Economic Implications of Single Cost Driver Systems

Rajiv D. Banker
University of Minnesota
and
Gordon Potter
University of Minnesota

Abstract: We subject claims about the benefits of activity-based costing systems to the scrutiny of analytical models incorporating rational behavior by users of product costing systems. We find that a monopolist is almost always strictly better off using multiple cost drivers as in an activity-based costing (ABC) system even when the system makes measurement errors in assigning overhead costs to activities. More importantly, we show that this result replicates for firms competing in an oligopoly when the cost and demand parameters are in steady state. The firms are strictly better off using a direct labor based single cost driver (SCD) system, however, if the demand for the overcosted labor intensive product is expected to grow sufficiently relative to the demand for the undercosted setup intensive product. This suggests that, facing imperfect competition, it is sometimes optimal for firms to persist in using a single cost driver system rather than switching to an activity-based cost system.

I. INTRODUCTION

The recent management accounting literature provides several case studies of multi-product firms whose product costs are distorted because they allocate overhead costs to products on the basis of a single volume-related variable. Johnson and Kaplan [1987] report that most of the firms that they have personally studied use simple cost accounting systems that assign overhead costs to products based on the direct labor hours expended on each product. Cooper and Kaplan [1987] describe three multi-product firms with complex manufacturing processes that rely mainly on direct labor hours to allocate overhead costs to products. They document how the apportionment of (long term) variable overhead costs on the basis of a single cost driver led these firms to overcost some high-volume products and undercost most low-volume products. Cooper [1987] argues that such obsolete single cost driver based systems often lead to improper pricing and distort "the strategy selected by the firm....tempting management to focus incorrectly on low-volume, specialty business." Shank and Govindarajan [1988] echo this, stating "volume-based costing can seriously

Helpful comments and suggestions from an anonymous referee and seminar participants at University of California (Berkeley), University of California (Los Angeles), Michigan State University, University of Minnesota, Rice University, San Diego State University, Stanford University, University of Wisconsin (Madison), and the 1991 Annual Management Accounting Research Conference are gratefully acknowledged.
distort the way a firm ... assesses the profit impact of its pricing and product emphasis decision." This assumption that it is sub-optimal for firms to persist in the use of single cost driver systems seems to be an axiom in the cost driver literature, but it has received little attention in the analytical management accounting literature.

A recent survey of 566 controllers by the National Association of Accountants [Schiff, 1991] indicated that 11 percent have implemented an activity-based costing (ABC) system, 19 percent are considering it and 70 percent of the firms are not currently considering switching to an activity-based costing system. Miller and Kim [1990] find that 22 percent of the 200 large manufacturing firms they surveyed used ABC—but it was the second least effective action in terms of 1988-89 payoff. While this represents a faster rate of adoption than that observed for previous innovations in management accounting, such as the use of discounted cash flow analysis for capital budgeting, it is not apparent that all firms in all industries perceive that the benefits from using an ABC system outweigh the costs of implementing and operating it.

One explanation for the continued use of single cost driver (SCD) systems may be the high cost of switching to an ABC system that recognizes multiple cost drivers. Such costs may include personnel, consulting, training and software costs for installing an ABC system, as well as organizational costs resulting from consequent realignment of strategies espoused by managers based on information provided by SCD systems. Costs of computerizing and operating complex multi-purpose cost systems, however, have declined sharply in recent years. In any case, such costs must be weighed against the presumed benefits from a multiple cost driver system. In this paper therefore, we ask the questions whether and to what extent a firm will benefit more if it switches from a SCD to an ABC system for its product mix decisions. We also identify conditions under which the use of a SCD system leads to higher expected profits than an ABC system even when the costs of implementing an ABC system are negligible.

We consider two different possibilities. First, we recognize that a multiple cost driver system may introduce errors because precise identification of the overhead costs associated with specific cost drivers may not be possible. Such accounting inaccuracies introduced in multiple cost driver systems need to be traded off against the imprecision resulting from the pooling of all overhead costs into a single cost driver system. Second, we

---

1An important empirical question is whether overhead costs depend on activity variables other than volume-related variables like direct labor. Cooper [1987], Johnson and Kaplan [1987], Miller and Vollman [1985] and others assert that a significant portion of manufacturing overheads varies with transactions such as setups, purchase orders, engineering change orders and material movements. Using a sample of 37 plants owned by one firm, Foster and Gupta [1990], however, find that manufacturing overhead is more frequently significantly correlated with volume-related drivers than complexity or efficiency-related drivers. Banker and Johnston [1993] find a similar highly significant relation between overheads and volume measures for a sample of airline firms, but they also find several operating strategy variables to be significant drivers of overhead costs. Using a sample of 31 plants from firms in the electronics, machinery and automobile components industries, Banker et al. [1992] confirm the high correlation between manufacturing overheads and direct labor, but they find that complexity variables representing transactions identified by Miller and Vollmann are also significant. This empirical question is important because if cost drivers other than direct labor-type volume measures are insignificant, then any benefits from switching from a single volume-based cost driver system are likely to be minimal.
recognize the possibility that the implications of product costs for a firm's pricing and product mix decisions may be quite different when it competes in an oligopoly than when it operates as a monopolist. In particular, we note that explicit collusion between firms is precluded by anti-trust statute in most oligopolistic environments, and therefore, we examine whether product mix decisions based on distorted product costs can lead to an equilibrium that is more favorable to all the firms in an oligopoly than that attained when such decisions are based on accurate product costs.2

A standard result in oligopoly theory is that competitive prices are strictly less than collusive prices; collusive prices yield higher profits for all firms. Equilibrium prices are increasing in the perceived product costs. An obvious result, therefore, is that all firms are better off if they commit to using cost systems that inflate all product costs by some positive amount.3 Such an observation, however, has little descriptive value in comparing actual product costing systems that are institutionally constrained to base product cost estimates on actual costs. That is, in SCD, ABC and most common cost accounting systems, if the cost of one product is overstated then it must be offset by understated costs for some other products. It is no longer obvious, therefore, that distortion of product costs increases expected profits. This analytical tension from both over- and understatement of product costs makes the problem of evaluating the expected profit impact of product costing systems interesting, and forms the focus of our paper.

The remainder of this paper has the following structure. The next section develops our basic model for a monopolist firm producing two products, and compares an imperfect activity-based costing system with a single cost driver system. Section III extends our model to identify conditions under which the use of a single cost driver based system by all firms in an oligopoly dominates their use of a perfect activity-based costing system. Section IV concludes the paper.

II. PRODUCT MIX PROBLEM IN A MONOPOLY

We begin by considering a monopolist firm that produces two products, j = 1, 2. The true expected (long term) variable cost $c_j$ for a unit of product $j$ is represented as the sum of the expected direct cost $v_j$ and the expected (long term) variable overhead $h_j$:

$$c_j = v_j + h_j \quad (1)$$

where $c_j =$ total expected (long term) variable cost per unit of product $j$

$v_j =$ expected direct cost per unit of product $j$

$h_j =$ expected (long term) variable overhead cost per unit of product $j$

The activity-based costing literature [e.g., Cooper and Kaplan, 1987] prescribes the use of long term variable costs for evaluating product profitability.4 In our model, the total actual (long term) variable overhead costs

---

2A possibility that we do not consider in this paper is whether the finer information provided by activity analysis is valuable for monitoring and incentive purposes.

3Alles [1991] also discovered this same result.

4Banker and Hughes [1991] show that expected equilibrium product prices in imperfectly competitive markets are functions of their corresponding "long term variable" costs.
(x) consist of two activity cost pools represented by the symbols L and M; one varying with the total number of direct labor hours \( (b_L) \) and the other varying with the total number of machine setups \( (b_M) \):

\[
x = x_L + x_M
\]

Since, \( x_L \) and \( x_M \) vary proportionately with their respective cost drivers \( b_L \) and \( b_M \), we write

\[
x_i = \theta_i b_i \text{ for } i = L, M,
\]

where \( \theta_i \) represents the variable overhead cost per unit of the cost driver. The parameters \( \theta_i \) follow first order stationary processes inasmuch as

\[
E(\theta_i) = \theta_i^0
\]

where \( \theta_i^0 \) is the realized value of the parameter \( \theta_i \) in the preceding period, which we refer to as the base period and distinguish with the superscript \( ^0 \). This assumption in (4) reflects a critical aspect of common product costing systems that base the estimation of the product costs in the current period on an analysis of the actual costs in the preceding (or base) period.

The two products impose different demands (in general) on the two overhead activities as reflected in the following equation (5) and inequality (6):

\[
b_i = \sum_{j=1}^{2} \lambda_{ij} q_j \text{ for } i = L, M,
\]

where \( q_{ij}, j=1,2, \) are the number of units of the two products, and

\[
\lambda_{L1}/\lambda_{L2} > \lambda_{M1}/\lambda_{M2}.
\]

Thus, product 1 requires proportionately more direct labor related overhead activity per unit produced, and product 2 requires proportionately more machine setup related overhead activity per unit produced. We therefore refer to product 1 as the labor intensive product and product 2 as the setup intensive product.

From (2), (3) and (5), it follows that the total variable overhead cost can be written as

\[
x = \sum_{i=L,M} \sum_{j=1}^{2} \theta_i \lambda_{ij} q_j
\]

and the true expected variable overhead cost per unit of a product \( j \) is given by

\[
h_j = \sum_{i=L,M} E(\theta_i) \lambda_{ij} = \sum_{i=L,M} \theta_i^0 \lambda_{ij} = \sum_{i=L,M} \lambda_{ij} \theta_i^0 / b_i^0.
\]

---

\(^5\)Our analysis extends directly to the case when a non-zero inflationary growth is expected for the overhead rate.
The firm managers responsible for product mix decisions, however, do not know the true expected cost per unit of product j. The only cost information available to them is that measured by the firm's cost accounting system. While direct costs, \( v_j \), are measured accurately by all the cost accounting systems we consider, the variable overhead cost estimates, \( \hat{h}_j \), depend on the method used for allocating overheads. Therefore, the accounting costs differ, in general, from the true expected costs represented in equation (8) above. In this paper, our focus is on product mix decisions given a firm's choice of its product costing system. The product managers base their product mix decisions on the costs reported to them by the firm's accounting system and demand information observed by them. Actual demand is realized based on equilibrium market prices and actual costs are incurred in accordance with the true cost function. Product managers' compensation functions are assumed to be increasing in the firm's reported profit, and therefore, risk neutral product managers choose product quantities to maximize the expected reported profit based on the chosen product costing system [Fershtman and Judd, 1987]. We consider, in particular, the following three cost accounting systems:

(i) **System Z**: a system that allocates costs on the basis of a single cost driver: direct labor hours \( (b_L) \) only,

(ii) **System I**: an imperfect activity-based system employing both cost drivers \( b_L \) and \( b_M \) for overhead allocation, but which introduces an error in analyzing \( x \) into its components \( x_L \) and \( x_M \), and

(iii) **System P**: a perfect activity-based costing system that employs both cost drivers \( b_L \) and \( b_M \) for overhead cost allocation, and accurately analyzes the total overhead costs into its components \( x_L \) and \( x_M \).

We refer to System Z as the single cost driver (SCD) system, System I as the imperfect ABC system, and System P as the perfect ABC system.

System Z considers the actual variable overhead costs \( (x^o) \) for the preceding period, and assumes that all of these costs vary in proportion to actual direct labor hours \( (b_L^o) \) for the same period. An important assumption in System Z is that the total variable overhead costs in a period are proportional to the direct labor hours in the same period. Thus,

\[
x = \hat{\theta} b_L \quad \text{with} \quad \hat{\theta} = 9 \quad \text{and} \quad \theta = x^o / b_L^o
\]

where \( \hat{\theta} \) is the presumed proportionality parameter. A single overhead rate \( (=x^o/b_L^o) \) is employed for product costing purposes. Because each unit of product \( j \) requires \( \lambda_{tj} \) units of direct labor hours (cost driver \( b_L \)), the estimated expected variable overhead costs per unit of product \( j \) under this system are:

\[
\hat{h}_j^z = \frac{x^o}{b_L} \lambda_{tj} \quad \text{for} \quad j = 1, 2, \ldots
\]

Formal analysis of a firm's product costing system choice is discussed by Banker and Potter [1992].
where the superscript o, as before, represents the base period. It also follows immediately from (6) that product 1 is overcosted \((h_1^o > h_1)\) and product 2 is undercosted \((h_2^o < h_2)\).

System I divides the total variable overhead costs into two activity-based cost pools. While the total variable overhead costs \((x^o)\) for the base period are observed accurately, we allow for the possibility that the division of the costs into two pools introduces some measurement error \((e^o)\) randomly distributed with mean \(\bar{e}^o\) and variance \(\sigma^2_o\). Thus, the observed overhead cost pools \(\hat{x}_i^o\), \(i = L, M\), are equal to:

\[
\hat{x}_L^o = x_L^o - e^o \\
\hat{x}_M^o = x_M^o + e^o
\]

The basic assumption in System I is that the observed overhead costs in each pool are proportional to the corresponding cost driver, so that

\[
\lambda = \sum_{i=L,M} \hat{\theta}_i b_i \quad \text{with} \quad (14)
\]

\[
E(\hat{\theta}_i) = \theta_i = \frac{x_i^o}{b_i} \quad \text{for} \quad i = L, M, (15)
\]

where \(\hat{\theta}_i\) are the presumed proportionality parameters. Thus, two overhead cost rates \(= \frac{x_i^o}{b_i}, i = L, M\) are employed for product costing purposes. This imperfect activity based costing system then estimates unit variable overhead costs as:

\[
h_j = \sum_{i=L,M} \lambda_{ij} \frac{\hat{x}_i^o}{b_i} \quad \text{for} \quad j = 1, 2. (16)
\]

System I undercosts the labor intensive product 1 (and overcosts the setup intensive product 2) if and only if \(e^o > 0\). Product managers observe only \(\hat{h}_j\) if System I is in place, they do not know the actual \(e^o\), \(\bar{e}^o\) or \(\sigma^2_o\).

The perfect activity based costing system, System P, operates just like System I except that the actual measurement error \(e^o\) is always equal to zero in this case, and hence it provides unbiased estimates of the true variable overhead costs.

While the cost structures outlined above are abstractions, they capture the most important difference between direct labor based (System Z) and activity-based (Systems I and P) costing systems. The true overhead costs are driven by more than one factor, and this fact is captured accurately only by System P. System I reflects the potential for measurement error in an accounting system that assigns costs to multiple overhead pools, and System Z reflects the observed phenomenon of firms allocating overhead costs to products based on direct labor alone.

The inverse demand functions for the monopolist firm in the two product markets are represented by the linear forms:
where $p_j$ is the price of product $j$, $\alpha_j$ and $\beta_j > 0$ are the estimated parameters of the inverse demand function, and $\epsilon_j$ reflects the residual uncertainty when estimating the demand relation. It is also assumed implicitly in the above expression that the values of $q_j$ are constrained to ensure that $p_j$ remains positive.

The following equation then specifies the expected reported profit when a cost accounting system $s$, $s = Z, I, P$, is employed, and the product mix represented by the quantities $q_1$ and $q_2$ is chosen:

$$E(\pi^s) = \sum_{j=1}^{2} (\alpha_j - \beta_j q_j - \nu_j - h_j) q_j - f \quad \text{for} \ s = Z, I, P.$$  

(18)

where $f \geq 0$ represents fixed costs and $h_j$ is the estimated unit variable overhead cost for product $j$.

The firm managers with the delegated responsibility for product mix decisions, choose $q_j$ to maximize the expected reported profit in equation (18). The first order optimality condition yields the following perceived optimal quantity:

$$q_j^s = \frac{(\alpha_j - \nu_j - h_j)}{2\beta_j} \quad \text{for} \ s = Z, I, P.$$  

(19)

The true expected profit when the quantities $q_j^s$ are chosen is then given by:

$$E(\pi'^s) = \sum_{j=1}^{2} (\alpha_j - \beta_j q_j^s - \nu_j - h_j) q_j^s - f \quad \text{for} \ s = Z, I, P.$$  

(20)

Note that whereas the optimal quantity in (19) is based on the perceived costs $h_j^s$, the term $h_j$ in (20) represents the true expected unit variable overhead costs. (In particular, of course, $h_j^s = h_j$ because the perfect activity based costing system yields unbiased estimates.)

Comparing the optimal expected profit under System Z or System I with that under System P, we obtain for $s = Z, I$,

$$E(\pi'^P) - E(\pi'^s) = \sum_{j=1}^{2} \left( (\alpha_j - \nu_j - h_j)(q_j^P - q_j^s) - \beta_j(q_j^P - q_j^s)^2 + q_j^s \right)$$  

$$= \sum_{j=1}^{2} \frac{h_j^s - h_j}{4\beta_j} \left[ 2(\alpha_j - \nu_j - h_j) - 2(\alpha_j - \nu_j) + (h_j^s + h_j) \right]$$  

$$= \sum_{j=1}^{2} \frac{h_j^s - h_j}{4\beta_j}^2 > 0.$$  

(21)

The residual $\epsilon_j$ is assumed to be statistically independent of the corresponding random variable representing the uncertainty remaining when estimating the (long term) variable costs of product $j$.

The second order conditions for maximization are also satisfied, so that the first order condition as in (19) always characterizes the optimal solution for $\alpha_j - \nu_j - h_j^s \geq 0$. 
We formalize this relation in the following:

**Proposition 1(a):** The monopolist firm is always strictly better off basing its product quantity choice on costs generated by System P rather than either System I or System Z.

Next, we employ the expressions in (3), (8), (11) and (16) to compare the estimated unit variable overhead costs \( h_j \), \( s = Z, I, j = 1, 2 \), with the true costs \( h_j \). Thus

\[
\hat{h}^z_j - h_j = x_M^0 \left[ \frac{\lambda_{ij}}{b_{ij}} - \frac{\lambda_{Mj}}{b_{Mj}} \right], \quad \text{and} \quad (22)
\]

\[
\hat{h}^I_j - h_j = -e^0 \left[ \frac{\lambda_{ij}}{b_{ij}} - \frac{\lambda_{Mj}}{b_{Mj}} \right]. \quad (23)
\]

Therefore, using (21) and (23), and taking expectation over \( e^0 \), we obtain

\[
E(\pi^I) - E(\pi^z) = \sum_{j=1}^{2} \frac{1}{4\beta_j} \left[ \sigma_e^2 + (e^0)^2 \right] \left[ \frac{\lambda_{ij}}{b_{ij}} - \frac{\lambda_{Mj}}{b_{Mj}} \right]^2 > 0 \quad (24)
\]

Thus, the expected value of the imperfect ABC system is strictly decreasing in the *a priori* bias \( e^0 \) and noise \( \sigma_e^2 \). While this result is intuitive for a single decision maker, we shall see in the next section that it holds in an oligopolistic environment only under specific conditions.

The difference between the optimal expected profits under the imperfect activity-based costing system, System I, and the direct labor based system, System Z, is given by:

\[
E(\pi^I) - E(\pi^z) = [E(\pi^IP) - E(\pi^z)] - [E(\pi^P) - E(\pi^Z)]
\]

\[
= \sum_{j=1}^{2} \frac{1}{4\beta_j} \left[ (x_M^0)^2 - \sigma_e^2 - (e^0)^2 \right] \left[ \frac{\lambda_{ij}}{b_{ij}} - \frac{\lambda_{Mj}}{b_{Mj}} \right]^2 \quad (25)
\]

The difference is positive provided \( (x_M^0)^2 - |e^0|^2 > \sigma_e^2 \). Thus, we have

**Proposition 1(b):** The monopolist firm is strictly better off using the cost estimates generated by System I rather than by System Z if and only if the variance of the measurement error \( e^0 \) is less than the difference in the squares of the true variable overhead cost \( x_M^0 \) and the *a priori* bias \( e^0 \).

Since the measured overhead cost pool amount \( x_M^0 \) in (13) cannot be negative, we have \( -e^0 \leq x_M^0 \). Therefore, a System I that systematically introduces a bias \( e^0 \) with certainty \( (\sigma_e^2 = 0) \) is preferred over System Z unless the bias is so large that it doubles the costs assigned to the overhead pool M. Such large bias might occur if cost pool \( x_M^0 \) is very large relative to \( x_M^0 \). Intuitively, System Z can be thought of as committing a 100 percent measurement error \( (e^0 = -x_M^0) \) with certainty \( (\sigma_e^2 = 0) \), and therefore it is strictly
inferior to a System I that introduces a smaller error \(|e^a| < x_M^0\) with certainty because the difference \(E(x^n) - E_x E(x^n)\) is strictly increasing in \(|e^a|\).

Equation (25) also reveals that the extent of the benefit obtained by switching to an imperfect activity-based costing system, from a single cost driver system, depends on three factors. One factor is the difference \((x^a_M)^2 - (e^a)^2 - \sigma^2\) discussed above. The benefit from switching to System I is decreasing in the a priori bias \(e^a\) and noise \(\sigma^2\). A second factor is the parameter \(\beta_j\) in the inverse demand function. As the demand elasticity increases, the benefit from activity-based cost information diminishes. A third factor is represented by the term \(\left[\frac{\lambda_{Lj}}{b_L^0} - \frac{\lambda_{Mj}}{b_M^0}\right]^2\) present in (25). This amount reflects the relative proportions of the two resources consumed by the two products and would equal zero if \((\lambda_{L1}/\lambda_{L2}) - (\lambda_{M1}/\lambda_{M2})\) were zero. In this case the firm would suffer no loss of expected profit by retaining the single cost driver system. Condition (6), however, precludes this possibility. The benefit from the activity-based costing system decreases as the demands of the two products on the two activities get close to proportional, and the difference \((\lambda_{L1}/\lambda_{L2}) - (\lambda_{M1}/\lambda_{M2})\) gets smaller.

III. PRODUCT MIX PROBLEM IN AN OLIGOPOLY

The prior section determined that a monopolist is generally better off with an imperfect activity-based cost system rather than a cost system that allocates costs to products based on direct labor alone. Consequently, absent competition or high switching costs it may be difficult to justify continuance of the single cost driver approach. We now consider a simple model of oligopolistic competition involving \(n\) symmetric competitors, indexed by the superscript \(r\), who compete in the same two product markets and have the same true cost functions as described in the previous section. Their common inverse demand functions are represented by

\[
p_j = \alpha_j - \beta_j \sum_{r=1}^{n} q_j^r + \varepsilon_j, \quad \text{with } q_j^r > 0, \quad j = 1, 2; \quad r = 1, \ldots, n. \tag{26}
\]

Similar to the monopolist case, all firms first select their product costing systems, and product managers seek to maximize the expected reported profit based on their accounting system. Parameters \((\alpha_j, \beta_j)\) of the expected inverse demand functions and product costs as reported by existing accounting systems are common knowledge to all product managers. Here we restrict attention to the case when all firms have decided to employ the same cost accounting system \(s\), \(s = Z, I, P\). Under a cost accounting system

---

9We assume identical firms only to make our analysis more transparent. Our results extend directly to the case of asymmetric firms. However, additional assumptions about how the key parameters become common knowledge are required. See Banker and Hughes [1991].

10This is in contrast to the information asymmetry in Milgrom and Roberts [1982] where potential entrants do not know the incumbent's costs, and the established firm may signal its costs with its pricing strategy to deter a potential entrant from entering the market.
that estimates the unit variable overhead costs to be \( \hat{h}_{ij} \), the expected reported profit of a firm \( r \) is given by:

\[
E(\pi_{rs}) = \sum_{j=1}^{2} \left[ \alpha_j - \beta_j \sum_{p=1}^{n} q_{jp}^r - v_j - \hat{h}_{ij} \right] q_{jr}^r - f, \quad r = 1, \ldots, n. \quad (27)
\]

First order conditions for the maximization of \( E(\pi_{rs}) \) yield the following Cournot-Nash equilibrium characterization of optimal quantities under a system \( s \):

\[
q_{jr}^r = \frac{\alpha_j - v_j - \hat{h}_{ij} - \beta_j \sum_{p=1}^{n} q_{jp}^r}{2\beta_j}, \quad r = 1, \ldots, n. \quad (28)
\]

Consistent conjectures equilibrium (CCE) is an alternative notion of equilibrium in oligopoly models, see e.g., Bresnahan [1981]. In our model of linear cost and inverse demand functions, the unique CCE is given by the Bertrand-Nash equilibrium, which implies that the prices are set equal to the (identical) marginal costs of the two firms. In the presence of (long term) fixed costs, however, the firms cannot survive under such a pricing regime. We have chosen, therefore, to employ the Cournot-Nash criterion to characterize the oligopoly equilibrium concept. Based on the assumed equilibrium concept, the optimal quantity for each firm \( r \) under a product costing system \( s \), is obtained by solving the \( n \) equations in (27):

\[
q_{jr}^r = \frac{(\alpha_j - v_j - \hat{h}_{ij})}{(n + 1)\beta_j}, \quad r = 1, \ldots, n. \quad (29)
\]

In this setting, the prices \( p_{js} \) are:

\[
p_{js} = \frac{1}{n + 1} \frac{n + n}{n + 1} (v_j + \hat{h}_{ij}). \quad (30)
\]

Each firm's true expected profit under the optimal quantity choice in (29) is given by:

\[
E(\pi_{rs}) = \sum_{j=1}^{2} \left[ \alpha_j - n\beta_j q_{jr}^r - v_j - \hat{h}_{ij} \right] q_{jr}^r - f \quad (31)
\]

Comparing the optimal expected profit under a system \( s \) with that under System P reveals the following difference:

\[
E(\pi_{rp}) - E(\pi_{rs}) = \sum_{j=1}^{2} (\alpha_j - v_j - \hat{h}_{ij})(q_{jr}^p - q_{jr}^s) - n\beta_j (q_{jr}^p - q_{jr}^s)(q_{jr}^p + q_{jr}^s)
\]

\[
= \sum_{j=1}^{2} \left( \frac{\hat{h}_{ij}}{(n + 1)^2 \beta_j} \right) [(n + 1)(\alpha_j - v_j - \hat{h}_{ij}) - 2n(\alpha_j - v_j)] + n(\hat{h}_{ij} + h_j)]
\]

\[
= \sum_{j=1}^{2} \frac{(\hat{h}_{ij} - h_j)^2}{(n + 1)^2 \beta_j} - \frac{(n - 1)(\hat{h}_{ij} - h_j)(\alpha_j - v_j)}{(n + 1)^2 \beta_j} \quad (32)
\]

\[\text{In the present case, this expression follows immediately from (27) by appealing to the symmetry resulting from identical firms.}\]
It follows from (8) that the actual overhead costs $x^o$ in the base period are equal to
\[ \sum_{j=1}^{2} h_j r_{j}^{\text{rso}} = \sum_{i=L,M} x_i \sum_{j=1}^{2} \lambda_{ij} q_j^{\text{rso}} / b_i^o. \]
By construction, the estimated overhead costs $\hat{h}_j^s$ are also such that
\[ \sum_{j=1}^{2} h_j^s q_j^{\text{rso}} = x^o. \]
This is verified easily by referring to the definition of $h_j^s$, $s=Z,I$, in (11) and (16). Therefore, we have:
\[ \sum_{j=1}^{2} (h_j^s - h_j) q_j^{\text{rso}} = 0. \]
We introduce next a concept of inter-period consistency of the cost accounting system to further simplify the expression in (32). In a steady state where the parameters $(\psi_j, \lambda_{ij}, \theta_i, \alpha_i, \beta_j)$ of the cost and demand functions do not change, the quantities produced of the two products also should remain unchanged. That is, in a steady state, the quantity $q_j^{\text{rso}}$ in the base period is equal to the quantity $q_j^{\text{rs}}$ in the current period:
\[ q_j^{\text{rso}} = q_j^{\text{rs}} = \frac{(\alpha_j - \psi_j - \hat{h}_j^s)/(n + 1)\beta_j}{\lambda_{ij} - \lambda_{ij}^s} = q_j^{\text{rs}} = (\alpha_j - \psi_j - h_j)/\beta_j \]
and therefore, we have
\[ \sum_{j=1}^{2} (h_j^s - h_j) q_j^{\text{rs}} = 0. \]
That is, estimates of total overhead costs based on the (possibly distorted) product cost estimates $(\hat{h}_j^s)$ from a system $s$ are, in fact, unbiased estimates of the true total overhead costs in steady state.

We refer to the above conditions in (34) and (35) as the Axiom of Consistency in Steady State (ACSS). Under ACSS, therefore, the second term in (32) vanishes and the expression reduces to
\[ E(\pi^{r_1}) - E(\pi^{r_2}) = \sum_{j=1}^{2} (h_j^s - h_j) \beta_j > 0 \]
and further, proceeding as in the monopoly case, we have
\[ E(\pi^{r_1}) - E(\pi^{r_2}) = \sum_{j=1}^{2} \frac{1}{(n + 1)^2 \beta_j} \left[ (\chi_{ij}^0)^2 - \sigma_2^2 - (e_0)^2 \right] \left[ \frac{\lambda_{ij}^s - \lambda_{ij}}{\lambda_{ij}} - \frac{\lambda_{ij}^s}{\lambda_{ij}} \right]^2 \]

12 It can be shown that the time series of optimal quantities converges exponentially if the demand and cost functions do not change. Our simulation studies also indicate rapid convergence (in two to three periods) to steady state following a perturbation in expected inverse demand or true cost parameters.

13 An anonymous referee noted that steady state for cost functions is the same as the common standard costing conditions where variances are charged directly against income.

14 ACSS was not required in the previous section as $n - 1 = 0$ for a monopoly.
Therefore, paralleling Propositions 1(a) and 1(b), we have

**Proposition 2:** If the Axiom of Consistency in Steady State is satisfied then in a Cournot-Nash oligopoly,

(a) System P is preferred over System Z and System I, and

(b) System I is preferred over System Z if \( \sigma^2 < (x_{\text{opt}}^2 - \psi^2)^2 \).

With the term \((n+1)^2\) in the denominators of the right hand side expressions in (35) and (36), it is evident that the gain from switching from System Z to System I or System P decreases with the number of competitors in the industry.

Under fluctuating demand, however, ACSS invoked in the above proposition is unlikely to be satisfied, and consequently the expression for \( E(\pi^r_Z) - E(\pi^r_Z) \) cannot be signed easily. In fact, it is possible to identify conditions under which \( E(\pi^r_Z) < E(\pi^r_Z) \). Consider a setting in which steady state prevailed until the base period, the cost parameters remained unchanged but demand expectations \( (\alpha_j) \) changed in the current period. The inverse demand function for each product \( j \) in the base period is given by:

\[
p_j^* = \alpha_j^0 - \beta_j \sum_{p=1}^{n} q_j^{\text{PSO}}
\]

where, in general, \( \alpha_j^0 \neq \alpha_j \). The equilibrium product quantities in the base period are then given by:

\[
q_j^{\text{PSO}} = \frac{(\alpha_j^0 - \nu_j - \hat{h}_j^s)}{(n + 1)\beta_j}
\]

Since, \( \sum_{j=1}^{2} (h_j^s - h_j) q_j^{\text{PSO}} = 0 \) from (33), (32) reduces to the following for \( s = Z \):

\[
E(\pi^r_P) - E(\pi^r_Z) = \sum_{j=1}^{2} \frac{\hat{h}_j^Z - \hat{h}_j^s}{(n + 1)^2 \beta_j} - \frac{(n - 1)(\hat{h}_j^Z - h_j)(\alpha_j - \alpha_j^0)}{(n + 1)^2 \beta_j}
\]

Since \( \hat{h}_1^Z > h_1 \) and \( \hat{h}_2^Z < h_2 \), the above expression can be negative if \( \alpha_1 \) is sufficiently greater than \( \alpha_1^0 \) and/or \( \alpha_2 \) is sufficiently less than \( \alpha_2^0 \). For instance, \( E(\pi^r_P) < E(\pi^r_Z) \) if \( \alpha_2 \leq \alpha_2^0 \) and

\[
\alpha_1 > \alpha_1^0 + \frac{\beta_1}{(n - 1)(\hat{h}_1^Z - h_1)} \sum_{j=1}^{2} \frac{\hat{h}_j^Z - h_j}{\beta_j}
\]

The parameters on the right hand side are base period values that determine the minimum growth required in product 1 so that System Z is preferable (in expected profit terms) to System P. Thus, the single cost driver system is preferred over the perfect activity-based costing system when the demand for the labor intensive product is expected to grow sufficiently while the demand for the setup intensive product is expected to remain unchanged (or decline).
Dividing the right hand side expression in (41) above by $\alpha_0^0$, we obtain the required growth rate in product 1.

$$g_1 = 1 + \frac{\beta_1}{\alpha_0^0(n-1)} \sum_{j=1}^{2} \frac{h_j - h_i}{\beta_j} (42)$$

We conducted a simulation study to obtain insight for the required growth rate $g_1^*$ expressed in (42) above. We sampled 5,000 observations, drawing values for parameters of our model from independent uniform distributions. The parameters $(\alpha_1^0 - v_1^0)$ and $(\alpha_2^0 - v_2^0) = (\alpha_2^0 - v_2^0)$ were assumed to be uniformly distributed on $[500,900]$, $\beta_1$ and $\beta_2$ on $[1.5]$, $\theta_1$ and $\theta_2$ on $[2.6]$, $\lambda_{m1}$ on $[1.3]$ and $\lambda_{m2}$ on $[3.5]$, while $\lambda_{L1}$ and $\lambda_{L2}$ were fixed at one. Pairwise correlation analysis between the required growth rate $g_1^*$ and the various parameters reported in Panel B of Table 1 indicates that $g_1^*$ is significantly negatively correlated with $(\alpha_1^0 - v_1^0)$, $(\alpha_2^0 - v_2^0)$ and $\lambda_{m1}$, and significantly positively correlated with $\lambda_{m2}$ and $\theta_1$. The distribution of $g_1^*$ for the 5,000 trials is depicted in the histogram in Figure 1A. The most striking feature is the relatively low value (median = 1.011, mean = 1.012) for $g_1^*$ in almost all observations. The simulation study was replicated first for $(\alpha - v)$ values drawn from the uniform distribution $[100,500]$ and next from $[50,90]$.

The correlation results were very similar in all cases, and consistent also with the results from regressing $g_1^*$ against all parameters for the entire pooled sample of 15,000 observations. The median $g_1^*$ value increased to 1.030 and 1.110 (mean: 1.036, 1.121) for the two replications, consistent with the negative correlation observed between $g_1^*$ and $(\alpha_1^0 - v_1^0)$. See Figures 1B and 1C.

The intuition behind this result identifying sufficient conditions for the Pareto superiority of the single cost driver system rests on the fact that the competitive equilibrium in an oligopoly is strictly Pareto inferior to a collusive choice of product quantities, as the rival firms can be strictly better off by implicitly coordinating their decisions. Product mix decisions based on distorted costs deviate from the optimal decisions in equilibrium, and such deviation is detrimental to a monopolist firm and to rival firms in an oligopoly in steady state. If the demand for the overcosted and overpriced labor intensive product is expected to grow sufficiently, however, the loss of sales due to overpricing are not so large as to offset the gains from the higher price, and all firms are better off using the single cost driver system. But, if the demand growth for the labor intensive product for such firms is expected to plateau off then they will be better off if they all switch from single cost driver systems to activity-based cost systems.

Observations for which the equilibrium quantities were negative were rejected, and simulation of new observations was continued until 5,000 cases with non-negative equilibrium quantities were obtained. The very low values for the equilibrium quantities when $(\alpha - v)$ values are low (see Panel A of Table 1) suggest limited applicability of the last replication.
### Table 1
Simulation Study to Examine Required Growth Rate $g_i^*$

#### Panel A: Descriptive Statistics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_i^*$</td>
<td>1.012</td>
<td>.007</td>
<td>1.011</td>
<td>.016</td>
<td>1.007</td>
<td>.025</td>
<td>1.019</td>
<td>.047</td>
</tr>
<tr>
<td>$q_1^P$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q_2^P$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q_1^Z$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q_2^Z$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range of Values for $(a_1^* - v_1^<em>)$ and $(a_2^</em> - v_2^*)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[500, 900]</td>
</tr>
<tr>
<td>$(a_1^* - v_1^*)$</td>
<td>-.1706</td>
</tr>
<tr>
<td>$(a_2^* - v_2^*)$</td>
<td>-.1791</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-.0243</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>.0192</td>
</tr>
<tr>
<td>$\lambda_{M1}$</td>
<td>-.5462</td>
</tr>
<tr>
<td>$\lambda_{M2}$</td>
<td>.5393</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>-.0233</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>.5452</td>
</tr>
</tbody>
</table>

*Significance levels are in parentheses
Panel C: Results of Regressing $g_i$ on Various Parameters for Pooled Observations

| Independent Variable | Coefficient Estimate | t-Statistic | Prob. > |t| |
|----------------------|----------------------|-------------|---------|---|
| Intercept            | .9967                | 330.27      | .0001   |  |
| ($\alpha_1 - v_1$)   | $-7.37 \times 10^{-5}$ | -32.01      | .0001   |  |
| ($\alpha_2 - v_2$)   | $-8.28 \times 10^{-5}$ | -36.05      | .0001   |  |
| $\beta_1$            | $3.01 \times 10^{-4}$ | 1.16        | .2481   |  |
| $\beta_2$            | $2.14 \times 10^{-4}$ | .83         | .4090   |  |
| $\lambda_{M1}$       | -.0280               | -53.88      | .0001   |  |
| $\lambda_{M2}$       | .0282                | 54.44       | .0001   |  |
| $\theta_1$           | $7.92 \times 10^{-5}$ | .30         | .7624   |  |
| $\theta_2$           | .0143                | 54.75       | .0001   |  |

*Adjusted $R^2 = .657$

Prob. > F = .0001

Figure 1

Simulation Study of Required Growth Rate ($g_i$) in Product 1 for System Z to be Preferred Over System P in a Duopoly

(A) Range of Values of ($\alpha - v$): [500, 900]
(B) Range of Values of ($\alpha - v$): [100, 500]

(C) Range of Values of ($\alpha - v$): [50, 90]
IV. CONCLUDING REMARKS

Many claims about the value of an activity-based costing system have been made by its advocates and opponents. These claims, however, have not been subjected to the scrutiny of a rigorous model that reflects rational behavior by users of product costing systems. We take a step in this direction by analytically examining how the inclusion of multiple cost drivers alters the value of a product costing system. Our analysis evaluates the impact a single cost driver and a multiple cost driver system have on the optimal expected profits of a monopolist, and alternatively firms competing in an oligopoly. We find that generally a monopolist firm can expect higher profit if it selects its product mix using a multiple cost driver system even when the system is imperfect in assigning overhead costs to the various activities. In particular, the value of an imperfect ABC system decreases with its noisiness. In an oligopoly setting, a similar result obtains if the cost and demand parameters do not change.

While these results ordering the value of the cost systems are intuitively appealing because they parallel those based on the coarseness of information partitions, such simple interpretations do not extend to the general competitive setting. In particular, we show that the equilibrium implemented by the rival firms using a direct labor-based cost accounting system is Pareto superior to that implemented under an accurate activity-based cost accounting system when the demand for the overcosted labor intensive product is expected to grow sufficiently relative to the demand for the undercosted setup intensive product. In other words, facing imperfect competition, it is sometimes optimal for firms to persist in using a single cost driver system rather than switching to an activity-based cost system. Our analysis also indicates that the benefits from an ABC system will likely be more pronounced in industries in which the demand growth for traditional direct labor-intensive products is small relative to that for products intensive in other cost drivers.
REFERENCES


Cooper, R., "Does Your Company Need a New Cost System?" Journal of Cost Management (Spring 1987), pp. 45-49.


