5. FOURIER TRANSFORM & BODE PLOTS

As we have seen, the Fourier transform can be used for aperiodic signals as well as for systems which could be filters or circuits. For this exercise you will be looking at the frequency characteristics of various first and second order circuits. To determine the frequency characteristics you will need to perform a series of calculations, some analysis, and some MATLAB experiments.

Given the electrical circuits shown below -

1. Determine the expression for the Laplace transfer function. This data will be used as input for MATLAB.

2. Determine the expression for the Fourier transfer function, the expression for the magnitude response and the phase response.

3. Sketch the log magnitude frequency response (Bode plot) of the Fourier transfer function. Note, you do not have to sketch the phase response but you must determine the mathematical expression for the phase.

4. Obtain the Bode plots (log magnitude in dB and phase response) using MATLAB – see below.

To obtain the Bode plots of the circuits above you must first determine the transfer function of each circuit. For example, Circuit #1 transfer function would be:
You will be using three new MATLAB functions, `fbode` (which has three different forms) and `semilogx` and `subplot`. The three forms of `fbode` are:

- `fbode(num, den)`
- `fbode(num, den, w)`

where `fbode` draws the Bode plot of the LTI (linear time-invariant) model. The frequency range and number of points are chosen automatically. Note, the second form of `fbode` shown above uses the user-supplied vector `w` of frequencies. The third form is:

```
[mag, phase, w] = fbode(num, den)
```

where - `mag` is the magnitude $|H(\omega)|$ and phase is the angle $\angle H(\omega)$, `w` is the vector `w` of frequencies if not supplied by the user on the right side of the equation (as shown in `fbode` command above), `num` is the numerator of the transfer function $H(s)$ in descending powers of $s$, and `den` is the denominator of $H(s)$ in descending powers of $s$. For example, given the following transfer function $H(s)$ –

$$H(s) = \frac{2000\pi}{s + 2000\pi}$$

You would enter the values of the numerator in the following way –

```
>> num = [ a1 a2 a3 a4 ];
```

Note - the use of brackets to identify that `num` is a vector!

You would then enter the values of the denominator in the following way –

```
>> den = [ b1 b2 b3 b4 ];
```

Also, note the use of the MATLAB function `semilogx`. This function is the same as `plot(...)`, except a logarithmic (base 10) scale is used for the X-axis.

```
semilogx(n, x))
```

where- `n` is the x-axis index and `x` are the vector values to be plotted.
The final MATLAB function is `subplot`. This function allows you to plot an \( n \times m \) array of plots number of plots together on the same page, identifying each plot with an index number \( p \). The form of the function is –

\[
\text{subplot}(n, m, p)
\]

Where – \( n \) is the number of rows, \( m \) is the number of columns, and \( p \) is the pane number. So, for a series of 6 plots, i.e., two rows and three columns, shown below, the darken pane would be plotted with the following command `subplot(2, 3, 4)`. The `subplot` command is followed by the appropriate `plot` command used to plot the data in pane 6.

![6 plots arranged in a 2x3 grid](image)

The following is the MATLAB code needed to generate the Bode plots for the transfer function (of circuit #1) derived on the previous page.

```matlab
>> num = [2000*pi]; % Define the numerator of H(s).
>> den = [1 2000*pi]; % Define the denominator of H(s).
>> fbode(num, den); % Calculate the data and construct the Bode plots.
>> title('Bode plot'); % Put a title on graph.
```

To plot the magnitude and phase characteristics of the filters in Hertz rather than radians per second do the following –

```matlab
>> [mag, phase, w] = fbode(num, den);
>> subplot(2, 1, 1), semilogx(w/2*pi, 20*log10(mag))
>> xlabel('Frequency [Hz]'), ylabel('Gain [dB]')
>> title('Bode Plot')
>> subplot(2, 1, 2), semilogx(w/2*pi, phase)
>> xlabel('Frequency [Hz]'), ylabel('Phase [degrees]')
>> grid;
```

**DESIGN PROBLEM** –

Using only a resistor and inductor, design a first order high pass filter with a cutoff frequency of 20 kHz. Justify your design using MATLAB – that's very possibly what you might do in industry!
QUESTIONS

1. Which of the following electrical components (resistor or capacitor) of circuits #2 and #3 above, affect the cutoff frequency, and how?

2. Which of the following electrical components (resistor, capacitor or inductor) of circuit #4 above, affect the cutoff frequency, and how?

3. Which of the following electrical components (resistor, capacitor or inductor) of circuit #4 above, affect the magnitude at cutoff, and how?

4. For the high pass filter design exercise above, what will be the roll-off of the filter?