.500 \text{ v})</td>
<td>0.4375 \text{ v}</td>
<td>1011</td>
</tr>
<tr>
<td>(0.500 \leq x(t) &lt; 0.625 \text{ v})</td>
<td>0.5625 \text{ v}</td>
<td>1100</td>
</tr>
<tr>
<td>(0.625 \leq x(t) &lt; 0.750 \text{ v})</td>
<td>0.6875 \text{ v}</td>
<td>1101</td>
</tr>
<tr>
<td>(0.750 \leq x(t) &lt; 0.875 \text{ v})</td>
<td>0.8125 \text{ v}</td>
<td>1110</td>
</tr>
<tr>
<td>(0.875 \leq x(t) \leq 1.000 \text{ v})</td>
<td>0.9375 \text{ v}</td>
<td>1111</td>
</tr>
</tbody>
</table>
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Analog-to-Digital and Digital-to-Analog Conversion

- *Sampling Baseband Analog Signals*
- Pages 392-399
• The analog signal $x(t)$ which is continuously, uniformly sampled is represented by:

$$x_s(t) = x(t) \sum_{k=-\infty}^{\infty} \delta(t - k T_S)$$

S&M Eq. 8.1

Multiplication in the temporal domain is convolution in the frequency domain and the frequency domain representation is:

$$X_s(f) = X(f) * \mathcal{F}\left\{ \sum_{k=-\infty}^{\infty} \delta(t - k T_S) \right\}$$

$$X_s(f) = X(f) * \left\{ f_s \sum_{k=-\infty}^{\infty} \delta(f - k f_S) \right\}$$

S&M Eq. 8.2

$$X_s(f) = f_s \sum_{k=-\infty}^{\infty} X(f - k f_S)$$
• Temporal and spectral representation of the continuous sampling process for a sum of three sinusoids.

\[ x_s(t) = x(t) \sum_{k=-\infty}^{\infty} \delta(t-kT_s) \]

\[ X_s(f) = f_S \sum_{k=-\infty}^{\infty} X(f-kf_s) \]

S&M Figure 8-3
• 2 V, 20° initial phase, 500 Hz sinusoid sampled at 5 k samples/sec

S&M Figure 8-4a,b
• Aliased samples can be reconstructed for a 4500 Hz and a 5500 Hz sinusoid that appears to be a 500 Hz sinusoid

S&M
Figure 8-4a,c,d
• The aliasing of the signal can be predicted by the magnitude spectrum of the original 500 Hz sampled signal.

If the 4500 Hz and 5500 Hz signals are then sampled at 5 k samples/sec, aliasing occurs at |4500 − 5000| and (5500 − 5000) Hz.

S&M Figure 8-4a,b
• The sum of three sinusoids does not have any aliased frequencies since the sampling frequency $f_s$ is greater than twice the highest frequency $f_{\text{max}}$

\[ f_s > 2 f_{\text{max}} \]
• The frequency $2f_{\text{max}}$ is called the **Nyquist frequency**.

Harry Nyquist, who contributed to the understanding of thermal noise while at Bell Labs, is also remembered in electrotechnology for his analysis of sampled data signals.

Harry Nyquist
1889-1976

Bell Laboratories
• The analog signal is *reconstructed* from the quantized samples by a DAC and a low pass filter (LFP).

S&M Figure 8-6
• For practical signals $f_S > 2f_{max}$ using a guard band for LPFs
• With *out-of-band noise* and sample signals, aliases of the noise now appear *in-band* and should be filtered before the sampling process.

S&M Figure 8-8
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Analog-to-Digital and Digital-to-Analog Conversion

• *Sampling Baseband Analog Signals*

• Pages 149-182
The periodic baseband signal consisting of three sinusoids is impulse sampled, sampled-and-held, processed by an 8-bit ADC-DAC and a quantizer in Simulink.
The periodic baseband signal is the sum of a 1 V 500 Hz, a 0.5 V 1.5 kHz and a 0.2 V 2.5 kHz sinusoid.

MS Figure 4.2
The power spectral density (PSD) of the periodic baseband signal has the expected peaks at 0.5, 1.5 and 2.5 kHz.
• The periodic baseband signal is overlaid with the continuous amplitude sample-and-hold signal with $f_s = 8$ kHz.

MS Figure 4.4

0.125 msec
• The analog signal $x(t)$ here is sampled and held rather than impulse sampled:

$$y_{s-h}(t) = \sum_{n} x(nT_S) \ h(t - nT_S) \quad h(t) = 1 \quad 0 \leq t \leq T_S$$

$$h(t) = 0 \quad otherwise \quad \text{MS Eq. 4.3}$$

The power spectral density ($\text{PSD}_{s-h}$) of the sample and hold operation is:

$$\text{PSD}_{s-h} = f_S^2 \sum_{k = -\infty}^{\infty} |X(f - k f_S)|^2 T_S^2 \text{sinc}^2 \left(2\pi f T_S\right)$$

$$\text{MS Eq. 4.4}$$
• However, if the analog signal $x(t)$ is impulse sampled:

$$x(nT_s) = \sum_{n} x(t) \delta(t - nT_s) \quad \text{MS Eq. 4.1}$$

Then the power spectral density (PSD) does not have a $\text{sinc}^2$ term:

$$\text{PSD} = f_s^2 \sum_{k = -\infty}^{\infty} |X(f - k f_s)|^2 \quad \text{MS Eq. 4.2}$$

The PSD$_{\text{s-h}}$ does have the $\text{sinc}^2$ term:

$$\text{PSD}_{\text{s-h}} = \sum_{k = -\infty}^{\infty} |X(f - k f_s)|^2 \text{sinc}^2 (2\pi f T_s) \quad \text{MS Eq. 4.4}$$
The PSD of the impulse sampled sum of three sinusoid signal with $f_S = 8$ kHz is:

$$\text{PSD} = f_S^2 \sum_{k=-\infty}^{\infty} |X(f - kf_S)|^2$$

MS Figure 4.5

MS Eq. 4.2
The PSD of the continuous amplitude sample and hold sum of three sinusoid signal with $f_S = 8$ kHz is:

$$\text{PSD}_{s-h} = f_S^2 \sum_{k = -\infty}^{\infty} |X(f - k f_S)|^2 T_S^2 \text{sinc}^2 \left(2\pi f T_S\right)$$

$$\text{PSD}_{s-h} = \sum_{k = -\infty}^{\infty} |X(f - k f_S)|^2 \text{sinc}^2 \left(2\pi f T_S\right)$$

MS Eq. 4.4

MS Figure 4.6
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• Sampling Bandpass Analog Signals
• Pages 399-400
A bandpass signal does not need to be sampled at $2f_2$. Nyquist’s bandpass sampling theory states that the sampling rate $f_S > 2(f_2 - f_1)$ which is substantially less than $2f_2$.
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• Sampling Bandpass Analog Signals
• Pages 180-181
• The *Simulink* simulation uses the DSB AM modulation block and the sum of three sinusoids source.

![Simulink Diagram](image)

**MS Figure 4-32**

![Spectrum Graph](image)

**MS Figure 4-33**
• The *Simulink* simulation initially uses a sampling rate of 5 MHz and results in $4 \times 194,304 = 2^{22}$ sampling points. The PSD shows the DSB-LC AM signal with the LSB and USB.

Scaled PSD $f_{\text{max}} = 50$ kHz

MS Figure 4-33
• The bandwidth of the bandpass signal is $f_2 - f_1 = 22.5 - 17.5 = 5$ kHz and the Simulink sampling rate is set to 50 kHz and results in only $32,768 = 2^{15}$ sampling points.

Scaled PSD $f_{max} = 50$ kHz

Aliased frequency range > 25 kHz

MS Figure 4-34
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- Quantizing Process: Uniform Quantization
- Pages 400-404
• The **quantizing process** divides the *range* (± full scale) into $2^n$ ($n = 4$ here) regions which are assigned an $n$-bit binary code.

![S&M Figure 8-10](image-url)
• The error associated with the quantizing process is assumed to have a uniform probability density function. The maximum error for uniform quantization is:

\[ q = \pm 0.5 \left( \frac{2V_{\text{max}}}{2^n} \right) = \pm \frac{V_{\text{max}}}{2^n} \]

The quantizer range is \( \pm V_{\text{max}} \) and the uniform quantizer voltage step size is:

\[ \Delta = \frac{2V_{\text{max}}}{2^n} = \frac{V_{\text{max}}}{2^{n-1}} \quad \text{MS Eq. 4.6} \]

The mean square quantizing \( E_q \) is the normalized quantizing noise power:

\[ E_q = \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} q^2 \, dq = \frac{\Delta^2}{12} = \frac{V_{\text{max}}^2}{3 \left(2^n\right)^2} = \frac{V_{\text{max}}^2}{3 \left(2^{2n}\right)} \quad \text{MS Eq. 4.7} \]
The **signal to quantizing noise power** (SNR<sub>q</sub>) is:

\[
\text{SNR}_q = \frac{12 P_S}{\Delta^2} = 3\left(2^{2n}\right)\frac{P_S}{V_{\text{max}}^2}
\]

MS Eq. 4.8

\(P_S\) is the normalized power of the signal that is quantized.

For the ADC here \(\Delta = 10\) mV and \(n = 8\). The sum of three sinusoids as the input signal has a peak amplitude of 1.1 V and the quantizing noise has a peak amplitude of 10 mV.

MS Figure 4.7
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• Quantizing Process: Nonuniform Quantization

• Pages 400-404
Nonuniform quantization divides the dynamic range of an analog signal into nonuniform quantization regions. Lower magnitudes have smaller quantization regions than high magnitudes.

The quantization of speech benefits from nonuniform quantization since the perception of hearing is logarithmical rather than linear.
• Uniform quantization (top) results in a large amount of error for small sample amplitude.

Non-uniform quantization (bottom) reduces the error for small sample amplitudes.
• Uniform quantization is simpler to implement so a *compressor* (a non-linear transfer function) is used before the quantizer. The $\mu$-Law compressor is used in telephony with $\mu = 255$. At the receiver, an expander has the inverse non-linear transfer function and results in *companding* (COMpressing and exPANDING).

\[
V_{\text{out}} = \ln \left( 1 + \mu \frac{V_{\text{in}}}{V_{\text{max}}} \right) \quad 0 \leq \frac{V_{\text{in}}}{V_{\text{max}}} \leq 1
\]
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• Companding
• Pages 157-159
• The µ-Law compander concept can be simulated in Simulink with the µ-Law Compressor and µ-Law Expander blocks. The A-Law Compressor and A-Law Expander blocks are included for comparison.
• The µ-Law compressor voltage transfer function is sigmoidal (S-shaped).

MS Figure 4.14
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• Pulse Code Modulation
• Pages 171-175
• The pulse code modulator (PCM) transmitter utilizes a *Simulink* μ-Law compressor block, an 8-bit ADC subsystem, an 8-bit DAC subsystem and a μ-Law expander block.

**MS Figure 4.21**
The **Simulink** 8-bit ADC subsystem has a sample-and-hold block controlled by a *sampling* pulse generator, an 8-bit encoder block, an integer-to-bit converter block which provides an 8-bit *vector* to a demultiplexer block and a multiport switch. An 8-level *staircase* subsystem sequences the multiport switch to select 1 of the 8 inputs for bit serial output.

**MS Figure 4.22**
• The 8-level *staircase Simulink* subsystem sequences the multiport switch with a 3-bit counter and a 3-bit DAC for the output.

MS Figure 4.22
• The 8-bit DAC *Simulink* subsystem for the PCM system uses a 8-bit *shift register* and an 8-bit DAC.
The 8-bit DAC *Simulink* subsystem for the PCM system uses a 8-bit *shift register* and an 8-bit DAC.
• Analog input signal to the PCM system
- 8-bit DAC output after 8-bit ADC and µ-Law Compressor
• μ-Law Expander block output of the PCM system

**MS Figure 4.25**

**MS Figure 4.21**
• LPF output of the PCM system

MS Figure 4.25

MS Figure 4.21
• Analog input signal to the PCM system

• LPF output of the PCM system

MS Figure 4.25
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- *Differential Pulse Code Modulation*

- Pages 407-411
Sampled speech data are highly correlated and differential pulse code modulation (DPCM) exploits this to lower the overall data rate. DPCM uses a predictor to subtract a predicted value from the input. The error difference is sent.
• The predictor is a recursive equation, for example:
\[ S(n) = 0.75 s(n-1) + 0.2 s(n-2) + 0.05 s(n-3) \]

where \( S(n) \) is the predicted value of the \( n \) th sample and \( s(n-i) \) is the \( n-i \) th sample. The error signal is \( s(n) - S(n) \).
• A typical continuous analog signal is sampled and results in a discrete signal $s(n)$, The discrete predicted signal $S(n)$ is recursively computed. The discrete error signal is transmitted and has less quantizing bits than the actual discrete signal.
A DPCM example of actual discrete values, predicted values and the error terms:

<table>
<thead>
<tr>
<th>Time (msec)</th>
<th>Predicted Values</th>
<th>Actual Values</th>
<th>Error Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.75(0) + 0.20(0) + 0.05(0) = 0v</td>
<td>0.23v</td>
<td>0.23v</td>
</tr>
<tr>
<td>0.125</td>
<td>0.75(0.23) + 0.20(0) + 0.05(0) = 0.1725v</td>
<td>0.38v</td>
<td>0.2075v</td>
</tr>
<tr>
<td>0.25</td>
<td>0.75(0.38) + 0.20(0.23) + 0.05(0) = 0.33v</td>
<td>0.56v</td>
<td>0.229v</td>
</tr>
<tr>
<td>0.375</td>
<td>0.75(0.56) + 0.20(0.38) + 0.05(0.23) = 0.5075v</td>
<td>0.73v</td>
<td>0.2225v</td>
</tr>
<tr>
<td>0.5</td>
<td>0.75(0.73) + 0.20(0.56) + 0.05(0.38) = 0.6785v</td>
<td>0.90v</td>
<td>0.2215v</td>
</tr>
<tr>
<td>0.625</td>
<td>0.75(0.90) + 0.20(0.73) + 0.05(0.56) = 0.849v</td>
<td>1.05v</td>
<td>0.201v</td>
</tr>
<tr>
<td>0.7</td>
<td>1.004v</td>
<td>1.2v</td>
<td>0.196v</td>
</tr>
<tr>
<td>0.875</td>
<td>1.155v</td>
<td>1.35v</td>
<td>0.195v</td>
</tr>
<tr>
<td>1</td>
<td>1.305v</td>
<td>1.48v</td>
<td>0.175v</td>
</tr>
<tr>
<td>1.125</td>
<td>1.440v</td>
<td>1.62v</td>
<td>0.18v</td>
</tr>
<tr>
<td>etc.</td>
<td>etc.</td>
<td>etc.</td>
<td>etc.</td>
</tr>
</tbody>
</table>
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Analog-to-Digital and Digital-to-Analog Conversion

• Differential Pulse Code Modulation
• Pages 175-180
• A 4-bit first order differential pulse code modulator (DPCM) can be simulated in Simulink.

MS Figure 4.26
• The first order linear predictor MetaSystem determines the error signal: $e(n) = s(n+1) - 2s(n) + s(n-1)$
The Simulink 4-bit ADC subsystem of the DPCM system is similar to the 8-bit ADC of the PCM system and illustrates design reuse.
The Simulink 4-bit DAC subsystem of the DPCM system is also similar to the 8-bit DAC of the PCM system and again illustrates design reuse.

MS Figure 4.29 4-bit DAC
The first order linear predictor Simulink subsystem reconstructs an estimate of the signal $s_e(n)$ from the error signal $e(n)$ received and past estimates:

$$s_e(n+1) = e(n+1) + 2s_e(n) - s_e(n-1)$$
• Analog input signal of the DPCM system
Output of the 4-bit first order DPCM system
• Analog input signal to the DPCM system

![Analog input signal to the DPCM system](image1)

• Output of the 4-bit first order DPCM system MS Figure 4.31

![Output of the 4-bit first order DPCM system](image2)
• Output of the 8-bit PCM system MS Figure 4.25

• Output of the 4-bit first order DPCM system MS Figure 4.31
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• *Delta Modulation*

• Pages 411-415
Delta modulation is an extreme example of DPCM using
1-bit data representing ±Δ:

\[
\begin{align*}
S(n) &= S(n-1) + \Delta \quad b_i = 1 \quad \text{if } S(n-1) \leq s(n-1) \\
S(n) &= S(n-1) - \Delta \quad b_i = 0 \quad \text{if } S(n-1) > s(n-1)
\end{align*}
\]

S&M Eq. 8.10

S&M Figure 8-18
• The reconstructed signal increments ±Δ on each transmitted bit.

\[ b_i = 1 \quad S(n) = S(n-1) + \Delta \quad b_i = 0 \quad S(n) = S(n-1) - \Delta \]
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- *Delta Modulation*
- Pages 72-75
Delta modulation (DM) can be simulated in Simulink. The DM receiver utilizes a sample and hold token as an accumulator and the step size $\Delta = 20 \text{ mV}$.

- $f_S = 2 \text{ kHz}$
- $T_S = 0.5 \text{ msec}$

**DM transmitter**
- Pulse Generator
- Sine Wave
- Sample and Hold
- S/H Signal
- DM T Binary Data
- Lookup Table
- Delay
- Gain

**DM receiver**
- Pulse Generator
- Sine Wave
- Sample and Hold
- S/H Signal
- DM R Signal
- Gain
- Delay

MS Figure 2.61
• DM can be subject to *slope overload* which occurs when:
  \[ \Delta / T_S < \max | d m(t) / dt | \]  
  SVU Eq. 2.61 modified

Here the sinusoid has \( A = 1 \) but \( f = 10 \) Hz and:

\[ \Delta / T_S = 20 \text{ mV} / 0.5 \text{ msec} = 40 < \max | d m(t) / dt | = 80\pi \]

and slope overload occurs.

![MS Figure 2.63](image)

**sinusoidal input signal**
Granular noise occurs in DM because if the input $m(t)$ is constant the received signal oscillates by $\pm \Delta$ because there is no 0 possible. Clocking occurs at the DM symbol interval $T_S = 0.5$ msec.
• The tradeoff between slope overload and granular noise is that a large value of $\Delta$ (to avoid slope overload) would increase granular noise. A decrease in $T_s$ (again to avoid slope overload) would increase the data rate $r_s$.

• The step size $\Delta = 20$ mV and $T_s = 0.5$ msec ($r_s = 2$ kb/sec) here.
• For a 10 Hz sinusoidal input signal to a DM:
  \[ m(t) = \sin (2\pi \times 10t) \]
  \[ \max | \frac{d m(t)}{dt} | = 20\pi \]

• If step size \( \Delta = 20 \text{ mV} \) and \( T_S = 0.5 \text{ msec} \) then
  \[ \frac{\Delta}{T_S} = 40 < \max | \frac{d m(t)}{dt} | = 20\pi \]
  so slope overload is predicted to occur.
• For the 10 Hz sinusoidal input signal to a DM
  \[ m(t) = \sin (2\pi 10t) \]
  \[ \max | \frac{d m(t)}{dt} | = 20\pi \]

  and the step size remains \( \Delta = 20 \text{ mV} \) but \( T_S = 0.25 \text{ msec} \)
  then \( \Delta / T_S = 80 > \max | \frac{d m(t)}{dt} | \) and slope overload is mitigated but \( r_S = 4 \text{ kb/sec} \).
• In comparison, an 8-bit PCM system sampling a 10 Hz sinusoid at a reasonable sampling rate of 500 Hz (50 sampling points/period) has $r_b = 8(500) = 4$ kb/sec or $r_b = r_s$ but PCM is more complicated than DM.
End of Chapter 8

Analog-to-Digital and Digital-to-Analog Conversion