Math 147: Review for Final Exam

Systems of Linear Equations
- solve a linear system
  - coefficient (augmented) matrix
  - consistent vs. inconsistent system
- elementary row operations
- row echelon form (ref) – existence of solutions
- reduced row echelon form (rref) – solutions
- pivots; pivot columns
- general solutions; free variables; \( x_p + x_h \)

Vector and Matrix Equations
- column vectors; vector equations
- linear combinations
- spanning sets; \( \text{Span} \{ x_1, x_2, \ldots, x_n \} \)
- linear combinations as matrix products
- matrix equations \( Ax = b \)
- homogeneous systems
  - number of solutions
  - general solution, geometric interpretation
- linearly independent set
- matrix transformations
- projections

Matrix Operations
- operations: \( A = B, A + B, A^k, \) and \( A^T \)
- multiplication
  - matrix times a vector
  - row-by-column (dot product)
- properties of \( AB \)
- inverse of a matrix
  - algorithm for computation
  - solution of \( Ax = b \) for \( x \)
- properties: \( (A^{-1})^{-1}, (AB)^{-1}, (A^T)^{-1} \)

Subspaces
- definition of subspace; Subspace Test closure
- basis, dimension, span, linear independence, basis for \( \text{ns}(A), \text{col}(A), \text{row}(A) \)

Determinants
- \( 2\times2 \) rule; \( 3\times3 \) rule; no \( 4\times4 \) rule
- properties
- via row operations

Eigen Stuff
- \( Av = \lambda v \)
  - \( \lambda \) via characteristic polynomial
  - \( v \) via \( \text{ns}(A - \lambda I) \)
- diagonalization
  - \( \lambda \) distinct vs. \( \lambda \) repeated
  - construction of \( P \) and \( D \)

Orthogonality
- dot product, norm, unit vector, distance
- orthogonal (orthonormal) set (basis)
- projections
- Gram-Schmidt process
- Spectral Theorem

Vector Spaces
- Vector space; subspace
- Linear combination, span, LI, basis, dimension

Function Spaces
- Inner products; orthonormal bases; projections
Be able to determine if a linear system has a solution or not. If it does, then find it.

Cases:
1. Homogeneous—find \( \text{ns}(A) \)
2. Nonhomog.—express as \( x_p + x_h \)

If \( Ax = b \) is consistent what is the connection to \( \text{col}(A) \)?

What is \( \text{rref} \) and why does it help in finding solutions of linear systems?

If a consistent system has 5 unknowns and 2 free variables how many leading ones are there in the \( \text{rref} \)? What are the connections here to \( \text{ns}(A) \), \( \text{col}(A) \), and \( \text{row}(A) \)?

Explain \( \text{span}(x_1, x_2, \ldots, x_k) \).

If \( S = \{v_1, v_2, v_3, v_4\} \) is a basis for a subspace \( W \), what is the \( \dim(W) \)? If \( T \) is a set of 5 vectors in \( W \) why can \( T \) not be a basis? Could \( \text{span}(T) = W \)? (Explain why or why not.) If \( Q \) were a set of 3 vectors from \( W \), could \( Q \) be basis? Could \( \text{span}(Q) = W \)? (Explain.)

How do you check to see if a set is a subspace?

How do you check to see if a set is linearly independent?

For vectors \( v \) and \( w \), explain how to compute \( \text{PROJ}_w v \). How do you find a vector orthogonal to \( w \) using the projection?

In \( \mathbb{R}^3 \) how do you find the projection of a vector onto the xy-plane?

What are matrix transformations? How do they affect the size of objects which are transformed?

If \( T: \mathbb{R}^3 \to \mathbb{R}^3 \) is a matrix transformation show that for vector \( v \) and \( w \) in \( \mathbb{R}^3 \) and scalar \( k \) that \( T(v + w) = T(v) + T(w) \) and \( T(kw) = kT(w) \).

Express \( Ax \) in terms of the columns of \( A \).

What does it mean to say \( A \) is nonsingular?

Name three ways you can test to see if \( A \) is nonsingular.

If \( A \) is nonsingular describe the solution set of \( Ax = b \) for any \( b \).

If \( C \) is singular describe the solution set of \( Cx = b \) for any \( b \).

State three properties of inverses.

How do we compute determinants? What good are they? What properties do they have?

How do you compute the eigenvalues of a matrix?
How do you compute the eigenvectors of a matrix?

What does it mean to say \( A \) is diagonalizable? If it is, what information do we know about matrix \( A \)? What kind of matrix is always diagonalizable? What do we call a matrix which is not diagonalizable?

Define an orthonormal set. What uses have we made of orthonormal sets?

Given a linearly independent set, how can you manufacture a related orthonormal set? (What is the key computational idea that is used in the process?)

What is a vector space? Name several.

What is an inner product? What is an inner product space. Name several.

In an inner product space how do we determine if vectors are orthogonal?

What is function space? Name several.

In a function space how do we determine projections? How do we determine orthonormal sets? How do we determine the distance from a vector to a subspace?
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Name three properties of matrix addition.
Name two properties of matrix multiplication that are very different from multiplication of real numbers.
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