Consumption Basket, Exchange Risk, and Asset Demand

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I. Introduction

Foreign exchange risk and hence the demand for foreign assets depend on the objective and habitat of investors. The investment objective, in turn, is contingent upon how the consumption basket or its price is defined. If the investor is "domestic" in the sense that he or she spends all income on domestic goods, then the domestic price index should be used in defining the investment objective in real terms, regardless of whether returns are generated at home or abroad. However, for an investor who consumes a mix of foreign and home products, or for multinational firms with extensive operations outside their home countries, some sort of world price reflective of the relative importance of home and foreign goods in their consumption basket is the proper deflator.

The controversy in international finance literature on the role of the exchange risk in international investment decisions centers around assumptions on consumption preferences of investors and on whether certain international parity conditions such as purchasing power parity hold. Solnik [19], [20], Roll and Solnik [18], and Stulz [22] recognize local consumption habitats of different national investors, and the exchange risk in their model is an important factor affecting international investment decisions. Grauer, Litzenberger, and Stehle [10] and others, on the other hand, assume identical consumption preferences and view the exchange risk as purely monetary, i.e., the exchange rate changes are completely determined by price changes à la purchasing power parity. The exchange risk, in real terms, is thus nonexistent. Nominal return models ([15], for instance) implicitly assume identical consumption preferences as well as the absence of inflation uncertainty. Adler and Dumas [2] provide a critical survey of this literature.

Recently, with wide application of the basket currency concept in currency floating, the global view of the exchange risk has gained attention. Various authors (see [13], [5], [7], and [6]), have argued that the exchange risk should be

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measured in terms of the variability of the real purchasing power of the returns generated from foreign investment over a global mix of goods.

This paper examines the relationship between the consumption basket and the demand for foreign assets. In particular, it asks a question, in a two-country framework, whether changes in the share of foreign goods in the consumption basket will cause changes in the optimum share of foreign assets in the investor's portfolio. This question is interesting in its own right -- should an individual (or a corporation) invest more abroad when he or she wants more foreign goods? The model also can be used to examine when the investor's consumption basket can be safely ignored; that is, the conditions under which consumption preferences can be separated from international investment decisions.

The organization of the paper is as follows. Section II specifies real returns on foreign and domestic assets for a risk-neutral investor. Section III develops the demand for foreign assets for a risk-averse investor and derives the conditions for the separation of the consumption basket and asset demand. Section IV examines implications of imposing purchasing power parity in international capital markets.

II. Real Returns on Assets

Assume that there are two countries—domestic and foreign—each producing only one good. International commodity trade, however, permits consumption of both goods by residents of both countries. Further, international firms regularly use both "domestic" and "foreign" goods and services for their operations. In an open economy, an investor is thus "global" not only in the sense of investment opportunities but also in consumption opportunities. The real returns that he or she can get from an investment at home and abroad are thus the nominal rates of return adjusted for change in the price index, \( P^* \), which reflects the consumption mix of that investor. Unlike home investment, the nominal returns from foreign assets, however, need to be translated by the exchange rate, \( e \); that is, the domestic-currency value of the foreign currency at the end of the period. It is assumed that there is no default risk or country risk due to government policy changes.

This two-country by two-commodity by two-asset scenario precludes the model's applicability to such issues as forming a market portfolio; it is assumed here that the market portfolio already has been formed in each country. However, the scenario sharpens the focus on a microeconomic international investment decision and its relationship with consumption preferences.
The real returns from foreign and domestic assets, respectively, in their rates of changes, are

$$\frac{dr^*}{r^*} = \frac{d \left( \frac{i^*}{e} \frac{1}{P^w} \right)}{\frac{i^*}{e} \frac{1}{P^w}}$$

(1)

$$= \frac{de}{e} \frac{1}{P^w} + \frac{di^*}{i^*} \frac{dP^w}{P^w} - \frac{di^*}{i^*} \frac{de}{e} \frac{1}{P^w} \frac{dP^w}{P^w} - \frac{di^*}{i^*} \frac{dP^w}{P^w} + \left( \frac{dP^w}{P^w} \right)^2$$

$$\frac{dr}{r} = \frac{d \left( \frac{i}{P^w} \right)}{\frac{i}{P^w}}$$

(2)

$$= \frac{di}{i} \frac{1}{P^w} - \frac{dP^w}{P^w} \frac{di}{i} + \left( \frac{dP^w}{P^w} \right)^2$$

where $r^*$ and $r$ are, respectively, one plus the real rates of return from foreign and domestic assets, and $i^*$ and $i$, respectively, are one plus the nominal rates of return from these two kinds of assets. The exchange rates and commodity prices at the beginning of the period are assumed to be known and set to unity.

Changes in the price deflator of the global investor’s consumption basket are

$$\frac{dP^w}{P^w} = a \frac{dP^*}{P^*} + (1-a) \frac{dP}{P}$$

(3)

where $0 \leq a < 1$ is the share of foreign goods in the consumption basket, and $P^*$ and $P$ are prices of foreign and home goods respectively. The two goods are distinct so there is no need to suppose any price linkage at this point.

Other than simplicity, the price deflator equation (3) is justifiable under two conditions. First, from the standpoint of a global investor, the consumption basket can be viewed as a composite commodity. This approach was taken by Kouri and de Macedo [13] and Eun [7]. Second, there are cases when the global investor can consume goods directly in the country of origin; that is, consume foreign goods in the foreign country rather than consuming them only after importation into the domestic market. This may be applicable not only to some footloose, globally-based individuals but also to multinational firms relying on local input supplies. In either case, an assumption of a homothetic utility function is implicit for this specification.\(^1\)

\(^1\) If “foreign” goods can be consumed only in the domestic market, then exchange rates should appear in the price deflator

$$\frac{dP^w}{P^w} = a \left( \frac{de}{e} \frac{1}{P^*} + \frac{dP^*}{P^*} \right) + (1-a) \frac{dP}{P}.$$ 

(3)

Consequences of this alternative specification are discussed in footnotes 3 and 7.
All variables are stochastic and assumed to follow Ito processes

\begin{align}
\frac{de}{e} &= M_e dt + S_e dZ_e \\
\frac{di}{i} &= M_i dt + S_i dZ_i \\
\frac{dp}{p^*} &= M_p dt + S_p dZ_p^* \\
\frac{dp}{p} &= M_p dt + S_p dZ_p.
\end{align}

The first terms in (4)-(8) are the trend or the expected rates of changes for a given unit of time. The second terms are the random unexpected component: over a short time interval, \( S_i \) can be interpreted as the instantaneous standard deviation of variable \( i \), and the \( dZ_i \) are Wiener processes.

Substitution of (4)-(8) in (1)-(3) and the subsequent application of Ito's lemma yield

\begin{align}
\frac{dr}{r^*} &= \left[ M_e - \left(aM_{p^*} + (1-a)M_p \right) + M_i - \left(aS_{p^*} + (1-a)S_{p^2} \right) \right] dt \\
&\quad + S_e dZ_e + S_i dZ_i - \left(aS_{p^*} dZ_{p^*} + (1-a)S_p dZ_p \right) \\
&\quad \equiv M_e dt + S_e dZ_e + S_i dZ_i - \left(aS_{p^*} dZ_{p^*} + (1-a)S_p dZ_p \right)
\end{align}

\begin{align}
\frac{dr}{r} &= \left[ M_i - \left(aM_{p^*} + (1-a)M_p \right) - \left(aS_{p^*} + (1-a)S_{p^2} \right) + S_w^2 \right] dt \\
&\quad + S_i dZ_i - \left(aS_{p^*} dZ_{p^*} + (1-a)S_p dZ_p \right) \\
&\quad \equiv M_i dt + S_i dZ_i - \left(aS_{p^*} dZ_{p^*} + (1-a)S_p dZ_p \right)
\end{align}

where

\[ S_w^2 = a^2 S_{p^*}^2 + (1-a)^2 S_p^2 + 2a(1-a)S_{pp^*} \]

and \( S_{ij} \) is the covariance between Wiener processes \( dZ_i \) and \( dZ_j \).\footnote{For an early application of stochastic calculus to finance, see [16]. For analyses of consumption and asset pricing in a domestic context, see [3] and [4].}

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When foreign goods are excluded from the consumption basket, that is, \( a = 0 \), then foreign inflation risk terms drop out in both (9) and (10). However, this result is based on the assumption that international trade is allowed in asset markets, but not in commodity markets. In general, the real returns on foreign assets depend on exchange risk, foreign interest rate risk, and domestic and foreign inflation risk. Domestic investments are subject to domestic interest rate risk, and domestic and foreign inflation risk.\(^3\)

### III. Demand for Foreign Assets

We now develop asset demand for a risk-averse investor. The flow wealth constraint is obtained by combining asset returns with a deterministic labor income, \( Y \)

\[
dV = Vx \frac{dr^*}{r^*} + V(1-x) \frac{dr}{r} + (Y-C) dt
\]

where \( dr^*/r^* \) and \( dr/r \) are given in (9) and (10), \( C \) is real consumption, \( V \) is real wealth given at a given time, and \( x \) is the share of foreign assets held in one's portfolio.

The investor's utility is given by time-additive von Neumann-Morgenstern expected utility function. The utility is generated by \( U[C(t), t] \); \( U \) is strictly increasing and concave in \( C \), and the time variable inserted as the second argument denotes the present-value discount factor. The investor, at a given time, selects the optimum portfolio between domestic and foreign assets, and maximizes consumption over time. The derived expected utility for the investor is

\[
J(V, t) = \max_{t} \int_{t}^{\infty} E[U_t | C_t(T), T] dT.
\]

This function is the present value of the maximum utility that the investor can expect from \( t \) onwards, when the real wealth at \( t \) is given at \( V \).

The decision problem is maximization of (12) subject to (11). With strictly concave \( U \) and an appropriate transversality condition,\(^4\) there exists a set of opti-

\(^3\) An alternative specification of the price deflator as (3)' (footnote 1) yields the following foreign and domestic returns

\[
(9)' \quad \frac{dr^*}{r^*} = M_r^w dt + (1-a)S_c dZ_c + S_p dZ_p - (aS_p dZ_p + (1-a)S_c dZ_p)
\]

\[
(10)' \quad \frac{dr}{r} = M_r^w dt + S_c dZ_c - a(S_p dZ_p + S_c dZ_p) - (1-a)S_p dZ_p
\]

where \( M_r^w \) and \( M_r^d \) are deterministic variables in the two return equations. Qualitatively, no changes are in order for foreign returns. Domestic assets, however, are now subject to exchange risk. This result was also obtained by Hodder [12] in a different model.

\(^4\) The transversality condition is

\[
\lim_{t \to \infty} EU(.) = 0.
\]

This requires a positive real discount rate.
minimum control \((C^*, x^*)\) for such a problem. To obtain the solution, form a Bellman function

\[
B(x, V, t) = U | C(t), t | + J_f + J_r | V x M_{r^*} + V (1 - x) M_r + Y - C |
\]

(13)

\[
+ \frac{J_x V^2}{2} \begin{bmatrix} x^2 S_{r^*} + (1 - x)^2 S_r^2 - 2x(1 - x) S_{r^*} \end{bmatrix}
\]

where

\[
S_{r^*} = S_e^2 + S_r^2 + 2S_{e^*} + S_w^2 - 2\left(aS_{e^p} + (1 - a)S_{r^p}\right)
\]

\[-2\left(aS_{e^p} + (1 - a)S_{r^p}\right)\]

\[
S_r = S_e^2 + S_w^2 - 2\left(aS_{e^p} + (1 - a)S_{r^p}\right)
\]

\[
S_{r^*} = S_{e^*} - \left(aS_{e^p} + (1 - a)S_{r^p}\right) + S_{r^*} + S_w^2
\]

\[-\left(aS_{e^p} + (1 - a)S_{r^p}\right) + \left(aS_{e^p} + (1 - a)S_{r^p}\right)\]

and \(J_f\) and \(J_r\) are the first and second derivatives of \(J\) with respect to \(i\).

Taking partial derivatives of (13) with respect to \(C\) and \(x\) the first-order conditions are\(^5\)

(14)

\[
U_x = J_x
\]

(15)

\[
x = \frac{1}{S} \left[ \frac{M_{r^*} - M_r}{A} + \left(S_r^2 - S_{r^*}^2\right) \right]
\]

where

\[
S = S_e^2 + S_r^2 - 2S_{r^*}, \text{ and }
\]

\[
A = -J_x V/J_x \] is the Arrow-Pratt measure of relative risk aversion.

Equation (14) is the familiar marginal condition on consumption and saving. The investor should consume until the marginal utility of consumption equals the expected marginal utility of increasing wealth. Equation (15) shows the optimum share of foreign assets in the investor's portfolio. The first term is the speculative demand for foreign assets dependent upon the investor's risk aversion. The second term is the hedging demand for foreign assets, which is independent of the investor's attitude toward risk.

\(^5\) The second-order condition for stability is

\[
U_{xx} + J_{xx} < 0
\]

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For a perfect risk averter \((A = \infty)\), the speculative demand disappears, but the hedging demand will remain. This is the case of a minimum variance portfolio. It is interesting that, due to the risk reduction possible with international diversification, even a complete risk averter may want to hold some foreign assets that are risky if \(S_{r'} > S_{r^*}\) (since \(S > 0\)), i.e., if the covariance between domestic and foreign returns is smaller than the variance of domestic returns. With a reversed sign, the investor would want to be short in foreign assets.

For an investor with \(A < \infty\), however, the speculative demand also matters and the optimum portfolio point shifts inside of the efficient frontier. This shift due to the speculative demand depends on the differential expected rates of return relative to the variability of portfolio returns, as well as on the degree of risk aversion. That is, the investor’s holding of foreign assets will increase, the greater the expected rates of return abroad than on home investment, the less the variability of portfolio returns, and the less risk-averse is the investor.

We are interested in the effect of changes in the consumption basket on the share of foreign asset demand

\[
\frac{\partial x}{\partial a} = \frac{1}{S} \left[ \frac{1}{A} \left( \frac{\partial M_{r^*}}{\partial a} - \frac{\partial M_r}{\partial a} \right) + \left( \frac{\partial S_r^2}{\partial a} - \frac{\partial S_{r^*}}{\partial a} \right) \right].
\]

This implies

\[
\frac{\partial x}{\partial a} \gg 0 \quad \text{as} \quad \frac{\partial M_{r^*}}{\partial a} \gg \frac{\partial M_r}{\partial a} \quad \text{and} \quad \frac{\partial S_r^2}{\partial a} \gg \frac{\partial S_{r^*}}{\partial a}.
\]

The first condition operates through the speculative demand. The second condition exerts its influence through the hedging demand. Combined, changes in the investor’s consumption preferences can affect international asset choices in two ways: through their effect on the expected real returns from foreign as opposed to domestic investments, and through the effect on the covariance of the two returns relative to the variance of the domestic returns. Thus, an increased demand for foreign goods by the American investor (or attraction to favorable local input supply conditions abroad by a U.S.-based multinational firm), ceteris paribus, should induce increased investment outflows from the United States, if the changing demand pattern, in terms of the new consumption deflator, leads to higher real returns abroad than in the United States, and if the covariance of the two returns is less affected than the variance of the domestic returns by this change in consumption preference.

It is interesting that the sign of the combined effect of consumption, \(\partial x/\partial a\), is dependent on the value of risk aversion. This result is seen by rewriting (16) in terms of original variables

\[(16)' \quad \frac{\partial x}{\partial a} = \frac{1}{S} \left[ \left( S_{e^p} - S_{p^*} + S_{r^p} - S_{r^*} - S_{r^p} \right) \left( \frac{1}{A} - 1 \right) \right].\]

This result is consistent with Krugman’s [14]. Krugman did not examine the con-
umption effect but did find the dependency of the direction of investment flows on whether or not A is greater than one.

If \( A = 1 \), \( \partial x/\partial a = 0 \); that is, investment decisions are independent of consumption preferences. This conclusion is what Adler and Dumas [2] called a logarithmic portfolio that is independent of the measurement currency; with a logarithmic utility function, the deflator can be separated from the real return objective and hence from asset demand as well.

There is, however, another sufficient condition for the separation of the investor's consumption basket from asset demand. That is immediately apparent from (16)\(^7\)

\[
(S_{ep^*} - S_{ep}) = (S_{ip^*} - S_{ip}) - (S_{ir^*} - S_{ir})
\]

An interesting feature of this condition is that it is a second-moment relationship. This contrasts with international parities such as purchasing power parity that are first-moment equilibrium relationships.

It is clear that the condition for the separation is rather strong. What is required is either \( A = 1 \), or that the covariance differential between exchange rates and two commodity prices should equal the covariance differential between domestic interest rates and two prices minus the covariance differential between foreign interest rates and two prices. A stricter set of the sufficient, but not necessary, conditions for the consumption neutrality is the pairwise equality in the three sets of the covariances in the bracket; i.e., \( S_{ep^*} = S_{ep} \), \( S_{ip^*} = S_{ip} \) and \( S_{ir^*} = S_{ir} \).

Consider two special cases where two commodity prices are not fully stochastic. With stochastic foreign but nonstochastic domestic prices, the separability condition reduces to \( S_{ep^*} = S_{ip^*} - S_{ir^*} \). This resembles the Fisher open equation, but only in terms of covariances with foreign commodity prices. If both prices are assumed to be nonstochastic, (17) is trivially met so that the separation is guaranteed. This observation suggests that the condition for the separation of the consumption basket from international investment decisions is not independent of assumptions on inflation risk.

It is often said that purchasing power parity eliminates the exchange risk in real terms and thus is capable of separating the consumption preferences from international investment decisions. This view is not supported by the present

\(^6\) Note that
\[
\frac{\partial S}{\partial a} = \frac{\partial S^2}{\partial a} + \frac{\partial S^2}{\partial a} - 2 \frac{\partial S}{\partial a} = 0
\]

upon substitution of original variables for partial derivatives.

\(^7\) For the case where the price deflator is specified as (3)' (footnote 1) and the return equations as (9)' and (10)' subsequently (footnote 3), we obtain the following separation condition.

The new term describes a parity between the exchange rates and nominal interest rates in terms of second moments.

\[
(S_{ep^*} - S_{ep}) = (S_{ip^*} - S_{ip}) - (S_{ir^*} - S_{ir}) - (S_{ir^*} - S_{ir})
\]

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analysis, however, because the separation condition above is concerned entirely with the covariance structure. A simple purchasing power parity describes a relationship between exchange rates and prices only in terms of expected values, and hence is not sufficient to ensure the separation of the consumption basket from asset demand.

IV. Purchasing Power Parity

To pursue the last remark on the implication of purchasing power parity (PPP) further, we now assume that the exchange rates are determined by the relative PPP

\[
\frac{de}{e} = \frac{dP}{P} - \frac{dP^*}{P^*} = (M_p - M_p^*) \, dt + S_p \, dZ_p - S_p^* \, dZ_p^* .
\]

Commodity arbitrage in tradeable goods and the constant relative prices between tradeables and nontradeable goods are sufficient to ensure the relative PPP.\(^8\) Substitution of (18) and (3)-(8) in (1) yields the real returns from foreign assets under PPP

\[
\left. \frac{dr^*}{r^*} \right|_{\text{PPP}} = \left[ aM_p - (1 + a)M_p^* + M_{i^*} + aS_{i^*p} - (1 + a)S_{i^*p^*} \right] \\
+ \left[ a^2 + (a^2)S_{p^*}^2 + (a^2 - a)S_p^2 + (1 - 2a^2)S_{pp^*} \right] \, dt \\
+ aS_p \, dZ_p - (1 + a)S_{p^*} \, dZ_{p^*} + S_{i^*} \, dZ_{i^*} \\
\equiv M_{r^*}^{\text{PPP}} \, dt + aS_p \, dZ_p - (1 + a)S_{p^*} \, dZ_{p^*} + S_{i^*} \, dZ_{i^*} .
\]

Compared with the general case (9), both in stochastic and nonstochastic components of the real returns, the exchange-rate related terms now have been replaced with terms with domestic and foreign prices. Does this mean that the foreign exchange risk is "irrelevant" with PPP? The answer is no. The PPP transforms the exchange risk to another form of risk, but does not eliminate it.

The transformation of the "foreign exchange risk" under PPP is more than a semantic point. As long as the exchange rates, prices, and the interest rates are stochastic, the foreign exchange risk will remain. In a stochastic world, like a law of conservation of values, the risk, once created, is preserved regardless of whether PPP holds; the exchange risk is not eliminated with PPP but simply takes a different form—inflation risk—which may add to or cancel out existing risk depending on the model structure. To say that the "exchange risk" is gone just because it does not appear in (19) is absurd since, then, substitution of any exchange rate equation—parity or not—would do the trick. The popular belief that the exchange risk is "irrelevant" with PPP is misleading if it suggests that such elimination of the exchange risk can be obtained without producing other effects or making additional assumptions. Rather, the exchange risk is made irrelevant

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\(^8\) Definition of PPP is independent of how the consumption basket is defined. See [17] for the review of PPP.
because of the assumption that inflation rates at home and abroad are nonstochastic. The PPP merely affects the form that the exchange risk takes; it does not eliminate the exchange risk since the exchange risk exists because the exchange rates are stochastic, not necessarily because there is a deviation from PPP.\footnote{Given the emphasis on the consumption basket, we examine only the PPP in this paper. It would not be appropriate nor fruitful to evaluate a nominal parity such as the Fisher open equation in a stochastic real-return model such as the present one.}

The exchange risk also depends on market conditions because with perfect markets such risk may be fully diversified away. However, consumption differences preclude such a possibility; see Solnik [19] who developed a three-fund theorem that admits of a systematic exchange risk in addition to the traditional market and nonmarket risk. Further, Adler and Dumas [2] point out that the currency risk is not an aggregate risk that can be shared fully by market participants: it is an asymmetric relative risk with both real and nominal dimensions. They further maintain that behaviorally the exchange risk is never borne alone but only in conjunction with purchasing power risk of a currency. An implication is that attempts to separate the two risks, say, by deviations from PPP are possible but probably misguided on a behavioral ground.

We now can develop the demand for foreign assets for a risk-averse investor under PPP. The solution process is identical to the one outlined in the previous section. We obtain the conditions, assuming \( A \neq 1 \), under which changes in the consumption basket are neutral to international investment decisions when PPP holds

\[
(S_p - S_{p*})^2 = (S_{p*} - S_{p}) - (S_{c_p*} - S_{c_p}).
\]

The difference from the general case, (17), is that \((S_{c_p*} - S_{c_p})\) now has been replaced by \((S_p - S_{p*})^2\). This result further supports the point made earlier regarding the transformation of exchange risk into inflation risks by PPP. This condition indicates that the international investment decisions can be divorced from the consumption basket, even under PPP, only if (barring \( A = 1 \)) the variance of goods prices and the covariance concerning the interest rates and prices meet the specified equality condition.

V. Conclusion

In this paper, we have been concerned with the relationship between the commodity market and asset market in terms of a global investor-consumer who is trying to make an optimal international investment decision in real terms. Using the stochastic optimum control theory, the demand for foreign assets for a risk-averse investor has been derived in a two-commodity, two-asset model where the interest rates, prices, and exchange rates are all stochastic. The demand for foreign assets includes both hedging and speculative motives that are shown to be dependent upon the exchange risk and risk aversion. We then analyzed the conditions under which international investment decisions can be separated from the consumption basket. Conditions for such separation in a stochastic
world are shown to be much more stringent than the ones provided by usual purchasing power parity. An observation is also made that the role of PPP should be viewed as that which effects the transformation of the exchange risk to inflation risk rather than its elimination.

References


