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The Journal of Political Economy, Volume 99, Issue 3 (Jun., 1991), 654-656.

Confirmations and Contradictions

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A recent empirical investigation by Diebold and Rudebusch (1990*b*) into the nature of business cycles reveals an apparent contradiction concerning the duration dependence of half and whole cycles. While expansions and contractions appear to be duration independent, completed cycles seem to be duration *dependent*. This implies, for example, that a long expansion is no more likely to terminate than a short expansion, whereas the termination probability for a whole cycle depends on its length. As discussed in Diebold and Rudebusch, this perplexing situation is feasible if the lengths of expansions and contractions are negatively correlated. However, they find only weak empirical evidence to support this conjecture. The purpose of this note is to resolve the puzzle by recognizing that it is impossible for expansions, contractions, and completed cycles to *all* follow the constant-hazard distribution. We then conclude that the (apparent) contradiction arises because the same statistical tests are applied to both half and whole cycles.

To illustrate, because data are reported on a monthly basis, we formulate two discrete-time tests for duration dependence. The first of these is applied separately to expansions, contractions, and completed cycles; the second is employed to test jointly the null hypothesis that expansions and contractions are both duration independent.

To derive the tests, initially assume that expansions and contractions have constant transition probabilities. If X is the random variable for the number of consecutive months of (say) expansion, p is the (constant) transition probability, and t_e represents the minimum

[*Journal of Political Economy,* 1991, vol. 99, no. 3]

maturity,[1] then X is distributed geometrically such that

$$P(X = t_e + i) = (1 - p)^i p, \quad i = 0, 1, 2, \ldots \tag{1}$$

(see Johnson and Kotz 1969, pp. 122–39). A test for duration-independent expansions is obtained by transforming the data to maturity zero, such that $Y = X - t_e$. As Y also follows a geometric distribution,

$$E(Y) = \frac{1 - p}{p}, \quad V(Y) = \frac{1 - p}{p^2}. \tag{2}$$

For small p, it follows that $\text{plim}[(\bar{y}/s_y) - 1] \sim 0$, where $\bar{y}$ and s_y are, respectively, the sample mean and standard deviation of the transformed data. Moreover, by the central limit theorem, the null distribution for the test statistic

$$z = \sqrt{T}\left(\frac{\bar{y}}{s_y} - 1\right) \tag{3}$$

is approximately standard normal for even moderately large samples.

This test can also be separately applied to contractions and completed cycles. However, it is important to realize that it is incompatible for expansions, contractions, *and* completed cycles all to follow the geometric (constant-hazard) distribution. In fact, if expansions and contractions are duration independent, then the probability distribution for whole cycles (WC) is given by

$$P(\text{WC} = t_e + t_c + i) = \sum_{k=0}^{i} (1 - p)^k (1 - q)^{i-k} pq, \tag{4}$$

$i = 0, 1, 2, \ldots$, where t_c and q are the parameters from the geometric distribution for contractions.

It is now clear that whole cycles that follow the constant-hazard distribution given in (1) imply some form of statistical dependence for the half-cycle components. If the z test in (3) is applied to completed cycles, then the null hypothesis incorporates the assumption that the length of an expansion (or contraction) has an effect on the termination probability of future contractions or expansions, or the termination probability varies within a given half cycle once the minimum maturity is realized, or both. Moreover, the empirical results of Diebold and Rudebusch are completely plausible since they apply analogous (but exact finite-sample) tests to both half and whole cycles. At certain values of t_e and t_c, they do not reject duration independence

[1] See McCulloch (1975) and Diebold and Rudebusch (1990*a*) for further discussion of the minimum maturity.

for expansions and contractions, though the implied null hypothesis for completed cycles is soundly rejected. The statistical conclusions concerning completed cycles then serve to reinforce those obtained from applying the tests separately to expansions and contractions. It is important to note, however, that applying z (or analogous tests) to completed cycles remains important since duration-dependent half cycles will not guarantee constant-hazard full cycles.

As an extension, it is also possible to test jointly for expansion and contraction duration dependence by using the completed cycles. Consider first applying the z test separately to expansions and contractions to obtain (say) z_e and z_c. When expansions and contractions are both duration independent, then

$$C = z_e^2 + z_c^2 \tag{5}$$

is asymptotically chi-square with two degrees of freedom.

The empirical results of the z and C tests closely agree with those reported in Diebold and Rudebusch.[2] Consider, for example, $t_e = 8$ and $t_c = 4$ for their full sample. We find that $z_e = 1.2416$, $z_c = 0.7107$, and $C = 2.0467$, so that none rejects duration independence of expansions and contractions at the 5 percent significance level. Applying the z test to completed cycles for peak-to-peak data yields a value of 3.4934; the corresponding trough-to-trough value is 4.4560. Since both of these reject the constant-hazard distribution given in (1), we have thus illustrated that the empirical conclusions from Diebold and Rudebusch are completely plausible once the implied null hypotheses for the various tests are recognized.

References

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[2] A complete set of empirical results is found in Mudambi and Taylor (1990). In that paper, we also formulate two other (asymptotic) tests for duration dependence, including a generalized method of moments test that operates unconditionally on the sample variance. All the tests perform comparably.