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EDITORIAL

The Nature of Randomness:

Part 1 – Knowable or Unknowable?

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When one looks for instances of randomness among the ordinary experiences of life, numerous examples come to mind: stock prices, coin tosses, weather patterns, industrial accidents, etc. The most salient aspect of these and similar phenomena is their unpredictability; that is, they are unknown ahead of time, and generally cannot be forecast with anything approaching perfect accuracy. There are, however, certain limited circumstances under which such phenomena can be predicted reasonably well – those in which the forecaster possesses what financial traders might call "insider" information.

Just as a stock trader who receives a confidential internal company report may be able to anticipate a significant upward or downward movement in the company's stock price, so: a magician who is skilled at tossing coins may be able to achieve "heads" with a high degree of certainty; a meteorologist with the latest satellite imagery may be able to forecast the emergence of a hurricane; and a worker who recognizes his employer's poor safety practices may foresee an accident "waiting to happen." This type of information is obviously available in different degrees to different observers in different contexts. Thus, whereas a privileged stock trader may know with 99 percent confidence that a yet-to-be-disclosed profit report will cause a firm's stock price to increase by at least 5 percent during the next trading day, an informed meteorologist may be only 80 percent confident that a category 5 hurricane will emerge within the next 48 hours.

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Moreover, whereas an experienced magician may be 95 percent confident that his next coin toss will result in "heads", a novice magician may be only 60 percent confident of this outcome.

God's Dice

To be clear, therefore, about what I mean by "insider" information, let us say that it consists of any and all information potentially available to the most privileged, persistent, and conscientious observer, whether or not such an observer exists in practice. With regard to a given random variable, the amount of uncertainty that can be eliminated through such information will be called *knowable complexity* (KC). Any residual uncertainty, which cannot be dispelled by even the ideal observer, will be called *unknowable complexity* (UC).

To illustrate the difference between these two types of uncertainty – and indeed, to provide a concrete example of the latter form – consider what happens when a person tosses a coin onto a table. At the moment of releasing the coin, the individual imparts a certain velocity and rotation to it at a given distance above the table. As the coin travels through the air, it is slightly affected by friction from air molecules. When the coin strikes the surface of the table, the impact causes a succession of bounces and rolls that ultimately results in the coin's coming to rest with one of its sides showing.

Clearly, if one could measure the exact position and velocity of every molecule, every atom, and every sub-atomic particle in the coin, the air, and the tabletop at the moment of the coin's release, then the outcome of the toss could be determined with complete accuracy. Under this assumption, all sources of uncertainty would constitute KC. However, such an assumption would be wrong.

No magician, no matter how skillful, can force "heads" to come up on every given try. This is because the coin's trajectory and final resting place are affected by events at the atomic level – for example, the minute change in momentum associated with the release of an alpha particle from a trace radioactive isotope composing the coin's mass – and, according to Heisenberg's uncertainty principle, an observer cannot know simultaneously both the exact position and the exact momentum of an elementary particle. Consequently, there will always be some residual uncertainty, or UC, associated with the behavior of the particles making up the coin, the air, and the table.

This UC affects not only coin tosses, but also stock prices, weather patterns, industrial accidents, and all other physical phenomena. In one sense, its existence is disturbing because it is difficult to believe that every coin and other object in the universe is composed of particles that move about in unpredictable ways – making decisions on their own, so to speak.[[1]] Alternatively, however, one could view UC as an attractive "spice of life," and take comfort in the fact that, as technology provides ever-greater amounts of "insider" information, UC is the only thing preventing a bleakly deterministic future. Regardless of esthetic considerations, I will equate UC with *true randomness*.

Incompressibility

In the absence of overt "insider" information – in other words, when the relevant KC is actually "unknown" to the observer – probability theory can be used to model KC just as well as UC. This is apparent even without any knowledge of modern quantum physics, and was noted by British philosopher David Hume in his famous treatise, *An Enquiry Concerning Human Understanding* (1748):

> It is true, when any cause fails of producing its usual effect, philosophers ascribe not this to any irregularity in nature [i.e., what I am calling UC – author's note]; but suppose, that some secret causes, in the particular structure of parts [i.e., what

> I am calling KC – author's note], have prevented the operation. Our reasonings, however, and conclusions concerning the event are the same as if this principle had no place. … [W]here different effects have been found to follow from causes, which are to *appearance* exactly similar, all these various effects must occur to the mind in transferring the past to the future, and enter into our consideration, when we determine the probability of the event.[2]

Hume goes on to assert that "… chance, when strictly examined, is a mere negative word, and means not any real power which has anywhere a being in nature."[3] In other words, the concept of chance denotes only the absence of information about underlying causes, and does not distinguish between UC and KC. So what, then, is the purpose of defining these two sources of uncertainty separately? And more pointedly, what is the purpose of seeking the meaning of "true randomness"?

The answer is that in real life, even in the absence of overt "insider" information, we often possess more than one observation of a given random variable. Although we cannot tell whether the uncertainty associated with a single outcome involves UC or KC, it may be possible to do so with a sequence of outcomes. Specifically, we can develop statistical methods to try to determine whether or not the observations evince the systematic behavior expected of KC.

In the 1960s, two remarkable mathematicians, Russian Andrey Kolmogorov and American Gregory Chaitin, independently proposed a formal definition of randomness that comports well with the present distinction between UC and KC. Working with (possibly infinite) sequences of integers, these researchers suggested that a sequence of integers of a given length is *algorithmically random* if and only if it cannot be encoded into another sequence of integers that is substantially shorter than the original sequence. Such a sequence is said to be *incompressible*.

For simplicity, and without loss of generality, Kolmogorov and Chaitin employed sequences of 0s and 1s, which can be viewed as binary computer code. To illustrate their concept, consider the infinite sequence

0101010101… ,

formed by endlessly alternating the digits 0 and 1, beginning with an initial 0. Clearly, such a sequence is not in any sense "random"; but to show that it is indeed compressible, we must find a means of mapping it to a shorter (finite) sequence. There are many ways to do this, and one of the most straightforward – but not necessarily most efficient – simply entails matching each letter of the English alphabet with its ordinal number expressed in base 2 (i.e., A = 00001, B = 00010, C = 00011, …, Z = 11010), and then assigning the remaining five-digit base-2 numbers to other useful typographical symbols (i.e., capital-letter indicator = 11011, space = 11100, comma = 11101, semi-colon = 11110, and period = 11111). Using these letters and symbols, we can write virtually any English-language sentence, and so it follows that the sentence "The infinite sequence formed by endlessly alternating the digits 0 and 1, beginning with an initial 0." can be transformed quite nicely into a finite sequence of 0s and 1s, thereby compressing the original sequence dramatically.

It is often convenient to treat (finite or infinite) sequences of 0s and 1s as base-2 decimal expansions between 0 and 1.0; for example, the sequence 0101010101… may be interpreted as

$0.0101010101\ldots_{\text{base 2}}$,

which equals

$$\frac{0}{2}+\frac{1}{4}+\frac{0}{8}+\frac{1}{16}+\frac{0}{32}+\frac{1}{64}+\frac{0}{128}+\frac{1}{256}+\frac{0}{512}+\frac{1}{1024}+\ldots=0.333\ldots_{\text{base 10}}.$$

Viewing things this way, it can be shown that *almost all* of the real numbers between 0 and 1.0 are incompressible. In other words, if one were to draw a real number from a continuous uniform probability distribution over the interval [0,1), then he or she would obtain an incompressible number with certainty. However, in the paradoxical world of the continuum, nothing is straightforward; and so, while *almost every* real number in [0,1) is incompressible, Chaitin has shown that it is impossible to prove that *any particular* infinitely long decimal is incompressible![4]

References

Chaitin, G. (2006), *Meta Math! The Quest for Omega*, Vintage Books, New York, NY.

Hume, D. (1748), *An Enquiry Concerning Human Understanding*.

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[1] Albert Einstein famously expressed his aversion to this fundamental randomness by declaring, "I am convinced that He [God] does not play dice." (Letter to Max Born, December 4, 1926 in *Einstein und Born Briefwechsel*, 1969, p. 130.)
[2] See Hume (1748), Section VI.
[3] See Hume (1748), Section VIII, Part I.
[4] See Chaitin (2006), Chapter 5. Perhaps Chaitin's result should not be viewed as too surprising. After all, to show that an infinitely long sequence is incompressible seems to presuppose the ability to describe, or at least apprehend, the sequence in intimate detail; but it also seems reasonable to believe that such familiarity is possible only if the sequence is compressible.