In the context of the simple linear regression model

\[ y_t = \beta_1 + x_t \beta_2 + u_t, \]

\[ t = 1, 2, \ldots, 10 \]

we have found the following data generating process

\[ x_t = \begin{cases} 
5 + a & \text{for } t = 1, 2, 3, 4, 5 \\
5 - a & \text{for } t = 6, 7, 8, 9, 10 
\end{cases} \]

1. Write down the 10x2 matrix X.
2. Calculate X'X.
3. Calculate \((X'X)^{-1}\).
4. Calculate \((X'X)^{-1}\) for \(a = 10, 5, 1, .1, 0\). How does this relate to the problem of collinearity? Your answer should be intuitive.
5. Compute the characteristic roots of each X'X, for the original data and for the normalized data. To normalize the data each vector should have length one. When you examine the roots are your suspicions of question 4 confirmed?