Financial Market Dynamics: Implications of Prospect Theory for Asset Prices and Trading Volume

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Abstract

Does prospect theory produce price momentum and return-volume co-movement via the disposition effect? The answer to this question requires a general equilibrium analysis. This paper provides a general equilibrium model and finds that different components of prospect theory play different roles. Diminishing sensitivity drives a disposition effect that, in turn, leads to price momentum and a positive return-volume correlation; loss aversion predicts exactly the opposite. In a calibrated economy with prospect theory preference parameters set at the values estimated by earlier studies, our model can generate price momentum ranging from 1% to 7% on an annual basis.

Keywords Prospect Theory; Disposition Effect; Momentum; Reversal; Turnover; Equity Premium.

JEL classification G11, G12.
I. Introduction

The disposition effect describes the greater tendency of investors to sell assets that have risen in value since purchase than those that have fallen.\footnote{This effect has been repeatedly observed in many markets for both retail and professional investors. Odean (1998), Grinblatt and Keloharju (2001), and Feng and Seasholes (2005) find disposition effects in the stock markets of the U.S., Finland, and China, respectively. Grinblatt and Keloharju (2001), Shapira and Venezia (2001), Wermers (2003), Garvey and Murphy (2004), Coval and Shumway (2005), Locke and Mann (2005), Frazzini (2006), and Scherbina and Jin (2006) document the disposition effect in the trading of professionals who trade on behalf of their firms; Genesove and Mayor (2001) and Heath et al. (1999) document disposition effects in the housing market and in the exercise of executive stock options, respectively. Webber and Camerer (1998) and Oehler et al. (2002) uncover disposition effects with experimental data. See Feng and Seasholes’ (2005) Appendix A for more empirical studies on the disposition effect.} The literature suggests that prospect theory is potentially a useful ingredient in explaining the disposition effect.\footnote{For empirical studies, see Shefrin and Statman (1985), Odean (1998), among many others; for theoretical models, see Gomes (2005) and Kyle et al. (2006).} In addition, some empirical studies suggest that the disposition effect can generate momentum in stock returns (Grinblatt and Han, 2005; Shumway and Wu, 2007), can induce post-earnings announcement drift (Frazzini, 2006), and can contribute to a positive correlation between returns and volumes (Statman et al., 2006; Griffin et al., 2007).

Therefore, it seems natural to argue that, through the disposition effect, prospect theory can produce price momentum and a positive return-volume correlation. However, does this argument survive rigorous examination? If it does, how much can prospect theory explain the existing pricing and volume patterns? If it does not, then why not? What are the precise implications of prospect theory for the dynamics of asset prices and trading volume?

We address these questions by providing a dynamic general equilibrium model in which prospect theory investors trade financial assets with each other. We find that the link between prospect theory preferences and asset prices and volumes is not as simple as initially thought: under some parameter values, prospect theory indeed produces price momentum and a positive return-volume correlation; but under other parameter values, prospect theory produces price reversal and a negative return-volume correlation. Our analysis further identifies the key to understanding these mixed results as realizing that prospect theory has various components and that different components make different predictions regarding trading behavior, asset prices, and volumes.

As a prominent theory of decision-making under uncertainty, prospect theory was first
proposed by Kahneman and Tversky (1979) and extended by Tversky and Kahneman (1992). Under prospect theory, (i) investors evaluate outcomes not according to final wealth levels but according to their perception of gains and losses relative to a reference point, typically the purchase price; (ii) investors are risk-averse for gains and risk-seeking for losses (diminishing sensitivity); (iii) investors are more sensitive to losses than to gains of the same magnitude (loss aversion); and (iv) investors use transformed rather than objective probabilities to calculate expectations (probability weighting).

Our model demonstrates that the diminishing sensitivity component predicts a disposition effect in equilibrium, which in turn drives price momentum and a positive return-volume correlation (see Subsection IVB). However, the loss aversion component predicts a reversed disposition effect in individual trading, namely that investors are more inclined to sell stocks with prior losses than those with prior gains; this resulting reversed disposition effect leads to a reversal in the cross-section of stock returns, and to a negative return-volume correlation (see Subsection IVC). In a calibrated economy, when preference parameters are set at the values estimated by earlier studies, our model can generate an annual momentum ranging from 1% to 7% (see Subsection IVD).

The intuition behind the implications of diminishing sensitivity is as follows. When a stock experiences good (bad) news and increases (decreases) in value relative to its purchase price, investors who previously purchased it will be keen (reluctant) to sell it, due to the concavity (convexity) of the value function of prospect theory in the region of the gains (losses). Their selling (holding) increases (decreases) volume. The resulting selling (holding) pressure makes the stock price underreact to the initial good (bad) news in equilibrium; that is, although the stock price rises (drops), it does not rise (drop) to the level that would prevail in the absence of disposition-related selling (holding), and starting from this lower (higher) base, subsequent returns will be higher (lower).

The intuition for the implications of loss aversion is a different story. Loss aversion

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3 For a review of prospect theory, see Barberis and Thaler’s (2003) Section 3.2.1 or Barberis and Huang’s (2008) Section 2.

4 Throughout this paper, we follow the literature in using the terms “diminishing sensitivity” and “concavity/convexity” interchangeably to refer to this S-shaped value function of prospect theory.

5 We do not incorporate probability weighting feature of prospect theory in our model for the reasons discussed in Section II.
means that the prospect theory value function has a kink at the origin. Investors are afraid of holding stocks if they are close to the kink. It is well understood that loss aversion produces positive equity premiums in equilibrium (e.g., Benartzi and Thaler, 1995; Barberis and Huang, 2001, 2007). It follows that, in equilibrium, good (bad) news will push investors far from (close to) the kink, making them more likely to hold (sell) stocks when facing gains (losses). This resulting reversed disposition effect, in turn, leads to a negative return-volume correlation and to a reversal in the cross-section of returns: when a stock experiences good (bad) news and increases (decreases) in value relative to the purchase price, investors, according to the reversed disposition effect, want to hold (sell) stocks, which reduces (raises) the trading volume; the increased (decreased) demand resulting from the reversed disposition effect causes the stock price to “overreact” to the initial good (bad) news, leading to lower (higher) subsequent stock returns.

It is not an easy task to build a general equilibrium model to examine the pricing and volume implications of prospect theory via the (reversed) disposition effect because (i) a prospect theory investor’s decision involves solving an optimal stopping time problem with a non-smooth and partially convex objective function; and (ii) the state vector in the general equilibrium model is highly dimensional, including the distribution of asset holdings and the reference points. Our trick is to use an overlapping-generation (OLG) setup to simplify an investor’s optimal stopping time problem and to reduce the dimension of the state vector. In our model, investors trade stocks and a risk-free asset over their lifetimes; at the end of their final periods, they receive prospect theory utility based on their trading profits. The behavior of investors who bought stocks in previous periods can potentially exhibit the (reversed) disposition effect, which, in turn, drives price momentum/reversal and the return-volume correlation.

Our paper contributes to the asset pricing literature on prospect theory. Previous research has focused primarily on the loss aversion component, showing that it can increase the equity premium, i.e., the mean of stock returns in excess of the risk-free rate (e.g., Benartzi and Thaler, 1995; Barberis et al., 2001; McQueen and Vorkink, 2004; Grünea and Semmler, 2008). Our model demonstrates that loss aversion also has implications for return pre-

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6 Barberis and Huang (2008) show that the probability weighting feature of prospect theory can cause a
dictability and trading volumes: it drives a reversal in the cross-section of stock returns and a negative return-volume correlation (see Subsection IVC). In addition, our paper shows that the diminishing sensitivity component of prospect theory helps to explain the momentum effect and the positive return-volume correlation in financial markets (see Subsection IVB). Over and above these results, Subsection IVB shows that diminishing sensitivity alone, in the absence of loss aversion, can raise equity premiums.

Recent papers by Barberis and Xiong (2009) and Hens and Vlcek (2009) also point out the possibility that prospect theory can drive a reversed disposition effect. Although these studies sharpen our understanding of the implications of prospect theory for the disposition effect, the partial equilibrium feature of their proposed models prevents these authors from pinning down which feature of prospect theory is driving the reversed disposition effect, and as a result, these authors turn to the exogenous equity premium for an explanation. In contrast, our general equilibrium analysis shows that it is the loss aversion feature of prospect theory, rather than the equity premium itself, that is responsible for the reversed disposition effect. In addition, our model pushes one step further by examining the implications of the resulting reversed disposition effect for the dynamics of asset prices and trading volume.

The remainder of the paper is organized as follows. Section II describes the model, and Section III characterizes the equilibrium. Section IV solves the price-dividend ratios and uses simulated data to analyze the implications of diminishing sensitivity and loss aversion for individual trading behavior, asset prices and trading volumes. In particular, Subsection IVD conducts a quantitative analysis using parameters estimated by earlier studies to evaluate how effectively prospect theory preferences can generate the disposition effect, momentum and return-volume correlation. Section V concludes the paper. The appendices describe the numerical algorithm used for computing the equilibrium and discuss the robustness of our results to certain modeling assumptions.
II. An OLG Model with Prospect Theory Preferences

Let us consider an OLG model with one consumption good. Time is discrete and indexed by \( t \). In each period, there are three generations (age-1, age-2 and age-3), each with a unitary mass. We adopt an OLG setup simply to reduce the dimension of the state vector. In the context of the disposition effect, the reference points usually relate to the purchase prices, which enter the state of the economy via the disposition effect, making the state history-dependent. In an OLG setup, investors live for a finite period of time; as a result, their purchase prices involve only a finite number of periods, effectively reducing the dimension of the state vector. The OLG setup should therefore not be interpreted literally. Generations should be understood as generations of transactions, not generations of people. Since the average holding periods of stocks are six months to one year, one generation corresponds to six months to one year.

Why are there three generations in each period? First, in order to study the disposition effect, which concerns selling decisions, we need at least three generations. In the standard two-generation models, old investors always sell stocks whether facing good news or bad, thereby automatically ruling out the disposition effect. A model with more than two generations, on the other hand, allows some investors to decide when to liquidate stocks they bought in previous periods. Second, if there were more than three generations, the state vector would be highly dimensional, making the model intractable. In Subsection IVE, we argue intuitively that our results might still hold in a setup with more than three generations.

Financial Assets

There are two traded assets: a risk-free bond and a risky stock. The bond is in perfectly elastic supply at a constant gross interest rate \( R_f > 1 \). The stock pays a random dividend \( D_t > 0 \) in period \( t \). The dividend growth rate \( \theta_{t+1} \triangleq \frac{D_{t+1}}{D_t} \) is i.i.d. over time and follows a distribution given by

\[
\theta_{t+1} = \begin{cases} 
\theta_H, & \text{with probability } \frac{1}{2}, \\
\theta_L, & \text{with probability } \frac{1}{2},
\end{cases}
\]

with \( 0 < \theta_L < \theta_H \). (1)
The stock is in limited supply (normalized as 1) and is traded in a competitive market at price $P_t$. Let $R_{t+1}$ be the gross return on the stock between time $t$ and $t+1$, i.e., $R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}$.

Investors can buy or short bonds at any level, but they cannot short stocks, and if they buy stocks, they can hold exactly 1 unit in each period. For several reasons, we assume that people hold either zero or one unit of stock. First, this specification is realistic in the sense that the lower (upper) bound of the holding position captures the shorting (borrowing) constraints in stock trading. Second, the assumption that investors buy at most one unit of stock at one time captures the idea that they tend to form different mental accounts for the same stock bought at different prices. Third, a binary choice in stock holdings simplifies an investor’s decisions because otherwise, due to the convexity of the Tversky and Kahneman (1992) value function in the loss domain, it is very difficult to characterize the investor’s demand function. Finally, a binary choice and an OLG setup combine to reduce the complicated optimal stopping problem of an age-2 investor owning a stock to a simple problem of choosing between an early liquidation and a late liquidation. In Appendix B4, we relax this assumption by allowing investors to hold intermediate levels of stocks and verify that our results are still valid.

**Beliefs** In order to study the impact of the disposition effect on trading volume, we make two assumptions regarding investors’ beliefs. First, investors hold heterogeneous beliefs about the dividend growth rate within one period. Due to this cross-sectional heterogeneity in beliefs, investors, particularly young investors, will make different investment decisions; more optimistic investors will purchase a stock, while more pessimistic investors will not. Second, an investor’s one-period-ahead dividend forecast changes during his lifetime. The time-variation in an investor’s belief will motivate the selling of a stock by a middle-aged investor who purchased the stock when he was young. With these two assumptions, we ensure that in each period, there is always a group of middle-aged investors who bought stocks last period and want to sell them this period. It is the behavior of this group of investors that can potentially exhibit a disposition effect. Of course, these two assumptions are simply a modeling device; any other trading motive, such as liquidity shock (e.g., Kaustia, 2008), can also serve the same purpose.
As a matter of fact, in the informal arguments that have been used to link prospect theory and the disposition effect, investors are often assumed to experience belief changes; that is, time-variation in an investor’s beliefs is often maintained, as the following quotation from Odean (1998, p. 1777) illustrates:

“(S)uppose an investor purchases a stock that she believes to have an expected return high enough to justify its risk. If the stock appreciates and the investor continues to use the purchase price as a reference point, the stock price will then be in a more concave, more risk-averse, part of the investor’s value function. It may be that the stock’s expected return continues to justify its risk. However, if the investor somewhat lowers her expectation of the stock’s return, she will be likely to sell the stock. What if, instead of appreciating, the stock declines? Then its price is in the convex, risk-seeking, part of the value function. Here the investor will continue to hold the stock even if its expected return falls lower than would have been necessary for her to justify its original purchase. Thus the investor’s belief about expected return must fall further to motivate the sale of a stock that has already declined than one that has appreciated.” [Emphasis added as italics]

Formally, in period $t$, investor $i$ believes that the dividend growth rate $\theta_{t+1}$ follows a distribution given by

$$
\theta_{t+1} = \begin{cases} 
\theta_H, & \text{with probability } q_{i,t}, \\
\theta_L, & \text{with probability } 1 - q_{i,t},
\end{cases}
$$

(2)

where $q_{i,t}$ is a random variable with uniform distribution on $[0, 1]$ and $q_{i,t}$ is i.i.d. across investors (index $i$) and over time (index $t$). On average, investors have the correct beliefs, because the mean of $q_{i,t}$ is equal to the true probability of $\frac{1}{2}$ that the dividend growth rate was

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8In reality, an investor’s one-period-ahead dividend forecasts might be correlated. As a robustness check, we also try the following specification to capture this correlation: $q_{i,t+1} = \rho q_{i,t} + (1 - \rho) \varepsilon_{i,t+1}$ with $\rho \in (0, 1)$, where $q_{i,t}$ follows a beta distribution and $\varepsilon_{i,t+1}$ follows a uniform distribution. If $\rho = 0$, then we return to the specification in the main text in which his forecasts are independent over time; if $\rho = 1$, then an investor’s forecasts about dividend growth rate are constant over time.
takes a high value. Investors are forward-looking; therefore, we can apply the standard dynamic programming techniques to solve their optimal decision problems.

**Preference** An investor derives prospect theory utility from trading assets in the spirit of Kahneman and Tversky (1979) and Tversky and Kahneman (1992). When investor $i$ is born, he is endowed with $W_{1,i}$ units of consumption good. He can trade when he is young and when he is middle-aged, leaving his final wealth as $W_{3,i}$ and his capital gains/losses as $X_{3,i}$. Let $E^i_t$ denote the investor’s expectation operator at time $t$. His time $t$ utility, $U^i_t$, is then given by

$$U^i_t = E^i_t [v(X_{3,i})],$$

(3)

where

$$X_{3,i} = W_{3,i} - R^2_f W_{1,i},$$

(4)

$$v(x) = \begin{cases} 
  x^\alpha, & \text{if } x \geq 0, \\
  -\lambda (-x)^\alpha, & \text{if } x < 0,
\end{cases}$$

(5)

with $0 < \alpha \leq 1$ and $\lambda \geq 1$.

Here, the function $v(\cdot)$ is the standard value function of prospect theory proposed by Tversky and Kahneman (1992). The argument of $v(\cdot)$ is the capital gain/loss, $X_{3,i}$, not the final period wealth, $W_{3,i}$. Function $v(\cdot)$ is concave for gains and convex for losses, meaning that investors are risk-averse in the domain of gains and risk-seeking in the domain of losses; it has a kink at the origin, implying a greater sensitivity to losses than to gains of the same magnitude. Parameter $\alpha$ governs its concavity/convexity and parameter $\lambda$ controls loss aversion. For simplicity, we do not explore prospect theory’s probability weighting feature in the above preference specification but instead simply apply the standard expectation operator $E^i_t$. The primary effect of probability weighting is to overweight small probabilities; it therefore has its biggest impact on securities with highly skewed returns. Since most stocks are not highly skewed, we do not expect probability weighting to be central to the

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9We also considered a model, similar to Barberis et al. (2001), in which an investor derives two kinds of utilities — the standard consumption utility and prospect theory utility — and obtained similar results.
link between prospect theory and the disposition effect. Indeed, Hens and Vlcek (2009) find that probability weighting plays only a minor role in determining whether prospect theory predicts the disposition effect.

In equation (4), we follow the literature (e.g., Gomes, 2005; Barberis and Huang, 2008; Barberis and Xiong, 2009) and define the capital gain/loss as $X_{3,i} = W_{3,i} - R_f^2W_{1,i}$. That is, we take a reference point as an investor’s final wealth, which he could have accrued by investing in bonds both when young and when middle-aged. The gain/loss from a particular stock sale is calculated as the difference between the reference point and the investor’s final wealth resulting from buying and selling this stock. For example, if investor $i$ buys a stock at price $P^B$ at age 1, sells it at price $P^S$ and collects a dividend $D_{2,i}$ at age 2, and he then reinvests $P^S + D_{2,i}$ in bonds, getting back $R_f P^S + R_f D_{2,i}$ at age 3. If he had not bought the stock at age 1, but had invested $P^B$ in bonds and held them till age 3, then he would have collected $R_f^2 P^B$ at age 3. Therefore, the gain/loss from this stock sale is $X_{3,i} = R_f P^S + R_f D_{2,i} - R_f^2 P^B$. This definition reflects the idea that an investor usually begins to consider the stock investment a loss if he finds that he could have earned more by investing in the riskless bond. In Appendix B2, we further show that our results are robust to taking purchase prices as reference points.

**Timeline** To summarize, the exogenous random variables of the model are $\theta_t$ and $q_{i,t}$, and the exogenous parameters of the model are $\theta_H > 0$, $\theta_L > 0$, $R_f > 1$, $0 < \alpha \leq 1$ and $\lambda \geq 1$. The order of events in each period $t$ is shown in Figure 1. At the beginning of period $t$, age-1 investors are born and receive consumption good endowments. The dividend growth rate $\theta_t$ is realized, and all investors observe $\theta_t$. The idiosyncratic belief shock $q_{i,t}$ is realized, and investor $i$ observes $q_{i,t}$. All investors trade in the stock and bond market; age-2 and age-3 investors carry stocks to the market, while after trading, age-1 and age-2 investors hold stocks. At the end of period $t$, age-3 investors receive prospect theory utility and exit the economy.

**Figure 1**

Our OLG setup can be understood as a stylized way of describing how different types
of investors existing in real markets interact with each other. The model economy is linked to reality as follows. The potential buyers are represented by an age-1 investor and an age-2 investor without a stock who correspond, respectively, to a new participant and to a “wait-and-seer” who has been sitting in the market for some time. The potential sellers are represented by an age-3 investor and an age-2 investor who owns a stock; they correspond, respectively, to a pure noise investor who has no discretion with regard to the timing of his trade and to a discretionary liquidity investor who can determine when to trade.¹⁰

**Extension: A Multi-Stock Setting**  
So far, we have assumed that each investor holds just one risky asset, but our analysis has implications for the cross-section property of stock returns as long as the investor engages in mental accounting or narrow framing (Thaler, 1980, 1985), that is, as long as investors derive prospect theory utility separately from the trading profit on each distinct stock. Such a model has been proposed by Barberis and Huang (2001), who find that narrow framing at the individual stock level can generate a large value premium in the cross-section. In fact, a narrowing framing assumption is almost always present in the literature relating prospect theory to the disposition effect (e.g., Odean, 1998; Barberis and Xiong, 2009).¹¹ Formally, we can consider an economy with $N$ stocks, in which each stock has an i.i.d. dividend process with distribution given by equation (1), investors hold heterogeneous beliefs about the dividend growth rates of their stocks and experience belief changes in their lifetimes, and these investors derive prospect theory utility from accumulative trading profits at the level of individual stocks. We can then use the same conditions that characterize the equilibrium in the single stock setting — more precisely, equations (6) through (24) — to define an equilibrium, stock by stock, in this multi-stock setting. In Section IV, we conduct an analysis of this type and examine whether prospect theory drives the momentum effect in a multi-stock setting.

¹⁰The importance of differentiating a pure noise investor from a discretionary liquidity investor has been emphasized in the microstructure literature, for example, Admati and Pfleiderer (1988).

¹¹Kumar and Lim (2008) document that narrow framers indeed exhibit a more pronounced disposition effect.
III. Equilibrium Characterization

We now derive equilibrium asset prices. To ease exposition, the investors of age 2 who have (do not have) a stock when they enter the market are referred to as age-2-1 investors (age-2-0 investors). Let \( z_t \) be the mass of age-2-1 investors in period \( t \). Variable \( z_t \) captures the distribution of stocks at the moment when the market opens: \( z_t \) units of stocks are in the hands of age-2-1 investors and \( 1 - z_t \) units are in the hands of age-3 investors.

Let \( f_t = \frac{P_t}{D_t} \) denote the price-dividend ratio in period \( t \). Then in period \( t \), the state of the economy is \( S_t = (\theta_t, f_{t-1}, z_t) \), namely, the dividend news at time \( t \), the price-dividend ratio at time \( t - 1 \), and the holding of stocks by age-2 investors. In equilibrium, the stock price-dividend ratios will be a function of the state vector, \( f_t = f(S_t) \). The three variables \( \theta_t, f_{t-1} \) and \( z_t \) affect stock prices because (i) \( \theta_t \) and \( f_{t-1} \) affect age-2-1’s investment decisions through the disposition effect and because (ii) \( z_t \) relates to aggregate effect on prices of age-2-1 investors as a whole. We construct the price-dividend function \( f \) by solving investors’ optimal decisions backwards and using the market clearing condition. When solving the investors’ decision problems, we assume that the aggregate state vector \( S_t \) is observable to investors; therefore, their individual state vectors (or information sets) are the aggregate state vector spanned by their beliefs: \( \mathcal{F}_t^i = \{S_t, q_{i,t}\} \).

Age-3 Investors’ Decisions  The decision of an age-3 investor is simple: if he has a stock, he sells it and derives prospect theory utility from his trading profit; if he does not have a stock, he simply waits until the end of the period and receives prospect theory utility. Because \( 1 - z_t \) units of stocks are possessed by age-3 investors before the market opens, they sell \( 1 - z_t \) stocks during trading as a whole; after trade, they do not hold any stock.

Age-2-1 Investors’ Decisions  An age-2-1 investor decides whether to sell the stock. If he continues to hold the stock, then he will sell the stock at price \( P_{t+1} \) in the next period, resulting in a gain/loss

\[
P_{t+1} + D_{t+1} + R_f D_t - R^2_{f} P_{t-1} = G^{t+1}_{1-1} D_{t-1},
\]

with

\[
G^{t+1}_{1-1} = (f_{t+1} + 1) \theta_t \theta_{t+1} + R_f \theta_t - R^2_{f} f_{t-1}.
\] (6)
As a result, his expected prospect theory utility is

\[ U_{1-1} (S_t, q_{i,t}) = E_t^i \left[ v \left( G_{1-1}^{t+1} \right) \right] D_{t-1}^\alpha, \]  

(7)

where \( E_t^i \) is the subjective expectation operator conditional on investor \( i \)'s period \( t \) information set \( \mathcal{F}_t^i = \{ S_t, q_{i,t} \} \). Here, investor \( i \) takes expectation over the random variables \( \theta_{t+1} \) and \( f_{t+1} \) according to his subjective belief [equation (2)] and the transition law of the state vector [equation (23)].

If he sells the stock, what is his expected prospect theory utility? Since he sells at price \( P_t \), then his gain/loss is

\[ R_f P_t + R_f D_t - R_f^2 P_{t-1} = G_{1-0}^t D_{t-1}, \]

with \( G_{1-0}^t = (f_t + 1) R_f \theta_t - R_f^2 f_{t-1}. \)  

(8)

Therefore, his expected utility is

\[ U_{1-0} (S_t) = v \left( G_{1-0}^t \right) D_{t-1}^\alpha. \]  

(9)

If \( U_{1-1} (S_t, q_{i,t}) \geq U_{1-0} (S_t) \), then investor \( i \) will continue to hold the stock. That is, those with sufficiently large optimistic beliefs \( q_{i,t} \) will not sell their stocks.

In summary, the optimal decision of an age-2-1 investor is

\[ h_{21} (S_t, q_{i,t}) = 1_{U_{1-1}(S_t, q_{i,t}) \geq U_{1-0}(S_t)} = 1_{E_t^i [v(G_{1-1}^{t+1})] \geq v(G_{1-0}^t)}. \]  

(10)

The corresponding indirect value function is\[^{12}\]

\[ V_{21} (S_t, q_{i,t}) = \hat{V}_{21} (S_t, q_{i,t}) D_{t-1}^\alpha, \]

with \( \hat{V}_{21} (S_t, q_{i,t}) = h_{21} (S_t, q_{i,t}) E_t^i \left[ v \left( G_{1-1}^{t+1} \right) \right] + [1 - h_{21} (S_t, q_{i,t})] v \left( G_{1-0}^t \right). \)  

(11)

\[^{12}\text{Note that the indirect value function } V_{21} (S_t, q_{i,t}) \text{ is different from the value function of prospect theory } v (\cdot). \text{ Function } v (\cdot) \text{ corresponds to a standard Bernoulli utility function in the choice theory under uncertainty; however, function } V_{21} (S_t, q_{i,t}) \text{ is the indirect utility function, which takes into account the investor's optimal decisions.}\]
This indirect value function will be useful when we apply the dynamic programming technique to solve age-1 investors’ optimal decisions.

After trading, the fraction of age-2-1 investors who continue to hold their stocks is

$$ H_{21} (S_t) = \int_0^{z_t} h_{21} (S_t, q_{i,t}) \, di = z_t E [h_{21} (S_t, q_{i,t}) | S_t], \quad (12) $$

where the second equality follows from the law of large numbers and the expectation is taken over the random variable $q_{i,t}$, which follows a uniform distribution over $[0, 1]$.

Before trade, stocks are in the hands of age-2-1 and age-3 investors, while after trade, these two groups of agents own $H_{21} (S_t)$ stocks in total; therefore, the trading volume or the aggregate selling in period $t$ is

$$ Q (S_t) = 1 - H_{21} (S_t). \quad (13) $$

Because there is only one share outstanding, the volume and the turnover are the same.

**Age-2-0 Investors’ Decisions** An age-2-0 investor decides whether to buy a stock. If he decides to buy a stock, then he will have a gain/loss

$$ P_{t+1} + D_{t+1} - R_f P_t = G_{0-t}^{t+1} D_{t-1}, $$

with $G_{0-t}^{t+1} = (f_{i+1} + 1) \theta_t \theta_{t+1} - R_f f_t \theta_t, \quad (14)$

and will have expected prospect theory utility

$$ U_{0-t} (S_t, q_{i,t}) = E_t^i [v (G_{0-t}^{t+1})] D_{t-1}. \quad (15) $$

If he decides not to buy a stock, then his utility is 0. It follows that an age-2-0 investor’s optimal decision is

$$ h_{20} (S_t, q_{i,t}) = 1_{U_{0-t} (S_t, q_{i,t}) \geq 0} = 1_{E_t^i [v (G_{0-t}^{t+1})] \geq 0}, \quad (16) $$
and the corresponding *indirect* value function is

\[ V_{20}(S_t, q_{i,t}) = \hat{V}_{20}(S_t, q_{i,t}) D_{t-1}^\alpha, \text{ with } \hat{V}_{20}(S_t, q_{i,t}) = h_{20}(S_t, q_{i,t}) E_t^i \left[ v\left(G_{t+1}^{t+1}\right)\right]. \tag{17} \]

After trading, the aggregate stock holding of age-2-0 investors is

\[ H_{20}(S_t) = \int_0^{1-z_t} h_{20}(S_t, q_{i,t}) \, di = (1 - z_t) E \left[ h_{20}(S_t, q_{i,t}) | S_t \right]. \tag{18} \]

**Age-1 Investors’ Decisions**  An age-1 investor decides whether to buy a stock. If he buys a stock, then he becomes an age-2-1 investor in the next period, and by the logic of dynamic programming, his expected utility is

\[
U_1(S_t, q_{i,t}) = E_t^i \left[ V_{21}(S_{t+1}, q_{i,t+1}) \right] = \hat{U}_1(S_t, q_{i,t}) D_t^\alpha, \\
\text{with } \hat{U}_1(S_t, q_{i,t}) = E_t^i \left[ \hat{V}_{21}(S_{t+1}, q_{i,t+1}) \right]. \tag{19}
\]

If he decides not to buy a stock, he becomes an age-2-0 investor in the next period, and his expected utility is

\[
U_0(S_t, q_{i,t}) = E_t^i \left[ V_{20}(S_{t+1}, q_{i,t+1}) \right] = \hat{U}_0(S_t, q_{i,t}) D_t^\alpha, \\
\text{with } \hat{U}_0(S_t, q_{i,t}) = E_t^i \left[ \hat{V}_{20}(S_{t+1}, q_{i,t+1}) \right]. \tag{20}
\]

Therefore, his optimal decision is

\[ h_1(S_t, q_{i,t}) = 1_{\hat{U}_1(S_t, q_{i,t}) \geq \hat{U}_0(S_t, q_{i,t})}. \tag{21} \]

So after trade, age-1 as a whole will hold

\[ H_1(S_t) = \int_0^1 h_1(S_t, q_{i,t}) \, di = E \left[ h_1(S_t, q_{i,t}) | S_t \right]. \tag{22} \]

**Evolution of State Variables**  The state vector \( S_t \) evolves according to the following equation:

\[ S_{t+1} = (\theta_{t+1}, f_t, z_{t+1}) = (\theta_{t+1}, f(S_t), H_1(S_t)), \tag{23} \]
where functions $H_1(S_t)$ [given by (22)] and $f(S_t)$ are both endogenously determined. The random process $\{\theta_{t+1}\}_{t=1}^{\infty}$ is i.i.d. with distribution $\Pr(\theta_{t+1} = \theta_H) = \Pr(\theta_{t+1} = \theta_L) = \frac{1}{2}$ [i.e., equation (1)]. When investors make decisions, however, they believe that $\theta_{t+1}$ evolves according to $\Pr_1^t(\theta_{t+1} = \theta_H) = q_{t,t}$ [i.e., equation (2)]. Since $S_t$ is in the investors’ period $t$ information set, in period $t$ they know the other two variables in $S_{t+1}$ (i.e., $f_t$ and $z_{t+1}$) by applying functions $f(\cdot)$ and $H_1(\cdot)$.

**Market Clearing Condition** The market clearing condition is

\[
H_1(S_t) + H_{20}(S_t) + H_{21}(S_t) = 1, \tag{24}
\]

which states that the stock holdings from age-1, age-2-0, and age-2-1 add up to the total stock supply 1. An equilibrium price-dividend function $f$ is implicitly determined by equations (6) through (24).

We adopt the equilibrium concept of Radner (1972), known as “equilibrium of plans, prices, and price expectations.” An equilibrium is formally defined as follows.13

**Definition 1** An equilibrium consists of decision rules, $h_1(S_t, q_{i,t})$, $h_{20}(S_t, q_{i,t})$ and $h_{21}(S_t, q_{i,t})$ and a law of motion $S_{t+1} = (\theta_{t+1}, f_t, z_{t+1}) = (\theta_{t+1}, f(S_t), H_1(S_t))$ such that

1. the decision rules maximize investors’ expected prospect theory utility conditional on their information;
2. markets clear: $H_1(S_t) + H_{20}(S_t) + H_{21}(S_t) = 1$ for almost every realization of $S_t$; and
3. the law of motion is generated by decision rules.

As is well-known in the literature, it is difficult to establish general results on the existence and uniqueness of the equilibria in dynamic heterogeneous agent models. Prospect theory preferences further complicate this issue. Except for the special case of $\alpha = \lambda = 1$, the case in which the investors’ preferences are reduced to risk neutrality, we cannot establish the existence of the equilibrium. However, analysis of the static models in Barberis and Huang (2008) and De Giorgi et al. (2009) suggests that if there is a continuum of prospect theory

13 Note that the definition of equilibrium has implicitly incorporated prices into the price-dividend ratio function in the law of motion.
investors, which is true in our model, an equilibrium generally exists. Therefore, in the present paper, we simply begin our analysis with the assumption that an equilibrium exists and, except in the case of the special risk-neutral economy, use numerical methods to find this equilibrium. Rigorously speaking, a numerical method can never find the exact equilibrium; what it finds, if any, is the “ε-equilibrium” defined by Kubler and Schmedders (2003), who interpret the computed ε-equilibrium as an approximate equilibrium of some other economy with endowments and preferences that are close to those in the original economy.

Solving the equilibrium reduces to solving two functions: the price-dividend function $f(\cdot)$, which determines the stock returns and thus investors’ decisions, and age-1 investors’ aggregate stock demand function $H_1(\cdot)$, which determines the evolution of the endogenous state variable $z_t$. The other functions involved in the equilibrium, such as the decision rules, can be obtained from functions $f(\cdot)$ and $H_1(\cdot)$ and the equations defining the investors’ optimal decisions. The basic computational methodology is as follows: starting from an initial conjecture of $f^{(0)}(\cdot)$ and $H_1^{(0)}(\cdot)$, solve $f^{(1)}(S_t)$ and $H_1^{(1)}(S_t)$ on a grid of $S_t$ from equations (6)-(24) and continue this process until $f^{(n)}(\cdot) \to f(\cdot)$ and $H_1^{(n)}(\cdot) \to H_1(\cdot)$. A detailed description of our numerical method is given in Appendix A.

To ensure the numerical accuracy of equilibrium computation, we implement several accuracy checks. First, we verify that all defining parts of an equilibrium are met; that is, individual optimization and market clearing conditions hold with small errors. In our case, the maximum errors lie below $10^{-6}$, suggesting that the algorithm produces reasonably accurate solutions. Second, we set $\alpha = \lambda = 1$ to reduce the investors’ preferences to risk neutrality (see subsection IVA) where an analytical solution is available, then solve the risk-neutral economy numerically and verify that the numerical solution is the same as the analytical solution. Moreover, we try different initial values for the functions $f^{(0)}(\cdot)$ and $H_1^{(0)}(\cdot)$, and find that they all converge to the same functions $f(\cdot)$ and $H_1(\cdot)$.

IV. Results and Intuitions

In this section, we first analytically solve the equilibrium in an economy with risk-neutral investors and show that there is no disposition effect, no price momentum and no correla-
tion between returns and volume (Subsection IVA). This works as a benchmark economy to illustrate that all our results are driven entirely by prospect theory preferences. We then demonstrate that diminishing sensitivity drives a disposition effect, which in turn leads to momentum in the cross-section of stock returns and a positive return-volume correlation (Subsection IVB) and that loss aversion predicts exactly the opposite, namely, a reversed disposition effect and reversal in the cross-section of stock returns as well as a negative return-volume correlation (Subsection IVC). We conduct a quantitative analysis in order to examine how much prospect theory can explain price momentum (Subsection IVD). Finally, we derive testable empirical predictions, check the robustness of our results to certain modeling assumptions and discuss possible generalizations of our model to allow for the existence of more standard, risk-averse expected utility investors and/or heterogeneous prospect theory investors (Subsection IVE).

A. Benchmark Case: Standard Risk-Neutral Utility

Suppose $\alpha = \lambda = 1$. Diminishing sensitivity and loss aversion, two distinctive features of prospect theory, will vanish, reducing the preferences to a standard risk neutral-utility representation. We do not use a standard risk-averse preference, such as a power utility function, as the benchmark, because risk aversion per se can qualitatively generate a disposition effect through portfolio rebalancing, although Odean (1998) argues that portfolio rebalancing cannot quantitatively account for the disposition effect. Risk neutrality removes this contamination and therefore gives cleaner results.

In this risk-neutral economy, we can solve the equilibrium analytically. Both the price-dividend ratio function and the age-1 investors’ aggregate demand function $H_1(\cdot)$ are constant: $f(S_t) \equiv f = \frac{E(\theta_{t+1})}{R_t - E(\theta_{t+1})}$ and $H_1(S_t) = \frac{1}{2}$. This result can be obtained by examining equations (6) to (24). In fact, the constant price-dividend ratio is consistent with the simple Gordon rule: $P_t = \frac{E(\theta_{t+1})D_t}{R_t - E(\theta_{t+1})}$. Intuitively, the potential buyers of stocks are those age-1

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14If investors sell winners due to portfolio rebalancing, then they will partially reduce their position in a winning stock, rather than sell the entire position of the stock. Odean (1998) shows that the disposition effect still remains strong, even when the sample is restricted to transactions of investors’ entire holdings of a stock, i.e., to those transactions not motivated by portfolio rebalancing. This suggests that portfolio rebalancing cannot entirely account for the disposition effect.
and age-2 investors who hold optimistic views about next period’s dividend growth rate; the marginal buyer’s subjective belief, coinciding with the true distribution of the dividend process, brings the stock price equal to the sum of the discounted expected dividends.15

In this special case, we have an i.i.d. return process,

\[ R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{f + 1}{f} \theta_{t+1}, \]

with mean equal to \( R_f \); thus, there is no price momentum in stock returns. The age-2-1 investors do not exhibit a disposition effect, because half of them, those who have received pessimistic belief shocks (i.e., \( q_{t,t} < 1/2 \)), will always liquidate stocks whether they face gains or losses. Then, by equations (12) and (13) and the fact that \( z_t = \frac{1}{2} \), the trading volume is constant over time,

\[ Q_t = \frac{3}{4}. \]

This result makes sense: before the stock market opens, half of the stocks are held by age-3 investors and will be sold; the other half of the stocks are held by age-2-1 investors and only half of those stocks will be sold; thus, \( \frac{3}{4} \) of the stocks will change hands in total in each period. As a result of the constant trading volume, there is no correlation between returns and volumes.

In sum, there is no disposition effect, no price momentum, and no return-volume correlation in the risk-neutral economy, and we summarize our results in the following proposition.

**Proposition 1** [Risk-Neutral Economy] Suppose \( \alpha = \lambda = 1 \). An age-2-1 investor liquidates his stock if and only if his belief \( q_{t,t} \) is smaller than \( \frac{1}{2} \). The stock return is i.i.d. over time, and it is given by \( R_{t+1} = \frac{R_t}{E(\theta_{t+1})} \theta_{t+1} \). The trading volume is equal to \( \frac{3}{4} \) in each period.

The analysis in this benchmark economy also ensures that our results in the following subsections do not stem from heterogeneous beliefs — heterogeneous beliefs also exist in the

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15 We deliberately set the marginal buyer’s belief as the true distribution to remove the “over-valuation” effect emphasized in models with heterogeneous beliefs and short-sales constraints (e.g., Miller, 1977; Harrison and Kreps, 1978; Chen, Hong and Stein, 2002; Scheinkman and Xiong, 2003). In those models, the marginal investor holds a more optimistic view than the true distribution, and thus, the equilibrium price is biased upward.
risk-neutral economy — rather, they stem from prospect theory preferences. In the following three subsections, we consider economies with prospect theory investors, i.e., economies where either $\alpha \neq 1$ or $\lambda \neq 1$, or both. For these economies, we must numerically compute the equilibrium, and the numerical computation requires us to calibrate parameter values.

**Calibrating Technology Parameters** There are five exogenous parameters in our model: two preference parameters ($\lambda$ and $\alpha$) and three technology parameters ($\theta_H$, $\theta_L$ and $R_f$). Because we are interested in the implications of preferences, we allow the preference parameters to vary over a certain range. However, we calibrate the technology parameters as follows. We take one period to be one year and thus set the net risk-free rate to $R_f - 1 = 3.86$ percent, a choice adopted by Barberis and Huang (2001). Because the disposition effect refers to the behavior of individual stocks, we choose dividend parameters to match the mean and standard deviation of the dividend growth rate of a typical individual stock. Barberis and Huang (2001) estimate the moments of individual stock dividend growth using the COMPUSTAT database, and based on their results, we set $\theta_H = 1.28$ and $\theta_L = 0.76$, such that the mean and volatility of the net growth rate of the dividend are 2.24% and 25.97%, respectively. Table I summarizes our choice of technology parameters.

**Table I About Here**

**B. Implications of Diminishing Sensitivity**

We obtain the implications of diminishing sensitivity through comparative static analysis with respect to parameter $\alpha$, which governs the curvature of the value function. The lower the value of $\alpha$, the more curved the value function. To ensure that our results are completely driven by the concavity/convexity component of prospect theory, in this subsection we also set parameter $\lambda$ at 1 to remove the loss aversion feature of the preference. Table II presents the main results for a range of values of $\alpha$: 0.2, 0.5, 0.88 and 1. In particular, when $\alpha = 1$, the investor is risk-neutral, providing a benchmark for highlighting the fact that our results stem from prospect theory preferences. The value of 0.88 is the number estimated by Tversky and Kahneman (1992). Our results demonstrate that, in a general equilibrium
setting, diminishing sensitivity drives the disposition effect, the momentum effect and the return-volume co-movement. We also find that diminishing sensitivity alone, in the absence of loss aversion, raises equity premiums.

Table II About Here

Disposition Effects We use the following measure to test whether our model can generate a disposition effect:

\[
\text{DispEffect} = \frac{E \left[ \frac{z_t - H_2 (S_t)}{z_t} \right] \left| G_{t-1}^t > 0 \right]}{E \left[ \frac{z_t - H_2 (S_t)}{z_t} \right] \left| G_{t-1}^t < 0 \right]}. \tag{25}
\]

If \(\text{DispEffect} > 1\), then we conclude that investors exhibit the disposition effect in our model. The numerator of \(\text{DispEffect}\) is the average fraction of age-2-1 investors who close their positions facing a capital gain. This term is the theoretical analog to Odean’s (1998) “proportion of gains realized” (PGR), i.e., the number of gains that are realized as a fraction of the total number of gains that could have been realized. Similarly, the denominator of \(\text{DispEffect}\) is the average fraction of age-2-1 investors who realize losses and corresponds to Odean’s “proportion of losses realized” (PLR). Odean uses the difference between PGR and PLR to measure the disposition effect. Because decreasing the value of \(\alpha\) will increase the equity premium, in equation (25), we instead adopt a ratio of PGR to PLR to remove the effect of equity premiums on the magnitudes of PGR or PLR.\(^{16}\)

To obtain the two conditional moments in equation (25), we simulate a long time series \(\{\theta_t\}_{t=1}^\infty\) of 500,000 independent draws from the distribution described in equation (1). Then we use the solved functions \(f(\cdot)\) and \(H_1(\cdot)\) to calculate \(f_t\) and \(z_{t+1}\) and obtain the time series \(\{S_t\}_{t=1}^\infty\). When we do this, we also compute \(\{H_2 (S_t)\}_{t=1}^\infty\) and \(\{G_{t-1}^t\}_{t=1}^\infty\) using equations (12) and (8). We compute sample moments from these simulated data to serve as approximations of population moments.

Table II reports the results for different values of \(\alpha\). The case of \(\alpha = 1\) corresponds to a linear value function, when investors are risk-neutral; these risk-neutral investors do

\(^{16}\)Brown et al. (2006) also use the ratio of PGR to PLR to measure the disposition effect when examining Australian stock trading data.
not exhibit a disposition effect, so that \( DispEffect = 1 \). As we gradually decrease \( \alpha \) from 1 to 0.2, the value function becomes more curved and the value of \( DispEffect \) increases monotonically from 1 to 1.73, giving rise to an even stronger disposition effect. The mechanism behind this result is exactly Odean’s (1998) intuition: risk aversion (risk-seeking) for gains (losses) causes an age-2-1 investor more (less) likely to sell the stock.

Figure 2 graphs this intuition for the case of \( \alpha = 0.5 \). Here, from the simulated time series of state vectors, we randomly choose a realization of \((f_{t-1}, z_t) = (20.01, 0.50)\) and then graph the possible gains/losses together with the associated prospect theory utilities faced by an age-2-1 investor in periods \( t \) and \( t + 1 \). The period \( t \) gains/losses, as well as prospect theory utilities from liquidating the stock [i.e., \((G^t_{1 \rightarrow 0}, v(G^t_{1 \rightarrow 0}))\)], are marked with dots, while the period \( t + 1 \) gains/losses and prospect theory utilities from keeping the stock [i.e., \((G^{t+1}_{1 \rightarrow 1}, v(G^{t+1}_{1 \rightarrow 1}))\)] are marked with circles.

**Figure 2 About Here**

Good dividend news \((\theta_t = \theta_H)\) will bring an age-2-1 investor to the point of choosing a sure medium gain \((6.5, \text{Point Hi in the figure})\) versus a gamble that offers either a smaller \((1.18, \text{Point HL})\) or a larger gain \((14.45, \text{Point HH})\) with some probability. Whether an age-2-1 investor will continue to hold the stock depends on his one-period-ahead dividend forecast. In this example, those age-2-1 investors who believe with probability higher than 0.54 (i.e., \(\frac{v(6.5) - v(1.18)}{v(14.45) - v(1.18)}\)) that the next period dividend growth rate \((\theta_{t+1})\) will take a high value \((\theta_H)\) will continue to hold the risky stock.

What will happen if the dividend news is negative \((\theta_t = \theta_L)\) in period \( t \)? If an age-2-1 investor sells the stock, he experiences a sure loss \((-4.23, \text{Point L})\); if he continues to hold the stock, he faces the gamble of a smaller loss \((-0.28, \text{Point LH})\) or an even larger loss \((-8.18, \text{Point LL})\). In this example, those age-2-1 investors who believe \(\theta_{t+1} = \theta_H\) with probability lower than 0.35 (i.e., \(\frac{v(-4.23) - v(-8.18)}{v(-0.28) - v(-8.18)}\)) will liquidate their stocks. Note that the cutoff probability in the low dividend realization case, 0.35, is lower than that in the high dividend realization case, 0.54. This precisely supports the informal argument, which relies

\[\text{17} \text{The result is robust to the choice of } (f_{t-1}, z_t).\]
on prospect theory to explain the disposition effect: “the investor’s belief about expected return must fall further to motivate the sale of a stock that has already declined than one that has appreciated” (Odean, 1998, p. 1777).

Table II suggests that PGR and PLR respond to a change in $\alpha$ differently: as $\alpha$ falls from 1 to 0.2, PGR first increases from 0.50 to 0.56 and then decreases to 0.49, while PLR consistently decreases from 0.50 to 0.29. There are two forces at work here. As $\alpha$ becomes smaller, the value function is more concave for gains and more convex for losses, causing the investor to be more likely to sell winners and hold losers and hence generating a higher PGR and a lower PLR. However, as $\alpha$ falls, the expected stock return rises and the stock becomes more attractive to the investor, which will be discussed shortly in this subsection; this second force decreases the investor’s propensity to sell the stock no matter whether he is facing gains or losses and therefore leads to both a lower PGR and a lower PLR. In sum, as $\alpha$ decreases, both forces tend to lower PLR, while the first force tends to raise PGR and the second to lower PGR. As $\alpha$ falls slightly below 1, the first force dominates and we observe a higher PGR, but once $\alpha$ falls sufficiently, the second force catches up and we obtain a lower PGR.

**Momentum**  Following Barberis et al. (1998), who rely on a model with one risky asset to explain the cross-section of stock returns, we measure momentum as

$$MomEffect = E(R_{t+1}|\theta_t = \theta_H) - E(R_{t+1}|\theta_t = \theta_L),$$

(26)
i.e., the difference in the expected return following a positive shock and following a negative shock. If $MomEffect > 0$, then we claim that there is momentum in the stock returns. The two moments in equation (26) are obtained using simulations. The results are also reported in Table II. Since $MomEffect > 0$ for $\alpha < 1$, our model shows that the concavity/convexity feature of prospect theory preferences generates momentum in stock returns. Moreover, the momentum effect becomes stronger as we increase the curvature of the value function, i.e., as we decrease the value of $\alpha$. For example, $MomEffect$ increases from 1.06% to 11.37% as $\alpha$ decreases from 0.88 to 0.2.
The underlying reason for this momentum effect is simple. Following a positive shock ($\theta_t = \theta_H$), stock prices will rise, moving age-2-1 investors into their capital gain domain. Due to the concavity of the value function of prospect theory in the gain region, age-2-1 investors tend to close their stock positions, which depresses the stock price, generating higher subsequent returns. In mathematical terms, good dividend news (high $\theta_t$) has two effects: first, it generates a high current stock return $R_t$, because $R_t = \theta_t (1 + f_t) / f_{t-1}$; second, via the disposition effect, it generates a low price-dividend ratio $f_t$, which, because $E_t (R_{t+1}) = E_t [\theta_{t+1} (f_{t+1} + 1)] / f_t$, implies a high expected next period return $E_t (R_{t+1})$. As a result, a high current return $R_t$ is expected to be followed by a high average next period return $E_t (R_{t+1})$. The same logic applies to bad dividend news: a negative shock ($\theta_t = \theta_L$) will decrease the stock price, driving age-2-1 investors into their capital loss domain; convexity in the region of losses means that they are less likely to sell the stock absent a price premium; the stock price is therefore initially inflated, generating lower subsequent returns.

We also conduct a cross-section analysis and replicate the momentum effect in the empirical literature (e.g., Jegadeesh and Titman, 1993; Liu and Zhang, 2008). As discussed at the end of Section II, we can extend our model to an economy with $N$ stocks. We simulate dividend data on $N = 2,000$ independent stocks over $T = 10,000$ time periods and then compute the resulting equilibrium return sequence for each stock. We create the “winners-minus-losers” zero cost portfolios as follows. In each period, we sort stocks into two equal-sized groups based on their last period returns and record the equal-weighted return of each group over the next period; in particular, $R_{t, \text{winner}}$ ($R_{t, \text{loser}}$) is the return on the portfolio containing stocks with better (worse) performance. Repeating this each period produces long time series of returns on the winner and loser portfolios, namely $\{R_{t, \text{winner}}\}_{t=1}^T$ and $\{R_{t, \text{loser}}\}_{t=1}^T$. Our second measure of momentum is the difference in the average returns on these two portfolios:

$$WML = \frac{1}{T} \sum_{t=1}^{T} (R_{t, \text{winner}} - R_{t, \text{loser}}).$$

Table II reports the results for this alternative measure. We find that the two measures for momentum are almost identical, so that they behave in precisely the same way: both
MomEffect and WML are greater than 0 for $\alpha < 1$, and both decrease with $\alpha$.

**Turnover**  Empirical studies show that there is more trading in rising markets than in falling markets (Karpoff, 1987; Statman *et al.*, 2006; Hong and Stein, 2007; Griffin *et al.*, 2007). In our model, age-2-1 investors have a much greater propensity to sell stocks facing good news ($\theta_t = \theta_H$) than facing bad news ($\theta_t = \theta_L$). This will contribute to a positive correlation between turnover and stock returns. In Table II, we report the simulated correlations between stock returns $R_t$ and turnovers $Q_t$: $Corr(R_t, Q_t)$.\(^{18}\) Indeed, we have $Corr(R_t, Q_t) > 0$ so long as $\alpha < 1$. This demonstrates that diminishing sensitivity drives a positive correlation between returns and volume.

As we gradually decrease $\alpha$ from 0.88 to 0.2, $Corr(R_t, Q_t)$ decreases from 0.92 to 0.52. The reason for this pattern is as follows. When $\alpha$ is close to 1, both the stock distributions ($z_t$) and price-dividend ratios ($f_{t-1}$) are almost constant at their values in the benchmark economy (i.e., $\alpha = 1$), so that the state of the economy is captured only by dividend growth rates ($\theta_t$). Since the disposition effect causes returns and turnovers to vary with $\theta_t$ in the same direction, there is an almost perfect correlation between returns and volume. On the other hand, as $\alpha$ approaches 0, both $z_t$ and $f_{t-1}$ will change over time and influence trading behavior. However, returns and volumes respond to the variation in $z_t$ and $f_{t-1}$ in opposite ways. For example, a larger $z_t$ implies that more stocks are held by age-2-1 investors and fewer by age-3 investors; after trading, all age-3 investors will have to close their positions, even though this is not the case for age-2-1 investors; as a result, stock selling (i.e., trading volume $Q_t$) will decrease with $z_t$ but, at the same time, the decreasing selling pressure causes stock returns $R_t$ to rise with $z_t$. The variation in $z_t$ and $f_{t-1}$ will therefore attenuate the positive correlation between returns and volume generated by $\theta_t$. As a result, for $0 < \alpha < 1$, a lower $\alpha$ implies a lower $Corr(R_t, Q_t)$.

\(^{18}\) Although Statman *et al.* (2006) and Griffin *et al.* (2007) suggest that the disposition effect can explain a positive relationship between turnover and past returns, we examine the contemporaneous return-volume correlation for two reasons. First, their data are at higher frequencies than our target data, and after aggregation, a high frequency positive relationship between turnover and past returns implies a low frequency positive relationship between turnover and contemporaneous returns. Second, many studies, e.g., the surveys by Karpoff (1987) and Hong and Stein (2007), document a positive contemporaneous return-volume relationship.
Equity Premiums Our model also demonstrates that the S-shaped value function of prospect theory can help explain the equity premium puzzle. Table II reports the simulated equity premiums, $\mathbb{E} (R_t - R_f)$, as well as average stock purchases by young people, $\mathbb{E} [H_1 (S_t)]$. As $\alpha$ becomes smaller, the curvature of the value function becomes larger, and equity premiums become higher. Note that the positive equity premium is not due to loss aversion, since we have set $\lambda = 1$ in this subsection. Notably, a low $\alpha$ is also associated with a low $\mathbb{E} [H_1 (S_t)]$, suggesting that equity premiums are driven by the behavior of young people.

The young investor makes investment decisions by comparing the expected utility from buying a stock to that from not buying. These utility levels are determined by their belief $q_{i,t}$ (current belief about $\theta_{t+1}$) as well as by how they evaluate their future reactions to $q_{i,t+1}$ (future belief about $\theta_{t+2}$). Those who are extremely optimistic (pessimistic), i.e., those with extremely high (low) values of $q_{i,t}$, always buy (not buy) the stock. It is the young investor who have intermediate values of $q_{i,t}$ who care most about their future reactions to $q_{i,t+1}$. As it turns out, only high realizations of $q_{i,t+1}$ matter because there is no extra benefit from holding a stock from middle- till old-age when $q_{i,t+1}$ is low. Only when $q_{i,t+1}$ is high will holding the stock bring a perceived extra benefit; that is, a young investor who buys a stock now will enjoy a further gain by keeping the stock, and one who does not buy now will enjoy a new gain by buying the stock in middle-age.

How does this extra gain associated with high $q_{i,t+1}$ relate to the current purchasing decision and the value function’s curvature $\alpha$? Not buying now means that when evaluating this gain, the young investor will stay at the origin of the value function where the marginal utility is highest; the more curved the value function, the higher this marginal utility. In contrast, if he buys now, he will be pushed away from the origin because this gain has to be appended to an existing gain or loss, namely the gain or loss generated by holding the stock from youth until middle-age. In this case, the marginal utility is much smaller than that at the origin; the more curved the value function, the smaller the marginal utility.

To summarize, for young investors, waiting till middle-aged to buy a stock has some option value induced by their time-varying beliefs $q_{i,t+1}$. The higher the curvature of the value function, the less young investors value the potential gain associated with high realizations
of $q_{t,t+1}$ and the less they want to buy now, thereby depressing stock prices and raising equity premiums.

C. Implications of Loss Aversion

To obtain the implications of loss aversion, we conduct comparative static analysis with respect to the parameter $\lambda$. In Table III, we present the results for a variety of values of $\lambda$: 1, 2.25, 3 and 4. In particular, $\lambda = 1$ is still our benchmark economy when the investor is risk-neutral. The value of $\lambda = 2.25$ is the number estimated by Tversky and Kahneman (1992). To guarantee that our results are solely due to the loss aversion component, we always set parameter $\alpha = 1$ to remove the curvature feature of the prospect theory value function. Table III demonstrates that loss aversion drives a reversed disposition effect and reversal in the cross-section of stock returns, as well as a negative return-volume correlation. In addition, Table III produces a well-known result in the asset pricing literature: loss aversion raises equity premiums (e.g., Benartzi and Thaler. 1995; Barberis et al., 2001).

Table III  About Here

Reversed Disposition Effects  Again, when investors are risk-neutral, i.e., when $\lambda = 1$, they do not exhibit a disposition effect, so that $DispEffect = 1$. When investors are loss-averse, i.e., when $\lambda > 1$, we obtain a reversed disposition effect, since $DispEffect < 1$. Moreover, as we gradually increase $\lambda$ from 1 to 4, investors become more loss-averse, and the value of $DispEffect$ decreases monotonically from 1 to 0.83, giving rise to an even stronger reversed disposition effect.

What is the intuition behind this result? The mechanism works through a combination of two forces: one is the kink at the origin of the value function, which is a direct implication of loss aversion; the other is the positive equity premium, which is an indirect equilibrium implication of loss aversion preferences. Roughly speaking, when investors are close to (far from) the kink, they are reluctant (inclined) to take risk, and want to sell (keep) the stock; when the average stock returns are higher than the risk-free rate, bad (good) dividend news will bring investors relatively close to (far from) the kink, so that they are more (less) likely
to liquidate the stock.

Figure 3 illustrates an exercise that confirms this intuition for the case of $\lambda = 4$. Now that we have assumed $\alpha = 1$ to remove the curvature, the investor's value function becomes piecewise linear with a kink at the origin due to loss aversion. In a similar fashion as in the exercise illustrated in Figure 2, we randomly choose a realization of $(f_{t-1}, z_t)$ ($(7.75, 0.48)$ in this case) from the simulated time series of state vectors. We then graph an age-2-1 investor’s period $t$ gains/losses as well as prospect theory utilities from liquidating the stock [i.e., $(G^t_{1\rightarrow 0}, v(G^t_{1\rightarrow 0}))$] with dots and the period $t+1$ gains/losses and prospect theory utilities from keeping the stock [i.e., $(G^{t+1}_{1\rightarrow 1}, v(G^{t+1}_{1\rightarrow 1}))$] with circles.

Good dividend news ($\theta_t = \theta_H$) will bring the investor to Point H. Bad dividend news ($\theta_t = \theta_L$) will bring him to Point L, which is closer to the kink than is Point H. That is, the investor is more cautious in holding stocks at Point L than at Point H. Specifically, at Point H, if the investor liquidates the stock, he will lock in a medium gain of 3.29; if he keeps the stock, when he becomes old he will arrive either at Point HH, enjoying a large gain of 7.35, or at Point HL, enjoying a small gain of 1.46. Since both Point HH and Point HL are in the gain domain, the investor’s behavior at Point H can be described as risk-neutral. Of course, whether an age-2-1 investor will indeed continue to hold the stock depends on his one-period-ahead dividend forecast. In this example, those age-2-1 investors who believe that $\theta_{t+1} = \theta_H$ with probability higher than 0.31 (i.e., $v(3.29) - v(1.46)$) will continue to hold the risky stock.

At Point L, if the investor sells the stock, he will realize a loss of 1.48. If he keeps the stock, then he will arrive either at Point LH, enjoying a small gain of 1.01, or at Point LL, facing a large loss of 2.52. Because Points LH and LL straddle the kink, the investor is reluctant to take a risk at Point L relative to Point H where his behavior resembles risk neutrality. In this example, age-2-1 investors who believe that $\theta_{t+1} = \theta_H$ with probability lower than 0.37 (i.e., $v(-1.48) - v(-2.52)$) will liquidate their stocks.

In Table III, we also observe that both PGR and PLR decrease with $\lambda$. This is because loss aversion raises equity premiums, making the investor less likely to sell stocks whether
facing good news or bad news. We also observe that PGR decreases at a faster rate than PLR due to the reversed disposition effect.

**Reversal** As discussed above, when \( \lambda = 1 \) the investor is risk-neutral and there is neither momentum nor reversal in the cross-section of stock returns because both measures that capture momentum, \( \text{MomEffect} \) and \( \text{WML} \), are equal to zero. But as long as \( \lambda > 1 \), i.e., as long as the investor is loss-averse, we obtain reversal in the cross-section of stock returns because both \( \text{MomEffect} \) and \( \text{WML} \) are negative. In particular, as we increase \( \lambda \) from 1 to 4, reversal becomes stronger. This result demonstrates that the loss aversion feature of prospect theory has implications for return predictability.

The underlying reason for this result is as follows. When facing good dividend news, age-2-1 investors are more likely to hold stocks according to the reversed disposition effect. This generates extra buying pressure, which will inflate stock prices and lead to lower stock returns later. Similarly, facing bad dividend news, those investors are likely to sell stocks and depress prices, generating higher subsequent returns.

**Turnover** Table III also shows that loss aversion can generate a negative correlation between returns and volumes: \( \text{Corr} (R_t, Q_t) < 0 \) as long as \( \lambda > 1 \). This result is also driven by trading by age-2-1 investors, who, due to the reversed disposition effect, have a much greater propensity to sell stocks in down markets (\( \theta_t = \theta_L \)) than in up markets (\( \theta_t = \theta_H \)), contributing to a negative correlation between turnover and stock returns.

As we gradually increase \( \lambda \) from 1 to 4, \( \text{Corr} (R_t, Q_t) \) monotonically decreases from 0 to \(-0.94\). This pattern is different from the relationship between \( \text{Corr} (R_t, Q_t) \) and \( \alpha \) in Table II and can be understood as follows. In Table II, when we vary \( \alpha \) while fixing \( \lambda \), dividend news \( \theta_t \) contributes to a positive \( \text{Corr} (R_t, Q_t) \) via the disposition effect, while the other endogenous state variables, stock distributions (\( z_t \)) and price-dividend ratios (\( f_{t-1} \)), tend to generate a negative \( \text{Corr} (R_t, Q_t) \). These two forces counteract each other. By contrast, in Table III, when we vary \( \lambda \) and fix \( \alpha \), dividend news \( \theta_t \) also leads to a negative \( \text{Corr} (R_t, Q_t) \) through the reversed disposition effect, which strengthens the impact of the two endogenous state variables on \( \text{Corr} (R_t, Q_t) \).
Equity Premiums  Table III also reproduces the well-known result that loss aversion raises equity premiums (e.g., Benartzi and Thaler, 1995; Barberis et al., 2001). As we increase $\lambda$ from 1 to 4, equity premiums rise from 0 to 12%. This result is intuitive: loss aversion means that investors are more sensitive to losses than to gains, and since stocks often perform poorly and investors often face losses, a large premium is required to convince them to hold stocks. The asset pricing literature studying loss aversion has focused primarily on its implications for the equity premium, that is, the average level of stock returns. Our model, on the other hand, shows that loss aversion can lead to reversal in the cross-section of stock returns, suggesting, in turn, that loss aversion may also be a useful ingredient for equilibrium models trying to understand return predictability.

D. Quantitative Analysis: How Successful is Prospect Theory in Explaining Data?

So far, we have shown that two counteracting forces, diminishing sensitivity and loss aversion, influence the disposition effect, the momentum effect and the return-volume correlation. In order to understand the degree of success prospect theory can have in explaining real data, we set preference parameters at certain empirical values and examine which force will dominate and to what extent.

What are the empirical values of the preference parameters $\lambda$ and $\alpha$? Most empirical studies estimate $\lambda$ to be close to 2, either from experimental (e.g., Kahneman et al., 1990; Tversky and Kahneman, 1991, 1992; Novemsky and Kahneman, 2005) or from real data (e.g., Putler, 1992; Hardie et al., 1993; Levy, 2009). In the following analysis we therefore fix $\lambda$ at 2.25, the value estimated by Tversky and Kahneman (1992).

There is, however, not much evidence as to the value of $\alpha$. As far as we know, only two studies have estimated this parameter and the results differ markedly across the data sets used. By offering subjects isolated gambles in experimental settings, Tversky and Kahneman (1992) estimate $\alpha = 0.88$. Using a different experimental data set, Wu and Gonzalez (1996) estimate $\alpha = 0.52$; however, when they apply Camerer and Ho’s (1994) data, they find $\alpha = 0.37$. Due to the small sample size in the experiments, none of those studies can
estimate $\alpha$ with great precision. Therefore, our strategy is to report results for these three possible values of $\alpha$ in Table IV.

**Table IV About Here**

Table IV also presents the historical values for the disposition effect, the momentum effect and the correlation between returns and volumes. Unlike Odean (1998), who studies the disposition effect by aggregating across investors, Dhar and Zhu (2006) examine the disposition effect at the level of the individual. In their Table 2, they report that the means of PGR and PLR for all individuals are 0.38 and 0.17, respectively. We adopt these numbers as the empirical values of PGR and PLR. For the momentum effect, we use Jegadeesh and Titman’s (1993) estimate, that is, 8.60% on an annual basis. Using AMEX/NYSE data from 1926-2006 from CRSP, we find that the correlation between returns and volumes, $\text{Corr} \left( R_t, Q_t \right)$, and the equity premium, $E \left( R_t - R_f \right)$, for a typical firm are 0.28 and 7.84%, respectively. These historical values help us evaluate how well our model matches the data. One comment should be provided here. Because we are not confident of the actual value of $\alpha$ among real investors, this evaluation should be interpreted with caution. However, this does not weaken the significance of our quantitative analysis; the only way to examine how much prospect theory preference explains data is through a general equilibrium model such as the one provided here. The present work thus makes a methodological contribution. With such a model at hand, all that is required are more accurate estimates of the parameters of the prospect theory preferences.

Table IV demonstrates that, for all three possible values of $\alpha$, the diminishing sensitivity component of prospect theory dominates the loss aversion component. This is not surprising, given that in Table II, even when $\alpha = 0.88$ (the value function is slightly curved), the price impact of diminishing sensitivity as measured by $\text{MomEffect}$, is greater than that of loss aversion when $\lambda = 2.25$ in Table III. In Table IV, the case of $\alpha = 0.37$ is of particular interest, because in this case our model closely matches the historical data along the di-

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19 More precisely, we take all stocks in the CRSP database for which at least 11 consecutive years of return and turnover data are recorded, compute the correlation between real returns and volume as well as the mean returns in excess of the 30-day T-bill rate for each and then calculate the medians.
mension of the disposition effect, the momentum effect and the equity premium; it predicts that $DispEffect = 2.25$, $WML = 7.20\%$ and $E(R_t - R_f) = 8.14\%$, while the historical counterparts for these variables are 2.24, 8.60\% and 7.84\%, respectively. However, the model predicts too high a correlation between returns and volumes, i.e., $Corr(R_t, Q_t) = 0.84$, while the empirical value is 0.28.

E. Discussion

Testable Predictions  Table IV suggests that if prospect theory predicts a momentum effect at all, it also simultaneously predicts a positive return-volume correlation. Thus, we expect the momentum effect to be stronger among those stocks whose returns are positively correlated with their own trading volumes. We can rely on this prediction to differentiate our model from other explanations of price momentum such as the belief-based models proposed by Barberis et al. (1998), Daniel et al. (1998) and Hong and Stein (1999). Note that our prediction is different from that of Lee and Swaminathan (2000), who show that price momentum is more pronounced among stocks with higher levels of trading volumes, while our prediction relates momentum to the sensitivity of returns to volumes.

In addition, according to our model, the momentum effect is stronger for economies with higher degrees of diminishing sensitivity and lower degrees of loss aversion. Recent papers (e.g., Chui et al., 2010) have shown that the momentum effect is different in different countries. According to our model, this difference could potentially be driven by differences in the loss aversion and diminishing sensitivity parameters in different countries. To test this prediction, more accurate estimates of both diminishing sensitivity and loss aversion parameters in different countries are required.\footnote{Although studies estimating the loss aversion parameter tend to agree that it should be assigned a value of around 2, they overwhelmingly use data from Western countries. We have not found much evidence on the estimation of this parameter using data from Eastern countries. One such study is that of Liu (2008), who estimates that Chinese people have a loss aversion coefficient of 3.47. This suggests that investors in Eastern countries might be more loss-averse than investors in Western countries, which, according to our model, generates the weaker momentum effect in Eastern countries, as documented in the literature.}

Robustness Checks  In Appendix B, we analyze the sensitivity of our results to modeling assumptions regarding decision intervals, reference points, technology parameters and binary
holdings and find that our main results — diminishing sensitivity drives the disposition effect, price momentum, and the positive return-volume relationship, while loss aversion predicts the opposite — are robust to alternative assumptions.

Our analysis has so far specifically assumed that one period is one year; Appendix B1 analyzes the effect of changing this assumption by assuming the decision interval of an investor to be six months. As described in Section II, we suppose that investor \( i \) uses \( R^2_j W_{1,i} \) as a reference level of wealth when calculating gains and losses. Odean (1998) and Genesove and Mayer (2001) assume that the investor uses the original purchase price as a reference point. Appendix B2 examines whether our results hold under this alternative assumption regarding reference points. Appendix B3 examines the effect of varying the volatility of the dividend growth rate. Because in our current model investors can only hold 0 or 1 unit, we do not allow them to carry out partial liquidation. To account for intermediate levels of demand, we modify our model by allowing investors to hold 0, 0.5, or 1 share of a given stock and report the results in Appendix B4.

**Heterogeneity, Aggregation and Price Impacts**

In our model, all investors have prospect theory preferences, the preference parameters (\( \alpha \) and \( \lambda \)) are the same across investors, and the disposition investors (age-2-1 investors) frame gains/losses in the same way. In reality, investors are likely to be heterogeneous in a variety of ways. First, some investors might be better described by traditional, risk-averse expected utility preferences, for example, the standard power utility representation, and these investors might take advantage of prospect theory investors and negate their effects on prices. Second, even prospect theory investors are likely to differ in many dimensions, and this heterogeneity might cause the effects of their aggregate behaviors to cancel out. Recognition of these heterogeneities raises the question of whether the results of our model still hold in this more realistic world.

A full analysis of this issue poses significant technical hurdles, but there is good reason to believe that a more general model can deliver similar results. On the one hand, as pointed out in the literature dealing with limits to arbitrage, there might be limits to the ability and willingness of traditional expected utility maximizers, or arbitrageurs, to offset the
pricing effects of prospect theory investors because by exploiting prospect theory investors, arbitrageurs face fundamental risk, as well as noise-trader risk, over and above the significant implementation costs they have to bear. As a result, arbitrageurs will trade cautiously and only partially absorb the impact on prices of prospect theory investors, thereby allowing our results to persist.

On the other hand, even though prospect theory investors might be heterogeneous in many ways, their disposition-related trading is likely to be systematic and to have implications for stock prices. For example, empirical evidence documents that both institutional investors and individual investors exhibit a disposition effect, although the former do so to a smaller extent. For example, Grinblatt and Keloharju (2001), Shapira and Veneezia (2001), Werners (2003), Garvey and Murphy (2004), Coval and Shumway (2005), Locke and Mann (2005), Frazzini (2006) and Scherbina and Jin (2006) document the disposition effect in the trading of professionals who trade on behalf of their firms. This suggests that prospect theory can indeed capture the preferences of both types of investors, individual investors and institutional investors, albeit differently. Formally, we can model their preferences as prospect theory utility with different parameters ($\alpha$ and $\lambda$) or as a combination of consumption utility and prospect theory utility with different weights. There is no reason to believe that this kind of heterogeneity will “wash out” in the aggregate.

Another heterogeneity of prospect theory relates to the framing of gains/losses by investors. One might argue that because different investors buy into stocks at different prices and thus, some face gains and others face losses in a given period, their disposition-related tradings cancel out in aggregate. However, this argument is flawed because it ignores the updating of reference points. When an investor has held a stock over many periods, it is most reasonable for him to think of the reference point as some weighted average of the purchase price and other former prices. Once this updating process is taken into account, in a rising (falling) market, most investors holding the stock will accumulate gains (losses) regardless of when they bought into the stock or at what price, making their disposition-related trading systematic. This idea can be formalized in a setup with more than three generations.

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It will, however, exponentially increase the dimension of state vector, making the problem intractable.

V. Conclusion

To the best of our knowledge, this paper is the first to comprehensively examine the implications of prospect theory for individual trading behavior, asset prices and trading volume in a dynamic setting. Through the disposition effect, we establish the direct link between prospect theory and salient asset pricing features such as momentum and the co-movement between stock returns and volume. By providing both qualitative and quantitative results, we aim to deepen our understanding of the implications of prospect theory for financial markets.

Our general equilibrium model offers the advantage of isolating the specific roles played by different components of prospect theory preferences in explaining asset-pricing behaviors: the diminishing sensitivity component drives the disposition effect, which leads to momentum and a positive correlation between returns and volume, whereas the loss aversion component drives the reversed disposition effect, which leads to reversal and a negative correlation between returns and volume. In a calibrated economy, when prospect theory preference parameters are set at the values estimated in previous studies, our model can generate price momentum ranging from 1% to 7% on an annual basis.
References


Appendices (Not for Publication)

Appendix A describes the algorithm we adopted in computing the equilibrium for economies with prospect theory investors. Appendix B analyzes the sensitivity of our results to modeling assumptions regarding decision intervals, reference points, technology parameters, and binary holdings.

Appendix A. Numerical Algorithm

The basic idea of computing the equilibrium is to form a system of two unknown functions, \( f(\cdot) \) and \( H_1(\cdot) \). The steps are summarized as follows.

**Step 0:** Solve the price-dividend ratio for the risk-neutral economy: 
\[
\frac{f}{E(\theta_{t+1})} = \frac{E(\theta_{t+1})}{E(\theta_{t+1})}. 
\]
The value of \( f \) serves two purposes: first, it gives a rough idea of the range of the endogenous state variable \( f_{t-1} \); second, it provides a natural candidate for the initial conjecture of the price-dividend ratio function \( f^{(0)}(\cdot) \) when \( \alpha \) and \( \lambda \) are close to 1.

**Step 1:** Define a finite grid on \([0,1] \) for the state variable \( z_t \) and a finite grid on \([\gamma_1 f, \gamma_2 f] \) for the state variable \( f_{t-1} \), where \( 0 < \gamma_1 < 1 < \gamma_2 \) are two constants. This step gives a finite grid of \( S_t = (\theta_t, f_{t-1}, z_t) \) on \( \{\theta_H, \theta_L\} \times [\gamma_1 f, \gamma_2 f] \times [0,1] \).

**Step 2:** Choose \( f^{(0)}(\cdot) = f \) and \( H_1^{(0)}(\cdot) = \frac{1}{2} \) as initials for the price-dividend ratio function and the age-1 investors’ aggregate demand function. Then, on each grid point \( S_t \), follow steps 3-8 to form and solve a system of two equations of \( f_t \) and \( z_{t+1} \).

**Step 3:** Obtain \( H_{21}(S_t) \) in equation (24). Use \( f^{(u)}(\cdot) \) and equation (6) to calculate \( G_{1_{t-1}}^{t+1} \) as a function of \( \theta_{t+1}, (f_{t-1}, \theta_t) \) and \( (f_t, z_{t+1}) \):
\[
G_{1_{t-1}}^{t+1} = G_{1_{t-1}}(\theta_{t+1}, \theta_t, f_{t-1}; f_t, z_{t+1}) = \left[ f^{(u)}(\theta_{t+1}, f_t, z_{t+1}) + 1 \right] \theta_t \theta_{t+1} + R_f \theta_t - R_f^2 f_{t-1}. 
\]

Here, we intentionally do not express \( f_t \) and \( z_{t+1} \) as functions of \( S_t \), because we want to solve them later on and assign the solved values of \( f_t \) and \( z_{t+1} \) to the next round \( f(S_t) \) and \( H_1(S_t) \). To obtain \( E_t[v(G_{1_{t-1}}^{t+1})] \), which is useful for calculating age-2-1 investors’ individual demands and indirect value functions, plug the above expression of \( G_{1_{t-1}}^{t+1} \) into the prospect theory value function \( v(\cdot) \) and take expectation with respect to \( \theta_{t+1} \). Use equa-
tions (8) and (9) to obtain \( v(G_{1 \rightarrow 0}^t) \). Finally, use equations (10) and (12) to calculate \( H_{21}(S_t) = H_{21}(\theta_t, f_{t-1}, z_t; f_t, z_{t+1}) \), where again, to stress the fact that we have not replaced \( f_t \) and \( z_{t+1} \) with \( f^{(n)}(S_t) \) and \( H_1^{(n)}(S_t) \), we have separated the argument of \( (f_t, z_{t+1}) \) with a semicolon. We can also calculate the indirect value function of an age-2-1 investor, \( \hat{V}_{21}(\theta_t, f_{t-1}, q_{i,t}; f_t, z_{t+1}) \), which will be useful for calculating age-1 investors’ decisions.

**Step 4:** Obtain \( H_{20}(S_t) \) in equation (24). Use \( f^{(n)}(\cdot) \) and equation (14) to calculate \( G_{0 \rightarrow 1}^{t+1} \) as a function of \( \theta_{t+1}, \theta_t \) and \( (f_t, z_{t+1}) \):

\[
G_{0 \rightarrow 1}^{t+1} = G_{0 \rightarrow 1}(\theta_{t+1}, \theta_t; f_t, z_{t+1}) = [f^{(n)}(\theta_{t+1}, f_t, z_{t+1}) + 1] \theta_t \theta_{t+1} - R_f f_t \theta_t.
\]

Then, use equations (15), (16) and (18) to calculate \( H_{20}(S_t) = H_{20}(\theta_t, z_t; f_t, z_{t+1}) \). By equation (17), we obtain the indirect value function of an age-2-0 investor: \( \hat{V}_{20}(\theta_t, q_{i,t}; f_t, z_{t+1}) \).

**Step 5:** Use the market clearing condition, equation (24), to form the first equation:

\[
z_{t+1} + H_{21}(\theta_t, f_{t-1}, z_t; f_t, z_{t+1}) + H_{20}(\theta_t, z_t; f_t, z_{t+1}) = 1.
\]

**Step 6:** Calculate \( H_1(S_t) \) from equation (22). By the indirect value function obtained in step 3, functions \( f^{(n)}(\cdot) \) and \( H_1^{(n)}(\cdot) \), and equation (19), we can calculate the expected benefit of purchasing a stock:

\[
\hat{U}_1(q_{i,t}; f_t, z_{t+1}) = E_t^i \left[ \hat{V}_{21}(\theta_{t+1}, f_t, q_{i,t+1}; f_{t+1}, z_{t+2}) \right] = E_t^i \left[ \hat{V}_{21}(\theta_{t+1}, f_t, q_{i,t+1}; f^{(n)}(\theta_{t+1}, f_t, z_{t+1}), H_1^{(n)}(\theta_{t+1}, f_t, z_{t+1})) \right],
\]

where the expectation is taken over \( \theta_{t+1} \) and \( q_{i,t+1} \). Again, to stress the fact that \( \hat{U}_1 \) is a function of \( (f_t, z_{t+1}) \), we express these terms explicitly. Similarly, using the indirect value function obtained in step 4, functions \( f^{(n)}(\cdot) \) and \( H_1^{(n)}(\cdot) \), and equation (20), we can calculate the expected benefit of not purchasing a stock:

\[
\hat{U}_0(q_{i,t}; f_t, z_{t+1}) = E_t^i \left[ \hat{V}_{20}(\theta_{t+1}, q_{i,t+1}; f_{t+1}, z_{t+2}) \right] = E_t^i \left[ \hat{V}_{20}(\theta_{t+1}, q_{i,t+1}; f^{(n)}(\theta_{t+1}, f_t, z_{t+1}), H_1^{(n)}(\theta_{t+1}, f_t, z_{t+1})) \right].
\]
From expressions of $\hat{U}_1(q_{t,t}; f_t, z_{t+1})$ and $\hat{U}_0(q_{t,t}; f_t, z_{t+1})$ and equations (21) and (22), we can calculate $H_1(S_t) = H_1(f_t, z_{t+1})$.

Step 7: Use the evolution function of the endogenous state variable $z_{t+1}$ to form the second equation:

$$z_{t+1} = H_1(f_t, z_{t+1}).$$

Step 8: Combine the equations in steps 5 and 7 to solve $f_t$ and $z_{t+1}$. Set $f^{(n+1)}(S_t) = f_t$ and $H_1^{(n+1)}(S_t) = z_{t+1}$.

Step 9: Check whether the following stop criterion is satisfied:

$$\max_{S_t} \left\| \left( f^{(n+1)}(S_t), H_1^{(n+1)}(S_t) \right) - \left( f^{(n)}(S_t), H_1^{(n)}(S_t) \right) \right\| < \tau,$$

where $\tau$ is an error tolerance. If yes, then the algorithm terminates. Otherwise, reset the range of $f_{t-1}$ and $z_t$ as $[\min_S f^{(n+1)}(S_t), \max_S f^{(n+1)}(S_t)]$ and $[\min_S H_1^{(n+1)}(S_t), \max_S H_1^{(n+1)}(S_t)]$, increase $n$ by 1 and go to Step 3.

Appendix B. Sensitivity Analysis

Appendix B1. Sensitivity Analysis to Decision Interval

We have mentioned that generations in our model should be understood as generations of trades, so that one period corresponds to six months to one year. So far in our analysis, we have taken one period to be one year. Table B1 analyzes the effect of changing this assumption, by assuming the decision interval of an investor to be six months.

Table B1 About Here

In Table B1, we recalibrate dividend parameters as $\theta_H = 1.19$ and $\theta_L = 0.83$, so that the time-aggregated annual growth rate of dividends has the same mean and volatility as the data. We also reset $R_f - 1$ to be 1.91 percent to maintain a net annual risk-free rate of 3.86 percent. The loss aversion parameter is still set at $\lambda = 2.25$, and the diminishing sensitivity parameter $\alpha$ can take three values: 0.37, 0.52 and 0.88. The variable $WML2$ is
the simulated average cumulative annualized momentum portfolio returns:

$$WML2 = \frac{1}{T} \sum_{t=1}^{T} \left( R_t^{\text{winner}} R_{t+1}^{\text{winner}} - R_t^{\text{loser}} R_{t+1}^{\text{loser}} \right).$$

Comparing Table B1 with Table IV, where one period is assumed to be one year, we find that changing the length of the decision interval affects the momentum effect and the equity premium. When the decision interval becomes shorter, a typical investor will experience more losses in one year, and since he is averse to losses, he will demand a higher premium. The higher equilibrium equity premium or, equivalently, the lower price-dividend ratio, means that the disposition effect, i.e., age-2-1 investors’ different behavior facing good news versus bad news, will have a higher impact on the stock return predictability, thereby generating higher returns to the “winners-minus-losers” portfolio.

**Appendix B2. Purchase Prices as Reference Points**

As described in Section II, we suppose that investor $i$ uses $R_t^j W_{1,i}$ as a reference level of wealth when calculating gains and losses. Odean (1998) and Genesove and Mayer (2001) assume that the investor uses the original purchase price as a reference point. That is, if an investor buys a stock at price $P^B$ and sells at price $P^S$, he calculates gains/losses $X$ as follows: if he holds the stock one period and receives a dividend $D$, then he perceives $X = P^S + D - P^B$; if he holds the stock two periods and collects dividends $D$ and $D'$, then he perceives $X = P^S + D + D' - P^B$. Table B2 presents the results for this specification of gains/losses. We still take one period as one year, and the parameter values are fixed at $\theta_H = 1.28$, $\theta_L = 0.76$, $R_f = 1.0386$ and $\lambda = 2.25$. Comparing Table B2 with Table IV, we find that this alternative definition of gains/losses has virtually no effect on our results except to deliver a lower equity premium. The reason for the low equity premium is that stock returns do not need to beat the risk-free rate to be counted as gain, which in turn makes the investor more willing to purchase a stock.

Table B2 About Here
Appendix B3. Sensitivity Analysis w.r.t Dividend Growth Rate Volatility

The analysis in our main text is conducted for technology parameters fixed at the values in Table I. Table B3 examines the effect of varying the volatility of the dividend growth rate. For a binary distribution given by equation (1), the dividend growth rate has a mean equal to \( E(\theta_{t+1}) = \frac{\theta_{t+1} + \theta_{t}}{2} \), and a volatility equal to \( \sigma(\theta_{t+1}) = \frac{\theta_{t+1} - \theta_{t}}{2} \). In Table B3, we maintain \( E(\theta_{t+1}) - 1 \) at 2.24% and change \( \sigma(\theta_{t+1}) \) from 21% to 26% to 31%.\(^{22}\) The preference parameters are set at \( \alpha = 0.52 \) and \( \lambda = 2.25 \). Table B3 suggests that increasing \( \sigma(\theta_{t+1}) \) generates stronger momentum effects and higher equity premiums. Since a higher \( \sigma(\theta_{t+1}) \) is also associated with a higher return volatility, the momentum effect is expected to be stronger among stocks both with higher dividend volatility and with higher return volatility. This observation is consistent with Zhang’s (2006) finding that momentum profits are higher among firms with higher cash flow volatility or return volatility.

Table B3 About Here

Appendix B4. Results for Allowing for Intermediate Levels of Demand

Because, in our model in the main text, age-2 investors can only hold 0 or 1 unit of stock, when making selling decisions they either sell the whole unit or do not sell at all, making partial liquidation impossible. To accommodate this possibility, we modify our model by allowing investors to hold 0, 0.5, or 1 share of the stock. We solve this modified model and follow the same procedure described in Section IVB to calculate the disposition effect, the momentum effect, the correlation between stock returns and trading volumes, and the equity premium. The results are presented in Table B4.

Table B4 About Here

By comparing Table B4 with Tables II, III and IV, we can see that our main results obtained from the model in the main text still hold in this modified model. In the risk-
neutral world ($\alpha = \lambda = 1$), there is no disposition effect, no momentum effect and no return-volume correlation, and the equity premium is zero. When we shut down the loss aversion component and turn on the concavity/convexity component by letting $\alpha = 0.5$, $\lambda = 1$, prospect theory can generate the disposition effect ($DispEffect = 2.11$), a momentum effect ($MomEffect = 6.71\%$), and a positive correlation between returns and volumes ($Corr (R_t, Q_t) = 0.8$). This indicates that the diminishing sensitivity component helps to explain the momentum effect and the co-movement between stock returns and trading volumes through the disposition effect. When we set $\alpha = 1$, $\lambda = 2.25$, the model generates the reversed disposition effect ($DispEffect = 0.97$), a return reversal ($MomEffect = -0.05\%$) and a negative return-volume correlation ($Corr (R_t, Q_t) = -0.78$). The intuition is exactly what has been described in Subsections IVB and IVC. Using Tversky and Kahneman’s (1992) parameter values, this modified model can generate a momentum of 0.81\% per annum, comparable to the 0.87% generated from the benchmark model (see Table IV).
Table I  Technology Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free rate</td>
<td>1.0386</td>
</tr>
<tr>
<td>( R_f )</td>
<td></td>
</tr>
<tr>
<td>Dividend parameters</td>
<td></td>
</tr>
<tr>
<td>( \theta_H )</td>
<td>1.2821</td>
</tr>
<tr>
<td>( \theta_L )</td>
<td>0.7628</td>
</tr>
</tbody>
</table>

We take one period to be one year. Dividend parameters (\( \theta_H \) and \( \theta_L \)) are calibrated to generate a dividend growth rate with the mean and standard deviation equal to 2.24\% and 25.97\%, respectively.
## Table II  Implications of Diminishing Sensitivity

<table>
<thead>
<tr>
<th></th>
<th>$\alpha = 0.2$</th>
<th>$\alpha = 0.5$</th>
<th>$\alpha = 0.88$</th>
<th>$\alpha = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Disposition Effect</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PGR</td>
<td>0.49</td>
<td>0.56</td>
<td>0.53</td>
<td>0.50</td>
</tr>
<tr>
<td>PLR</td>
<td>0.29</td>
<td>0.36</td>
<td>0.47</td>
<td>0.50</td>
</tr>
<tr>
<td>$DispEffect$</td>
<td>1.73</td>
<td>1.56</td>
<td>1.12</td>
<td>1.00</td>
</tr>
<tr>
<td>(ii) Momentum Effect</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(R_{t+1}</td>
<td>\theta_t = \theta_H)$</td>
<td>1.1295</td>
<td>1.0934</td>
<td>1.0516</td>
</tr>
<tr>
<td>$E(R_{t+1}</td>
<td>\theta_t = \theta_L)$</td>
<td>1.0158</td>
<td>1.0439</td>
<td>1.0410</td>
</tr>
<tr>
<td>$MomEffect$</td>
<td>11.37%</td>
<td>4.95%</td>
<td>1.06%</td>
<td>0.00%</td>
</tr>
<tr>
<td>$WML$</td>
<td>10.91%</td>
<td>4.67%</td>
<td>1.06%</td>
<td>0.00%</td>
</tr>
<tr>
<td>(iii) Turnover</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Corr(R_t, Q_t)$</td>
<td>0.52</td>
<td>0.83</td>
<td>0.92</td>
<td>0.00</td>
</tr>
<tr>
<td>(iv) Equity Premium</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(R_t - R_f)$</td>
<td>3.43%</td>
<td>3.01%</td>
<td>0.82%</td>
<td>0.00%</td>
</tr>
<tr>
<td>$E[H_1(S_t)]$</td>
<td>0.43</td>
<td>0.46</td>
<td>0.49</td>
<td>0.50</td>
</tr>
</tbody>
</table>

PGR and PLR are the simulated “proportion of gains realized” and “proportion of losses realized.” We define $DispEffect = \frac{PGR}{PLR}$, and if $DispEffect > 1$, then a disposition effect exists. $MomEffect = E(R_{t+1}|\theta_t = \theta_H) - E(R_{t+1}|\theta_t = \theta_L)$. $WML$ is the simulated average momentum portfolio return in the multi-stock setting. If $MomEffect > 0$ and $WML > 0$, then a momentum effect exists. $Q_t = 1 - H_{21}(S_t)$ is the turnover, or aggregate selling, in period $t$. Technology parameter values are fixed at the values in Table I: $\theta_H = 1.2821$, $\theta_L = 0.7628$ and $R_f = 1.0386$. The preference parameter $\lambda \geq 1$ determines loss aversion; in this table, we deliberately set $\lambda$ as 1, so that the investor does not exhibit loss aversion.


<table>
<thead>
<tr>
<th></th>
<th>( \lambda = 1 )</th>
<th>( \lambda = 2.25 )</th>
<th>( \lambda = 3 )</th>
<th>( \lambda = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Disposition Effect</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PGR</td>
<td>0.50</td>
<td>0.39</td>
<td>0.35</td>
<td>0.31</td>
</tr>
<tr>
<td>PLR</td>
<td>0.50</td>
<td>0.41</td>
<td>0.39</td>
<td>0.38</td>
</tr>
<tr>
<td>( DispEffect )</td>
<td>1.00</td>
<td>0.97</td>
<td>0.91</td>
<td>0.83</td>
</tr>
<tr>
<td>(ii) Momentum Effect</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E(R_{t+1}</td>
<td>\theta_t = \theta_H) )</td>
<td>1.0386</td>
<td>1.1000</td>
<td>1.1255</td>
</tr>
<tr>
<td>( E(R_{t+1}</td>
<td>\theta_t = \theta_L) )</td>
<td>1.0386</td>
<td>1.1006</td>
<td>1.1311</td>
</tr>
<tr>
<td>( MomEffect )</td>
<td>0.00%</td>
<td>-0.06%</td>
<td>-0.56%</td>
<td>-1.13%</td>
</tr>
<tr>
<td>( WML )</td>
<td>0.00%</td>
<td>-0.23%</td>
<td>-0.85%</td>
<td>-1.48%</td>
</tr>
<tr>
<td>(iii) Turnover</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Corr (R_t, Q_t) )</td>
<td>0.00</td>
<td>-0.70</td>
<td>-0.91</td>
<td>-0.94</td>
</tr>
<tr>
<td>(iv) Equity Premium</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E(R_t - R_f) )</td>
<td>0.00%</td>
<td>6.17%</td>
<td>8.97%</td>
<td>11.89%</td>
</tr>
</tbody>
</table>

PGR and PLR are the simulated “proportion of gains realized” and “proportion of losses realized.” We define \( DispEffect = \frac{PGR}{PLR} \), and if \( DispEffect < 1 \), then a reversed disposition effect exists. \( MomEffect = E(R_{t+1}|\theta_t = \theta_H) - E(R_{t+1}|\theta_t = \theta_L) \). \( WML \) is the simulated average momentum portfolio return in the multi-stock setting. If \( MomEffect < 0 \) and \( WML < 0 \), then there is reversal in the cross-section of stock returns. \( Q_t = 1 - H_{21}(S_t) \) is the turnover, or aggregate selling, in period \( t \). Technology parameter values are fixed at the values in Table I: \( \theta_H = 1.2821 \), \( \theta_L = 0.7628 \) and \( R_f = 1.0386 \). Preference parameter \( \alpha \) controls the curvature of the value function. In this table, we deliberately set \( \alpha \) to be 1 to get rid of the diminishing sensitivity component of prospect theory.
<table>
<thead>
<tr>
<th></th>
<th>α = 0.37</th>
<th>α = 0.52</th>
<th>α = 0.88</th>
<th>Empirical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Disposition Effect</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PGR</td>
<td>0.40</td>
<td>0.41</td>
<td>0.41</td>
<td>0.38</td>
</tr>
<tr>
<td>PLR</td>
<td>0.18</td>
<td>0.23</td>
<td>0.37</td>
<td>0.17</td>
</tr>
<tr>
<td>DispEffect</td>
<td>2.25</td>
<td>1.75</td>
<td>1.10</td>
<td>2.24</td>
</tr>
<tr>
<td>(ii) Momentum Effect</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E (R_{t+1}</td>
<td>\theta_t = \theta_H)$</td>
<td>1.1575</td>
<td>1.1431</td>
<td>1.1091</td>
</tr>
<tr>
<td>$E (R_{t+1}</td>
<td>\theta_t = \theta_L)$</td>
<td>1.0822</td>
<td>1.0927</td>
<td>1.1004</td>
</tr>
<tr>
<td>MomEffect</td>
<td>7.54%</td>
<td>5.04%</td>
<td>0.87%</td>
<td>—</td>
</tr>
<tr>
<td>WML</td>
<td>7.20%</td>
<td>4.76%</td>
<td>0.76%</td>
<td>8.60%</td>
</tr>
<tr>
<td>(iii) Turnover</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Corr (R_t, Q_t)$</td>
<td>0.84</td>
<td>0.88</td>
<td>0.91</td>
<td>0.28</td>
</tr>
<tr>
<td>(iv) Equity Premium</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E (R_t - R_f)$</td>
<td>8.14%</td>
<td>7.94%</td>
<td>6.62%</td>
<td>7.84%</td>
</tr>
</tbody>
</table>

PGR and PLR are the simulated “proportion of gains realized” and “proportion of losses realized.” We define $DispEffect = \frac{PGR}{PLR}$ and $MomEffect = E (R_{t+1}|\theta_t = \theta_H) - E (R_{t+1}|\theta_t = \theta_L)$. $WML$ is the simulated average momentum portfolio return in the multi-stock setting. $Q_t = 1 - H_{21} (S_t)$ is the turnover, or aggregate selling, in period $t$. Technology parameter values are fixed at the values in Table I: $\theta_H = 1.2821$, $\theta_L = 0.7628$ and $R_f = 1.0386$. Loss aversion parameter $\lambda$ is set at 2.25, the value estimated by Tversky and Kahneman (1992). The empirical values of PGR/PLR and momentum are taken from Dhar and Zhu (2006) and Jegadeesh and Titman (1993), respectively. The empirical values of $Corr (R_t, Q_t)$ and $E (R_t - R_f)$ are based on AMEX/NYSE data from 1926-2006.
Table B1  Results for a Decision Interval of Six Months

<table>
<thead>
<tr>
<th></th>
<th>$\alpha = 0.37$</th>
<th>$\alpha = 0.52$</th>
<th>$\alpha = 0.88$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Disposition Effect</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PGR</td>
<td>0.40</td>
<td>0.41</td>
<td>0.40</td>
</tr>
<tr>
<td>PLR</td>
<td>0.18</td>
<td>0.24</td>
<td>0.37</td>
</tr>
<tr>
<td>$DispEffect$</td>
<td>2.15</td>
<td>1.68</td>
<td>1.07</td>
</tr>
<tr>
<td>(ii) Momentum Effect</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$WML2$</td>
<td>10.33%</td>
<td>6.59%</td>
<td>0.78%</td>
</tr>
<tr>
<td>(iii) Turnover</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Corr\left(R_tR_{t+1}, \frac{Q_t+Q_{t+1}}{2}\right)$</td>
<td>0.87</td>
<td>0.88</td>
<td>0.89</td>
</tr>
<tr>
<td>(iv) Equity Premium</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E\left(R_tR_{t+1} - R_f^2\right)$</td>
<td>11.27%</td>
<td>11.22%</td>
<td>9.69%</td>
</tr>
</tbody>
</table>

The decision interval of the investor is assumed to be six months. Dividend parameters are recalibrated as $\theta_H = 1.1913$ and $\theta_L = 0.8309$, so that the annualized dividend growth rate has a mean of 2.24% and a volatility of 25.97%. The risk-free rate is set as $R_f - 1 = 1.91\%$. Loss aversion parameter $\lambda$ is set at 2.25. PGR and PLR are the simulated “proportion of gains realized” and “proportion of losses realized.” We define $DispEffect = \frac{PGR}{PLR}$. $WML2$ is the simulated average cumulative annualized momentum portfolio return. $E\left(R_tR_{t+1} - R_f^2\right)$ is the annualized equity premium. $Q_t = 1 - H_{21}(S_t)$ is the turnover, or aggregate selling, in period $t$. 
Table B2  Results for Using Purchase Prices as Reference Points

<table>
<thead>
<tr>
<th></th>
<th>$\alpha = 0.37$</th>
<th>$\alpha = 0.52$</th>
<th>$\alpha = 0.88$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Disposition Effect</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PGR</td>
<td>0.40</td>
<td>0.41</td>
<td>0.49</td>
</tr>
<tr>
<td>PLR</td>
<td>0.18</td>
<td>0.24</td>
<td>0.43</td>
</tr>
<tr>
<td>$DispEffect$</td>
<td>2.24</td>
<td>1.74</td>
<td>1.16</td>
</tr>
<tr>
<td>(ii) Momentum Effect</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$MomEffect$</td>
<td>7.39%</td>
<td>4.91%</td>
<td>0.86%</td>
</tr>
<tr>
<td>$WML$</td>
<td>7.07%</td>
<td>4.65%</td>
<td>0.68%</td>
</tr>
<tr>
<td>(iii) Turnover</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Corr (R_t, Q_t)$</td>
<td>0.84</td>
<td>0.88</td>
<td>0.95</td>
</tr>
<tr>
<td>(iv) Equity Premium</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E (R_t - R_f)$</td>
<td>3.86%</td>
<td>3.77%</td>
<td>2.76%</td>
</tr>
</tbody>
</table>

The investor uses the purchase price as the reference point when calculating capital gains or losses. PGR and PLR are the simulated “proportion of gains realized” and “proportion of losses realized.” We define $DispEffect = \frac{PGR}{PLR}$ and $MomEffect = E (R_{t+1}|\theta_t = \theta_H) - E (R_{t+1}|\theta_t = \theta_L)$. $WML$ is the simulated average momentum portfolio return in the multi-stock setting. $Q_t = 1 - H_{21} (S_t)$ is the turnover, or aggregate selling, in period $t$. Technology parameter values are fixed at their values in Table I: $\theta_H = 1.2821$, $\theta_L = 0.7628$ and $R_f = 1.0386$. Loss aversion parameter $\lambda$ is set at 2.25, the value estimated by Tversky and Kahneman (1992).
Table B3  Sensitivity Analysis w.r.t Dividend Growth Rate Volatility $\sigma(\theta_{t+1})$

<table>
<thead>
<tr>
<th></th>
<th>$\theta_L = 0.81586$</th>
<th>$\theta_L = 0.7628$</th>
<th>$\theta_L = 0.70865$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta_H = 1.2289$</td>
<td>$\theta_H = 1.2821$</td>
<td>$\theta_H = 1.3362$</td>
</tr>
<tr>
<td>(i) Disposition Effect</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PGR</td>
<td>0.41</td>
<td>0.41</td>
<td>0.41</td>
</tr>
<tr>
<td>PLR</td>
<td>0.24</td>
<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
<td>$DispEffect$</td>
<td>1.71</td>
<td>1.75</td>
<td>1.79</td>
</tr>
<tr>
<td>(ii) Momentum Effect</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$MomEffect$</td>
<td>3.83%</td>
<td>5.04%</td>
<td>6.28%</td>
</tr>
<tr>
<td>$WML$</td>
<td>3.62%</td>
<td>4.76%</td>
<td>5.94%</td>
</tr>
<tr>
<td>(iii) Turnover</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr ($R_t, Q_t$)</td>
<td>0.88</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td>(iv) Equity Premium</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E (R_t - R_f)$</td>
<td>6.13%</td>
<td>7.94%</td>
<td>9.96%</td>
</tr>
</tbody>
</table>

PGR and PLR are the simulated “proportion of gains realized” and “proportion of losses realized.” We define $DispEffect = \frac{PGR}{PLR}$ and $MomEffect = E (R_{t+1}|\theta_t=\theta_H) - E (R_{t+1}|\theta_t=\theta_L)$. $WML$ is the simulated average momentum portfolio return in the multi-stock setting. $Q_t = 1 - H_{21}(S_t)$ is the turnover, or aggregate selling, in period $t$. The risk-free rate is set at $R_f = 1.0386$. The preference parameters are $\alpha = 0.52$ and $\lambda = 2.25$. 
Table B4 Results for Allowing for Intermediate Levels of Demand

<table>
<thead>
<tr>
<th></th>
<th>$\alpha = 1$, $\lambda = 1$</th>
<th>$\alpha = 0.5$, $\lambda = 1$</th>
<th>$\alpha = 1$, $\lambda = 2.25$</th>
<th>$\alpha = 0.88$, $\lambda = 2.25$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(i) Disposition Effect</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PGR</td>
<td>0.50</td>
<td>0.49</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>PLR</td>
<td>0.50</td>
<td>0.23</td>
<td>0.41</td>
<td>0.37</td>
</tr>
<tr>
<td>$DispEffect$</td>
<td>1.00</td>
<td>2.11</td>
<td>0.97</td>
<td>1.10</td>
</tr>
<tr>
<td><strong>(ii) Momentum Effect</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E (R_{t+1}</td>
<td>\theta_t = \theta_H)$</td>
<td>1.0386</td>
<td>1.0955</td>
<td>1.0958</td>
</tr>
<tr>
<td>$E (R_{t+1}</td>
<td>\theta_t = \theta_L)$</td>
<td>1.0386</td>
<td>1.0284</td>
<td>1.0963</td>
</tr>
<tr>
<td>$MomEffect$</td>
<td>0.00%</td>
<td>6.71%</td>
<td>-0.05%</td>
<td>0.81%</td>
</tr>
<tr>
<td><strong>(iii) Turnover</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Corr (R_t, Q_t)$</td>
<td>0.00</td>
<td>0.80</td>
<td>-0.78</td>
<td>0.95</td>
</tr>
<tr>
<td><strong>(iv) Equity Premium</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E (R_t - R_f)$</td>
<td>0.00%</td>
<td>2.33%</td>
<td>5.74%</td>
<td>6.29%</td>
</tr>
</tbody>
</table>

Investors are allowed to hold 0, 0.5, or 1 unit of stocks. PGR and PLR are the simulated “proportion of gains realized” and “proportion of losses realized.” We define $DispEffect = \frac{PGR}{PLR}$ and $MomEffect = E (R_{t+1} | \theta_t = \theta_H) - E (R_{t+1} | \theta_t = \theta_L)$. $WML$ is the average momentum portfolio return in the multi-stock setting. $Q_t = 1 - H_{21} (S_t)$ is the turnover in period $t$. Technology parameter values are fixed at the values in Table I: $\theta_H = 1.2821$, $\theta_L = 0.7628$ and $R_f = 1.0386$. 

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Figure 1 plots the order of events in period $t$. 
Figure 2 Diminishing Sensitivity Drives the Disposition Effect

Figure 2 graphs the possible capital gains/losses, as well as prospect theory utilities, faced by an age-2-1 investor. If this investor liquidates his stock, his capital gains/losses, together with his prospect theory utilities, are marked with dots; if he keeps the stock, then his possible future capital gains/losses and his prospect theory utilities are marked with circles. The two endogenous state variables are $f_{t-1} = 20.01$ and $z_t = 0.50$. The parameter values are $\theta_H = 1.2821$, $\theta_L = 0.7628$, $R_f = 1.0386$, $\lambda = 1$ and $\alpha = 0.5$. 

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Figure 3 graphs the possible capital gains/losses, as well as prospect theory utilities, faced by an age-2-1 investor. If this investor liquidates his stock, his capital gains/losses, together with his prospect theory utilities, are marked with dots; if he keeps the stock, then his possible future capital gains/losses and his prospect theory utilities are marked with circles. The two endogenous state variables are $f_{t-1} = 7.75$ and $z_t = 0.48$. The parameter values are $\theta_H = 1.2821$, $\theta_L = 0.7628$, $R_f = 1.0386$, $\lambda = 4$ and $\alpha = 1$. 