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EDITORIAL

Presbyter Takes Knight

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In his influential 1921 treatise, *Risk, Uncertainty, and Profit*, American economist Frank H. Knight (1885 – 1972) distinguished between the concepts of *risk* and *uncertainty* as follows:

> … Uncertainty must be taken in a sense radically distinct from the familiar notion of Risk, from which it has never been properly separated. The term "risk", as loosely used in everyday speech and in economic discussion, really covers two things which, functionally at least, in their causal relations to the phenomena of economic organization, are categorically different. … The essential fact is that "risk" means in some cases a quantity susceptible of measurement, while at other times it is something distinctly not of this character; and there are far-reaching and crucial differences in the bearings of the phenomenon depending on which of the two is really present and operating. … It will appear that a measurable uncertainty, or "risk" proper, as we shall use the term, is so far different from an unmeasurable one that it is not in effect an uncertainty at all. We shall accordingly restrict the term "uncertainty" to cases of the non-quantitive type.

In short, Knight argued that risks are predictable from empirical data using formal statistical methods, whereas uncertainties cannot be predicted because they have no historical precedent. In modern risk finance, this distinction is often used to explain: (1) uninsurability – that is, the refusal of insurance companies to underwrite certain types of exposures because of anticipated,

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but actuarially intractable, structural changes in loss distributions;[1] and (2) flights to quality – that is, the abandonment of certain asset or derivative markets by traders perceiving forthcoming, yet unforecastable, structural changes in return distributions (see, e.g., Ben-Haim, 2005). In both contexts, the structural changes often are described in terms of potentially unprecedented variations in the tail-heaviness of the relevant probability distributions.

While there certainly exists a *qualitative* difference between the concepts of predictable risk and Knightian uncertainty, it is not immediately clear why, from a *quantitative* standpoint, this difference is anything more than a simple distinction between "lesser risk" and "greater risk." In particular, one might ask: *Is it possible to forecast and make decisions regarding such uncertainties using formal statistical methods?* To examine this question more closely, let us consider a representative example from insurance underwriting.

One of the insurance industry's most worrisome bogeymen is pollution-liability risk, often modeled using Pareto-type loss distributions (i.e., distributions with a continuous, nonnegative sample space and power-law tail). For the purpose at hand, let us assume that the total loss amount covered by a particular insurance company's pollution-liability policy, recorded in millions of dollars, is captured by a Pareto-Lomax random variable,

$$X|\alpha \sim F_{X|\alpha}\left(x|\alpha\right) = 1-\left(\frac{100}{x+100}\right)^{\alpha}, \quad X \in \left(0,\infty\right),\ \alpha \in \left(0,\infty\right),$$

which possesses infinite variance if and only if $\alpha \leq 2$, and infinite mean if and only if $\alpha \leq 1$. To make the implications of Knightian uncertainty especially poignant, we will assume further that: (1) throughout the insurance company's historical experience, the value of α has always remained fixed at 2.5 (for which both the mean and variance are well behaved), but (2) the insurance company's actuaries now anticipate a major structural change in civil-justice awards that is likely to cause α to shrink dramatically, and in a manner without historical precedent.

For the insurance company to continue writing its pollution-liability coverage, its actuaries must be able to forecast the random variable X (i.e., to estimate $F_X(x)$) for the post-change period, and also to estimate an associated decision-making criterion, such as the expected utility, $E_X[u(W+P-X)]$ (where $u(\cdot)$ denotes the company's utility function, and W and P denote its initial net worth and total pollution-liability premiums, respectively). If the actuaries are frequentist statisticians, then they must forecast α with some value $\hat{\alpha}\in(0,2.5)$ to compute

$$F_X(x)=F_{X|\hat{\alpha}}(x|\hat{\alpha}) \text{ and} \tag{1}$$

$$E_X[u(W+P-X)]=E_{X|\hat{\alpha}}[u(W+P-X)|\hat{\alpha}], \tag{2}$$

whereas if they are Bayesian statisticians, then they must estimate $F_\alpha(\alpha)$ with some $\hat{F}_\alpha(\alpha)$, for $\alpha\in(0,2.5)$, to compute

$$F_X(x)=\int_0^{2.5}F_{X|\alpha}(x|\alpha)d\hat{F}_\alpha(\alpha) \text{ and} \tag{3}$$

$$E_X[u(W+P-X)]=\int_0^{2.5}E_{X|\alpha}[u(W+P-X)|\alpha]d\hat{F}_\alpha(\alpha). \tag{4}$$

Interestingly, it is not the computational differences between the frequentist approach of (1) and (2) and the Bayesian approach of (3) and (4) that are most critical in the context of Knightian uncertainty. Rather, it is the fact that frequentist actuaries *would be entirely unable to construct* $\hat{\alpha}$ because they have no formal procedure for saying anything about α in the absence of relevant historical data, whereas Bayesian actuaries *would have no trouble positing* $\hat{F}_\alpha(\alpha)$ because they are used to selecting subjective prior distributions based largely – and sometimes entirely – on judgment. Note further that, even in extreme cases in which $\hat{F}_\alpha(\alpha)$ places substantial weight on values of α less than or equal to 2 (for which $Var_X[X]$ and possibly even

$E_X[X]$ are infinite), there is no obstacle to calculating the expected utility as long as $u(\cdot)$ is bounded.[2] Thus, the qualitative difference between risk and Knightian uncertainty poses a quantitative conundrum for frequentists, but not for Bayesians.

As noted in previous editorials (see Powers, 2007 and Powers, 2008), insurance actuaries often use Bayesian methods precisely because of data limitations and the necessity of incorporating human judgment into the decision-making process. Therefore, in the context of the present discussion, one actually could argue that Bayesian methods are not only resistant to issues of Knightian uncertainty, but also specifically motivated by, and designed to resolve, these issues.[3] This principle applies equally well to the analysis of financial markets as to problems in insurance.

This leaves us with an interesting question: *To a Bayesian, is Knight's distinction between risk and uncertainty ever of real significance?* Somewhat surprisingly, I would argue that it is. Specifically, Knight's use of the terms "measurable" and "unmeasurable" to distinguish between risk and uncertainty, respectively, is quite valid in a Pickwickian sense.

Although Knight employs "unmeasurable" to mean a random outcome without historical precedent, a similar term – "nonmeasurable" – is used in probability theory to describe a mathematical set that cannot be used as the sample space of a proper random variable (because there is no way to distribute probability in a non-trivial way across such a space). Consequently, if one were to contemplate a random outcome α that must take values from a nonmeasurable set, then not even the boldest Bayesian could construct a legitimate subjective distribution $\hat{F}_\alpha(\alpha)$, and further analysis would be entirely impossible.

Fortunately, such problems have not (yet) arisen in insurance or financial markets – or other quotidian realms of human activity – where conventional discrete and/or continuous sample spaces appear quite sufficient. However, things are not so clear-cut at the epistemological edge of human speculation. For example, in scientific arguments associated with the celebrated "anthropic principle" (which, in its weakest form, states that a universe observed by conscious beings is likely to be somewhat different from a universe created at random), the idea that our familiar universe can be viewed as one particular realization of a random process requires the notion of a "sample space of all possible universes." *But how do we know that such a sample space is measurable?*

For a Bayesian, this is truly problematic "Knightian uncertainty" (… or perhaps I should say, "certain benightedness").

References

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[1] Other commonly cited causes of uninsurability include: (1) unavoidable problems of adverse selection; (2) unavoidable problems of moral and/or morale hazard; and (3) an insurance company's inability to hedge or diversify the risks under consideration.

[2] In conventional expected-utility analysis, boundedness is frequently required of utility functions. As shown by Menger (1934), even the famous "St. Petersburg Paradox" cannot be resolved entirely without making this assumption.

[3] The fact that the Reverend Thomas Bayes (c. 1702 – 1761) was a Presbyterian minister thus explains the title of the present editorial (which itself hints at a far cleverer title – at least from the perspective of chess enthusiasts – that might have prevailed if John Calvin had distinguished between the offices of "Presbyter" and "Bishop").