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EDITORIAL

Sharing Responsibility: What They Didn't Teach You in Kindergarten

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It was an otherwise ordinary day in Ms. Chauncey's kindergarten class. The students had just completed their finger-painting projects, and were getting ready for their mid-morning naps. Three boys, Mort, Lars, and Carl, were asked by the teacher to gather the plastic bottles of finger paints from around the classroom, and to put them neatly away. Unfortunately, because of one simple careless act, what should have been a tranquil transition from digital artistry to manufactured slumber became an awful awakening from finger-painting to finger-pointing.

Details are crucial whenever one attempts to assess blame. Therefore, we shall try to be as clear as possible in describing the events of this case.

First, we note that Ms. Chauncey had made the following assignments: Mort was to pick up all the bottles of blue finger paint, of which there were four; Lars was to pick up the bottles of red finger paint, of which there were two; and Carl was to pick up the single bottle of yellow finger paint. Although Ms. Chauncey had not stated explicitly where the bottles were to go, she had pointed in the general direction of the back of the classroom where paints, brushes, construction paper, and various other art/craft supplies were stored on a large set of shelves.

Next, we observe that each of the boys could carry only one bottle at a time, and so both Mort and Lars had to make more than one trip from the front of the class to the back. As things transpired, the boys could not immediately find sufficient space for the bottles on the art/craft shelves, and so they initially set their bottles on the floor in front of the shelves. Then, when all seven bottles had been transported there, the boys began looking for a place to put them. It was

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not long before they simultaneously noticed the same attractive free space: the flat top of a brand new computer scanner sitting on a small table next to the teacher's desk.

Each boy took his responsibility very seriously, and thus maintained close custody of the bottle or bottles entrusted to him. First, Mort placed each of his four blue bottles carefully on the scanner. It then was Carl's turn, and so he set down his single yellow bottle in a similar fashion. Unfortunately, it was this fifth bottle that "broke the scanner's back," so to speak. The moment Carl released his bottle, the scanner toppled over and crashed to the floor in a nasty pile of broken glass and running pigment.

In the interest of delicacy, we will provide no further details of that particular morning's events. However, suffice it to say that there was some discussion among the three boys and their teacher – as well as a few of the class' pre-lawyers – as to who was responsible for the disaster than preempted naptime. And that is indeed the question we shall now consider: exactly who was responsible for the paint/scanner tragedy?

We begin by pronouncing – by authorial fiat – that Ms. Chauncey bears no significant responsibility for the disaster. While we recognize that it has become vogue in certain quarters to assess primary responsibility to those with "deep pockets," and while it is clear that the adult Ms. Chauncey undoubtedly possesses considerably deeper "emotional pockets" than do her young charges, the purpose of this discussion is to assess responsibility among those who are truly responsible rather than among those who are most able to bear the burden of responsibility.

So, how should responsibility for the paint/scanner disaster be assessed? Clearly, a number of intuitive approaches come to mind:

- Hold Carl entirely responsible, since it was his individual action – placing the single yellow bottle on the scanner – that precipitated the disaster. (Using the notation $[m,\ell,c]$ to indicate the proportions of responsibility attributable to Mort, Lars, and Carl, respectively, this approach gives $\left[0,0,1\right]$.)

- Hold Mort entirely responsible, since it was his individual action – placing the first four bottles on the scanner – that initiated the sequence of events leading inevitably to the disaster (i.e., $\left[1,0,0\right]$).
- Hold Mort, Lars, and Carl equally responsible, since they all tacitly agreed to place the bottles on the scanner (i.e., $\left[\frac{1}{3},\frac{1}{3},\frac{1}{3}\right]$).
- Hold Mort, Lars, and Carl responsible in proportion to the numbers of bottles that they had placed on the scanner at the time of the disaster (i.e., $\left[\frac{4}{5},0,\frac{1}{5}\right]$).
- Hold Mort, Lars, and Carl responsible in proportion to the numbers of bottles entrusted to them, whether or not the bottles had the opportunity to be placed on the scanner (i.e., $\left[\frac{4}{7},\frac{2}{7},\frac{1}{7}\right]$).

Rather remarkably, none of the above *ad hoc* procedures is equivalent to the most prominent solution concept offered by cooperative game theory – i.e., the Shapley value. This allocation scheme, first proposed by Shapley (1953), would identify each boy's proportion of responsibility as the frequency with which his bottle causes the disaster when all possible sequences of bottle placements are considered. Thus, listing these sequences as

(a) Mort (4), Lars (2), Carl (1);

(b) Mort (4), Carl (1), Lars (2);

(c) Lars (2), Mort (4), Carl (1);

(d) Lars (2), Carl (1), Mort (4);

(e) Carl (1), Mort (4), Lars (2);

(f) Carl (1), Lars (2), Mort (4);

we find that Mort is implicated in cases (c), (d), (e), and (f), Lars in case (a), and Carl in case (b). This yields the allocation vector $\left[\frac{2}{3},\frac{1}{6},\frac{1}{6}\right]$.

Given that the Shapley value is defined in an intuitively plausible way, and that it also may be derived rigorously as the unique quantity satisfying a set of three reasonable axioms (often referred to as *symmetry*, *dummy*, and *additivity*), one might be tempted to settle on this particular allocation. However, the Shapley value does present one serious problem: If the boys are permitted to take turns placing their *individual* bottles on the scanner (rather than each placing all of his bottles together in one bunch), then the calculation would have to be revised.

Without listing all 105 ($=\frac{7!}{4!2!1!}$) different sequences of individual bottles, it is fairly easy to see that this modification would result in a Shapley-value allocation of $\left[\frac{4}{7},\frac{2}{7},\frac{1}{7}\right]$, because each boy's chance of setting down the critical fifth bottle would be given precisely by his share of the total complement of bottles. Since there was no actual requirement that each boy place all of his bottles in one bunch, and also since the vector is $\left[\frac{4}{7},\frac{2}{7},\frac{1}{7}\right]$ appeals particularly strongly to intuition, this seems like an ideal choice. However, before jumping to a final conclusion, it might be prudent to ask: What happens if the bottles are subdivided further? For example, what happens if the boys had decided to take turns pouring single drops of their respective finger-paint stores onto the scanner?

This line of thinking leads to the Aumann-Shapley value (first proposed by Aumann and Shapley, 1974), which would evaluate each boy's responsibility as the frequency with which his paint causes the disaster when his entire store of paint is treated as an infinitely divisible continuum. Without going into the necessary formalities from calculus, one can argue heuristically that the Aumann-Shapley approach yields an allocation of $\left[\frac{4}{7},\frac{2}{7},\frac{1}{7}\right]$, because on the margin, when the cumulative amount of finger paint poured onto the scanner is just short of five bottles' worth, each boy's chance of providing the next drop would be given by his share of the total number of bottles.[[1]]

That certainly seems like a considerable amount of work just to arrive at the obvious solution of $\left[\frac{4}{7},\frac{2}{7},\frac{1}{7}\right]$! But such precision is necessary, because the problem becomes even more complicated when transferred to the real world.

Consider, for example, an insurance company's portfolio of total losses composed of random loss amounts X_M, X_L, and X_C from motor insurance, liability insurance, and credit insurance, respectively. To maintain numerical consistency with our kindergarten story, let us assume that $E\left[X_M\right]=\$400\text{ million}$, $E\left[X_L\right]=\$200\text{ million}$, and $E\left[X_C\right]=\$100\text{ million}$, and that all three random variables have the same standard deviation. The problem then is to decide how to allocate the "risk" of the company's total loss portfolio across the three separate lines of business. This is especially important in two contexts: (1) determining how the company's capital (surplus) should be distributed among the three lines of business, and (2) determining what the premium loading should be for each line.

In most real-world applications, the distributions of X_M, X_L, and X_C would be approximately continuous. Thus, it is reasonable to proceed directly to the Aumann-Shapley value (see Powers, 2007). Interestingly, if one assumes that the three separate loss amounts are independent normal random variables, and uses the insurance company's single-period probability of insolvency as the measure of "risk", then the Aumann-Shapley approach gives an allocation vector of $\left[\frac{1}{3},\frac{1}{3},\frac{1}{3}\right]$, rather than $\left[\frac{4}{7},\frac{2}{7},\frac{1}{7}\right]$.

Why?

As previously mentioned, "Details are crucial whenever one attempts to assess blame." Although the three lines of business have very different *expected* loss amounts, they are assumed to have identical *standard deviations*. Therefore, assuming that each line's pure premium is set equal to its expected losses, we can say that all three lines present exactly the same threat to the insurance company's solvency.

References

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[[1]] Note that this formulation assumes that the scanner tips at exactly five bottles, rather than at some arbitrary point between five and six bottles. However, the exact tipping point is immaterial to the argument in the continuous case.