

Bandpass Sampling of MFSK in SystemVue

Bandpass sampling of a minimum frequency shift keyed (MFSK) signal can be demonstrated with the *SystemVue* model shown in Figure 1. MFSK occurs when the frequency deviation $f_{dev} = r_b/2 = 1/2 T_b$ (see page 158-159 and Figure 3.8, page 155).

The carrier frequency $f_c = 29.000$ MHz and the data rate $r_b = 50$ kb/sec here. The bandwidth of this MFSK signal then contains 95% of the total power in $2f_{dev} + 4/T_b = 5/T_b = 5r_b = 5(50 \text{ kHz}) = 250$ kHz (see Table 3.3, page 160 of the text).

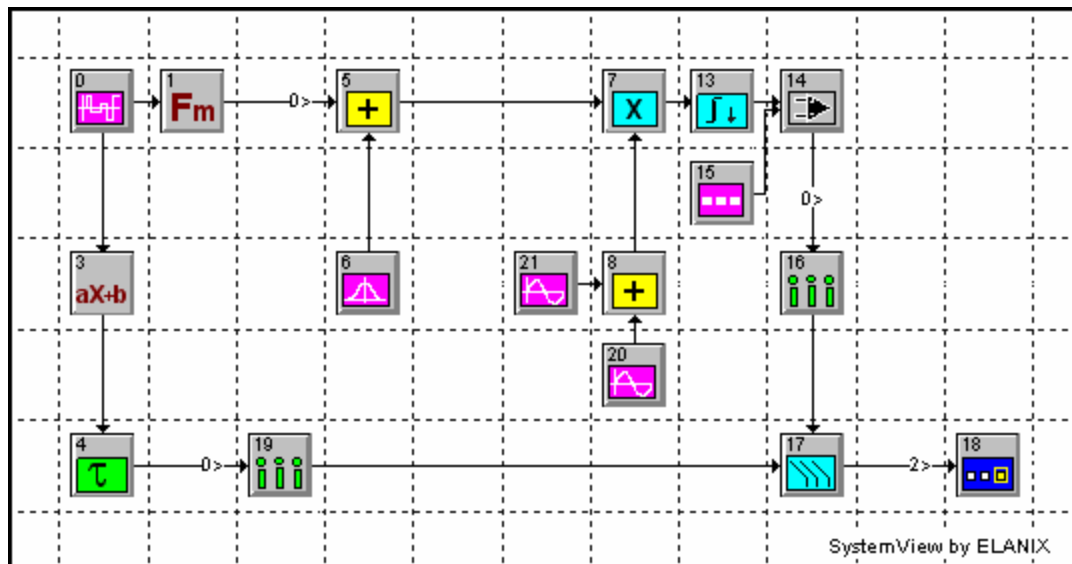


Figure 1

For this 95% bandwidth the lower frequency $f_L = 29.0 - 0.125 = 28.875$ MHz and the upper frequency $f_H = 29.0 + 0.125 = 29.125$ MHz. Bandpass sampling would occur for $f_s = 2(f_H - f_L) = 2(250) \text{ kHz} = 500$ kHz. However, the *SystemVue* bandpass sampling rate must be chosen carefully to elucidate both a nearly symmetrical aliased spectrum and a spectrum with definitive spectral nulls.

The center frequency of the bandpass sampling PSD is $f_{alias} = N \times f_{bandpass} - f_c$, where N is an integer. The spectral nulls of the MFSK signal appear at $f_{alias} \pm f_{dev} \pm r_b = f_{alias} \pm 1.5r_b$ (see page 158-159 of the text).

The *SystemVue* bandpass sampling rate must be a multiple of the data rate ($r_b = 50$ kb/sec here). Figures 2, 3 and 4 show a bandpass sampling rate greater than

the minimum of 500 kHz at 540, 550 and 560 kHz respectively. Note that only the bandpass sampling rate of 550 kHz shows the expected spectral nulls of the MFSK signal at $f_{alias} \pm 1.5r_b = 150 \text{ kHz} \pm 75 \text{ kHz}$. Here $N = 53$ and $f_{alias} = 53(0.55) \text{ MHz} - 29 \text{ MHz} = 150 \text{ kHz}$.

The transmitter amplitude is $A = 5 \text{ V}$ and for the asymmetrical MFSK signal the energy difference per bit $E_d = 2(A^2 T_b / 2) = (5)^2 \times (1/50\,000) = 5 \times 10^{-4} \text{ V}^2\text{-sec}$ (see Equation 3.10, page 145 of the text).

The BER measurement is performed with a delay of $20 \mu\text{sec}$ (1 bit at 50 kb/sec) for a total of 10 000 data bits. The observed BER with a carrier frequency here of 29.0 MHz and bandpass sampling agrees with the theoretical upper bound of the probability of bit error P_b and the observed BER obtained with a carrier frequency of 20 kHz and conventional sampling (see Table 3.4, page 161 of the text).

Using the bandpass sampling rates of 540 and 560 kHz produces somewhat erroneous BER results. This is because neither 540 or 560 kHz are multiples of the data rate here. The minimum bandpass sampling rate of 500 kHz, although it is a multiple of the data rate $r_b = 50 \text{ kb/sec}$, produces the *baseband* spectrum shown in Figure 5. This occurs because $f_{alias} = 58(0.5) \text{ MHz} - 29 \text{ MHz} = 0 \text{ Hz}$ and is incorrect.

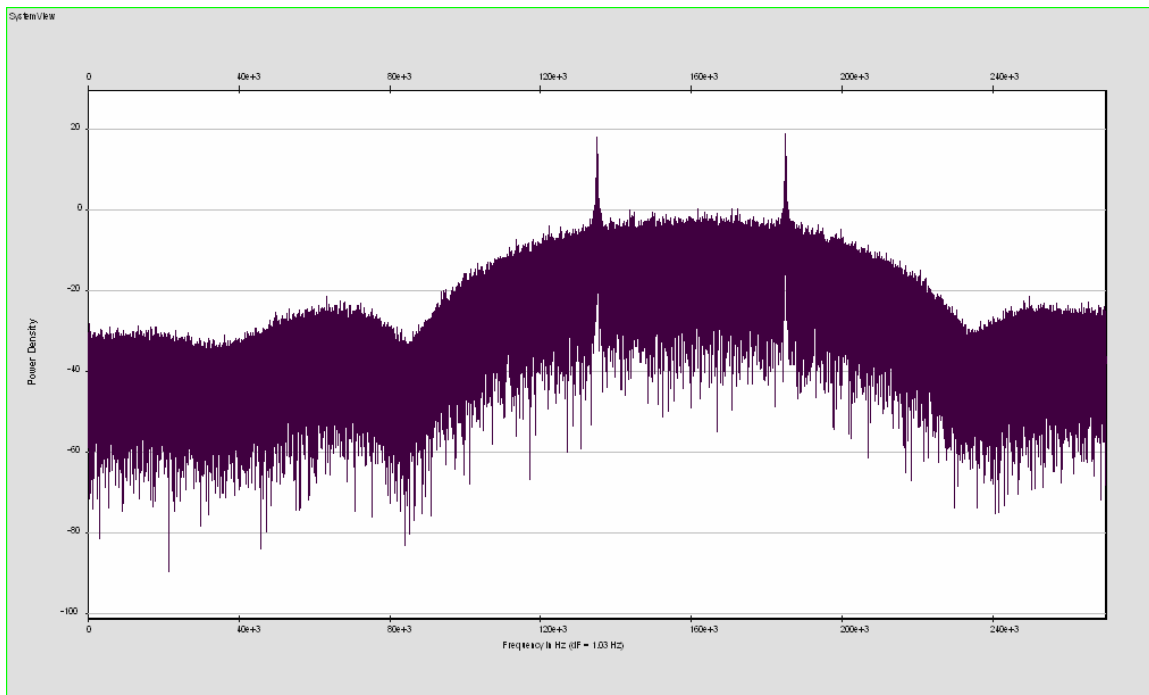


Figure 2

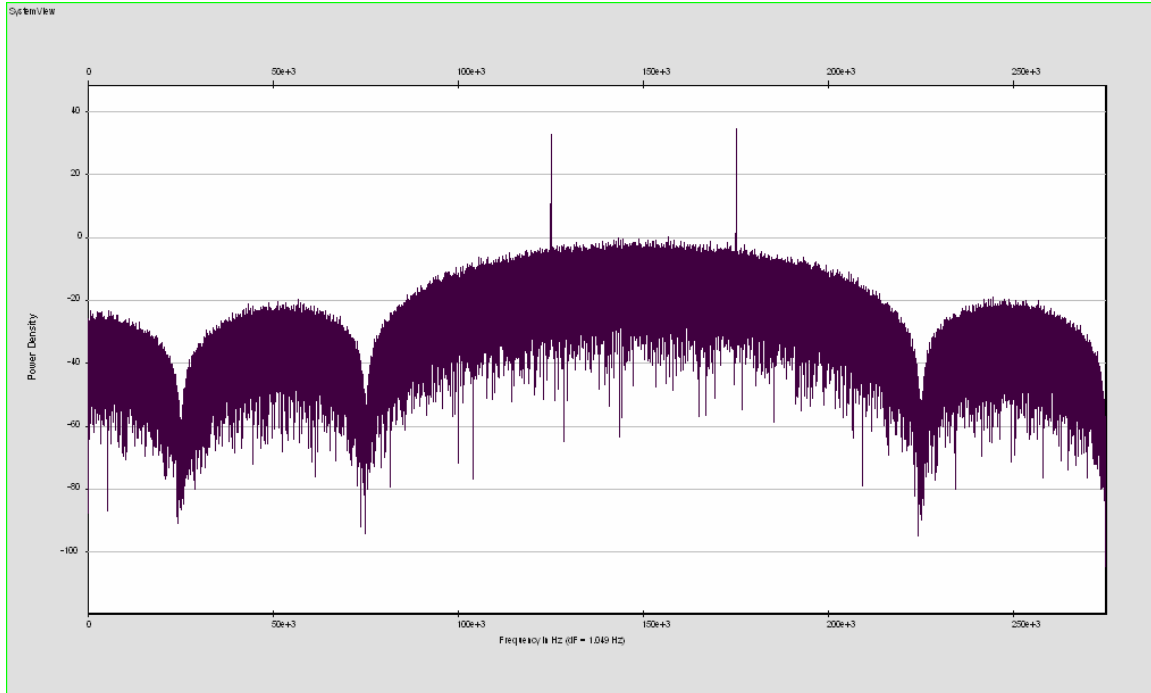


Figure 3

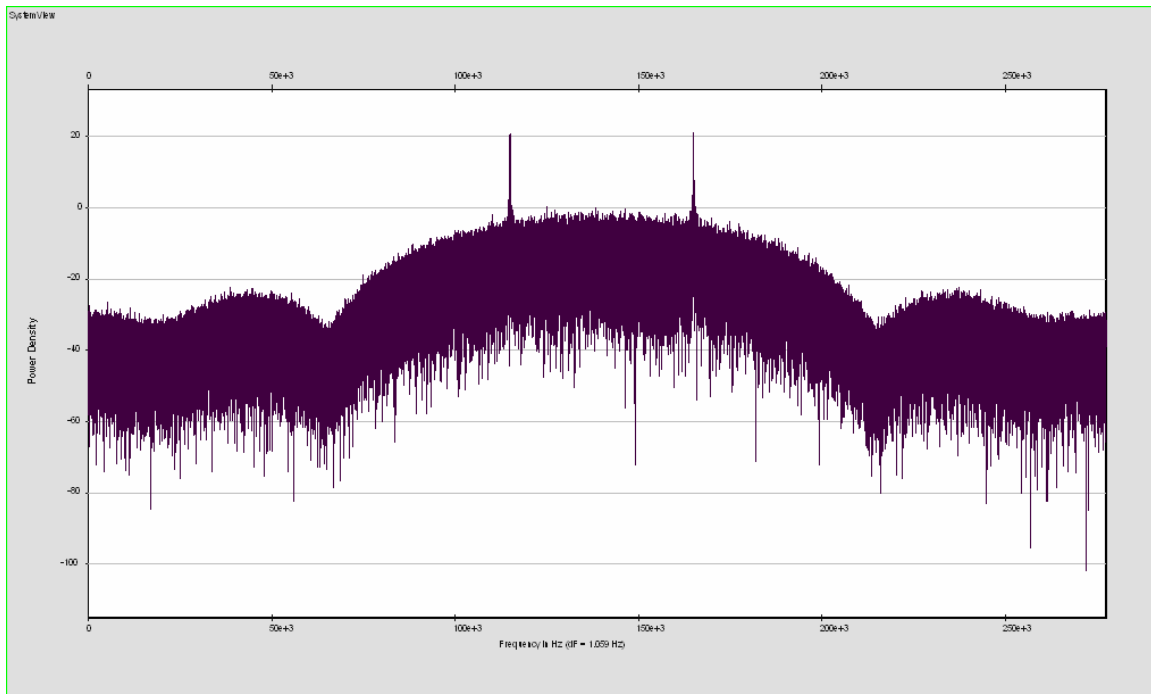


Figure 4

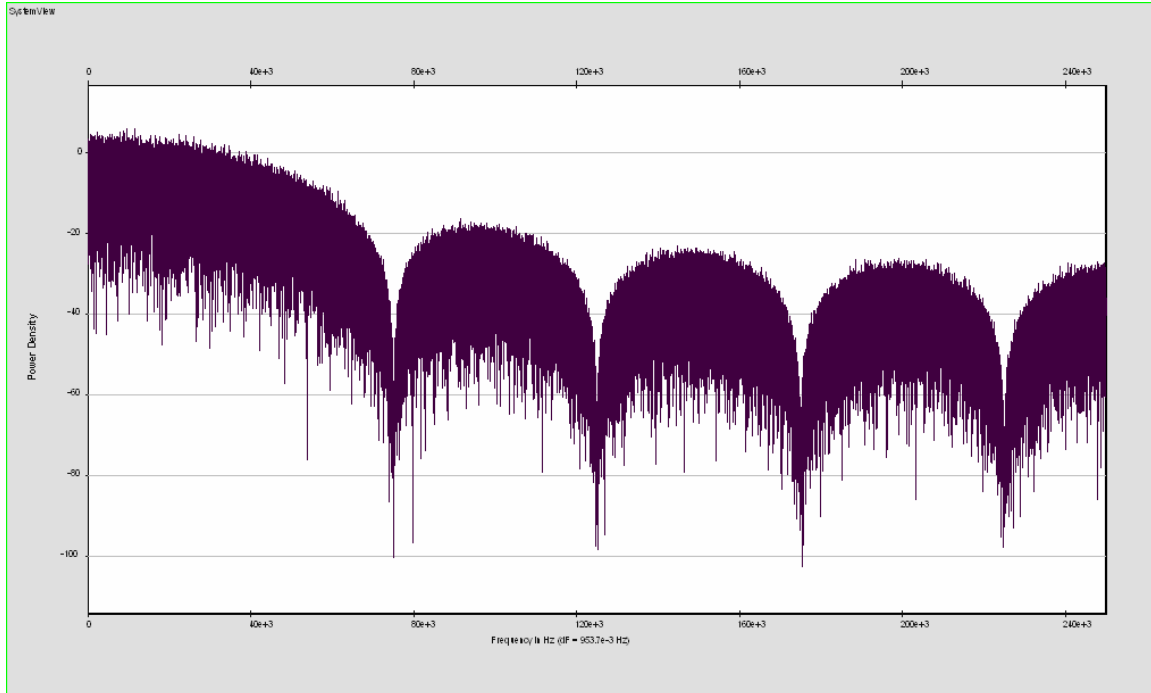


Figure 5

E_d/N_o dB	N_o V ² -sec $f_c = 20$ KHz	BER $r_b = 1$ kb/sec	N_o V ² -sec $f_c = 29$ MHz	BER $r_b = 50$ kb/sec	P_b
8	0	0	0	0	0
12	1.58×10^{-3}	3.3×10^{-3}	3.2×10^{-5}	3.5×10^{-3}	2.5×10^{-3}
10	2.50×10^{-3}	1.4×10^{-2}	5.0×10^{-5}	1.45×10^{-2}	1.25×10^{-2}
8	3.96×10^{-3}	3.72×10^{-2}	7.9×10^{-5}	3.99×10^{-2}	3.75×10^{-2}
6	6.28×10^{-3}	7.58×10^{-2}	1.26×10^{-4}	9.12×10^{-2}	7.98×10^{-2}
4	9.95×10^{-3}	1.346×10^{-1}	1.99×10^{-4}	1.416×10^{-1}	1.314×10^{-1}
2	1.58×10^{-2}	1.839×10^{-1}	3.15×10^{-4}	1.973×10^{-1}	1.872×10^{-1}
0	2.50×10^{-2}	2.369×10^{-1}	5×10^{-4}	2.521×10^{-1}	2.393×10^{-1}

The probability of bit error P_b is a theoretical upper-bound. The observed BER for the conventional and bandpass sampling tracks well until low signal to noise ratios (SNR). The system sampling here ($f_{sys} = 550$ kHz) does not consistently reproduce the peak amplitudes of either the transmitted signal or the correlation receiver reference signals.