The Financial Management of Extreme-Event Risk

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Extreme-Event Risks, Past and Present

- Nature – earthquakes, tsunamis, volcanoes, hurricanes, tornadoes, infectious diseases, toxic mold.
- Terrorism – product tampering, Tokyo subway attack, Oklahoma City, September 11, 2001.
- Socio-Economic Effects – medical malpractice, discrimination/harassment, alcohol, tobacco, financial market crises.
- Technological Effects – underground seepage, asbestos, computer viruses.

Perceptions of Risk

- Are there trends in these extreme risks? Are they becoming more *frequent*?

- "Given the lethality of terrorist groups such as the *Aum Shinrikyo*, why is it that the largest number of casualties caused by a single terrorist act is still around 300 to 400? Floods, famines, earthquakes, volcanic eruptions, political instability, revolutions, pillage, and plague notwithstanding, we are still here and are even insuring many phenomena.

  "Given the potentials for disaster, why are things so good?"

  — Martin Shubik (April 20, 2001)

Extreme-Event Risks, The Future??

- Nature – rising ocean levels, “mini” ice age, mutating bacteria/viruses, asteroid/comet impact, demise of nearby star.
- Terrorism – more ambitious suicide attacks, “dirty” bombs, nuclear/bio./chemical weapons.
- Socio-Economic Effects – communicable diseases, firearms, actions of offspring, high-fat foods, attorney malpractice.
- Technological Effects – pesticides, prescription drugs, database privacy.

Feasibility of Insurance

- How can we insure/reinsure – or even *price* – extreme risks?
- An alternative approach, with some "politically incorrect" views:
  - ✓ The decreasing marginal utility of net wealth does not directly explain the economics of insurance/reinsurance.
  - ✓ Pure risks and speculative risks are intrinsically different, and should be treated as such.
  - ✓ Mean-TCM-FCM optimization is more relevant to insurance/reinsurance than is mean-variance optimization.
  - ✓ Financial speculation is a legitimate insurance/reinsurance risk management tool.
  - ✓ The solvency regulation of insurers/reinsurers should be de-emphasized.

St. Petersburg Paradox

- Proposed by Nicholas Bernoulli, 1713; solved by Gabriel Cramer ~ 1728; solved by Daniel Bernoulli, 1738.

- Suppose a fair coin is tossed until it comes up “heads”. You receive:

  \$1 if “heads” occurs on the first toss,

  \$2 if “heads” occurs on the second toss,

  \$4 on the third toss,

  \$8 on the fourth toss, etc.

- Expected value, $E[X] = \sum x_i f(x_i) = 1/2 + 1/2 + 1/2 + \ldots$, is infinite.

St. Petersburg Paradox (cont.)

- How much would you be willing to pay to play this game?

  Classical answer: No more than a few dollars.

  My students: No more than a few dollars.

- Why is this game so unattractive?

  Classical answer:

  (1) Because people have decreasing marginal utility (D. Bernoulli).

  (2) Utility also must be bounded above (K. Menger, 1934).

  My students:

  Because we don't apprehend that low-probability, high-payoff outcomes will ever occur.

Expected Utility Principle

- Maximize:

  Expected Utility = $E_f[u(w_0+X)] = \sum u(w_0+x_i)\, f(x_i)$,

  where $u(w_0+x)$ is utility of net wealth $w = w_0+x$.

- Assume Risk Aversion, i.e.:

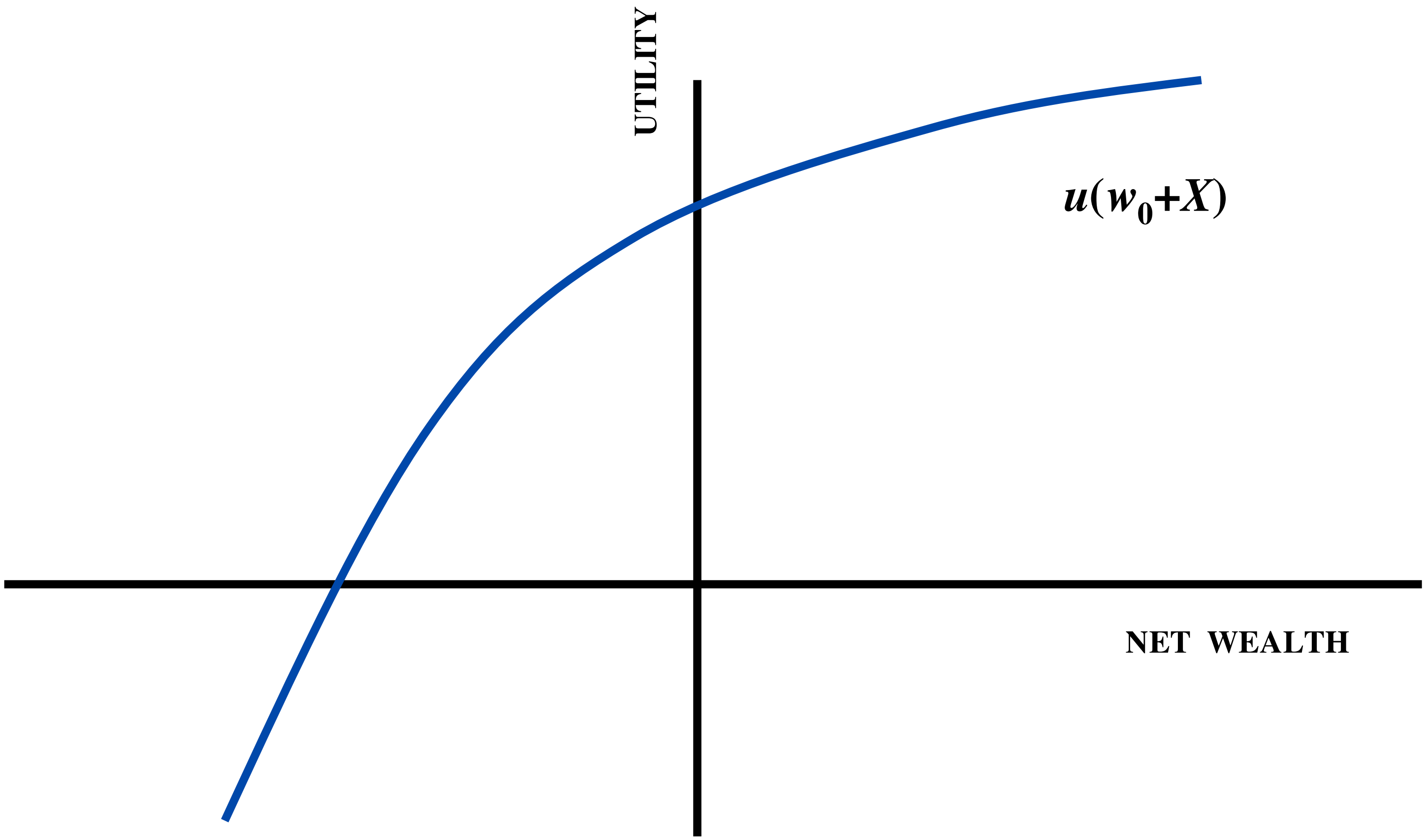
  Increasing utility of net wealth, $u'(w) > 0$;

  Decreasing marginal utility of net wealth, $u''(w) < 0$.

- Assume Prudence, $u'''(w) > 0$.

  (Implied by DARA)

Utility Function (Risk Aversion)



Expected Utility (Analysis)

$$E_f\left[u(w_0 + X)\right]$$

$$= E_f\left[u(w_0 + \mu) + u'(w_0 + \mu)(X - \mu) + \frac{1}{2}u''(w_0 + \mu)(X - \mu)^2 \right.$$

$$\left. + \frac{1}{6}u'''(w_0 + \mu)(X - \mu)^3 + \ldots\right]$$

$$= u(w_0 + \mu) + \frac{1}{2}u''(w_0 + \mu)Var_f[X] + \frac{1}{6}u'''(w_0 + \mu)TCM_f[X] + \ldots$$

Expected Utility/Moment Ordering

- Under certain regularity conditions (e.g., $u(w)$ is cubic polynomial), $E_f[u(w_0+X)]$ is:
  - ✓ Increasing in mean of net wealth,
  - ✓ Decreasing in variance of net wealth,
  - ✓ Increasing in TCM of net wealth,
  - ✓ Unaffected by higher moments.

Apprehended Value Principle

- Maximize:

  Apprehended Value = $E_{A*f}[w_0+X] = w_0 + \sum x_i A(x_i) f(x_i)$,

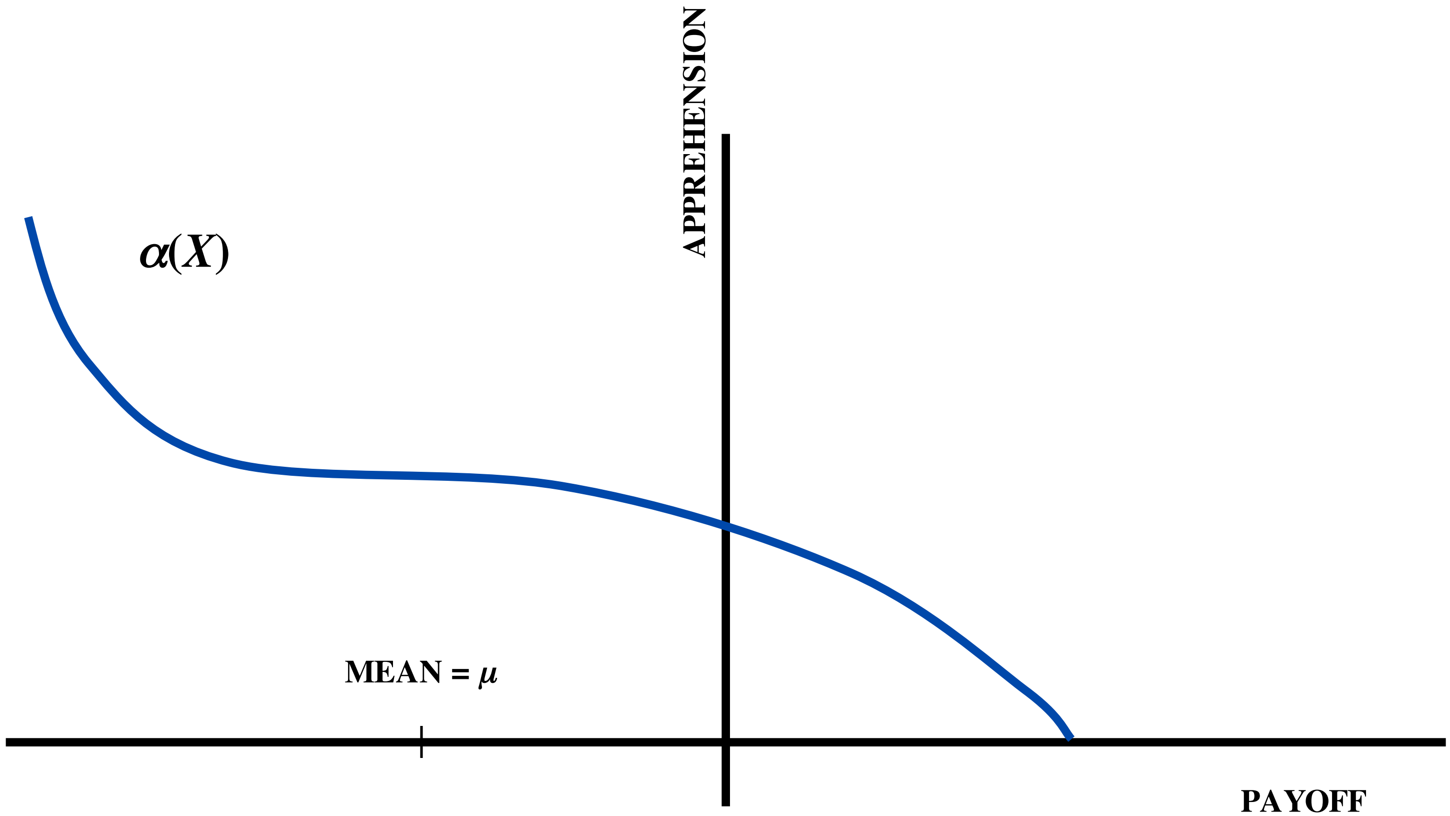
  where $A(x) = \alpha(x) / E_f[\alpha(X)]$, and $\alpha(x)$ is apprehension of payoff x.
- Similar to Prospect Theory (Kahneman/Tversky, 1979).
- Assume Risk Pessimism, i.e.:

  Positive apprehension of payoff, $\alpha(x) > 0$;

  Decreasing apprehension of payoff, $\alpha'(x) \leq 0$.
- Assume Impatience, $\alpha'''(\mu) < 0$, $\alpha'(\mu) = \alpha''(\mu) = 0$.

  Decreasing apprehension becomes less [more] pronounced below [above] mean payoff; no change in function/slope at mean payoff.

Apprehension Function (Risk Pessimism)



Apprehended Value (Analysis)

$$E_{Af}\left[w_0 + X\right]$$

$$= w_0 + E_f\left[A(\mu)X + A'(\mu)X(X-\mu) + \frac{1}{2}A''(\mu)X(X-\mu)^2 + \frac{1}{6}A'''(\mu)X(X-\mu)^3 + \ldots\right]$$

$$= w_0 + A(\mu)\mu + \left[A'(\mu) + \frac{1}{2}A''(\mu)\mu\right]Var_f[X] + \left[\frac{1}{2}A''(\mu) + \frac{1}{6}A'''(\mu)\mu\right]TCM_f[X]$$

$$+ \left[\frac{1}{6}A'''(\mu) + \frac{1}{24}A^{(4)}(\mu)\mu\right]FCM_f[X] + \ldots$$

Apprehended Value/Moment Ordering

- Under certain regularity conditions (e.g., $\alpha(x)$ is cubic polynomial), and for negatively skewed pure risks with $(Sk_f\,[X])^2\,/\,Ku_f\,[X] \geq 3/4$, $E_{A^*f}\,[w_0+X]$ is:
  - ✓ Increasing in mean of payoff,
  - ✓ Unaffected by variance of payoff,
  - ✓ Increasing in TCM of payoff,
  - ✓ Decreasing in FCM of payoff,
  - ✓ Unaffected by higher moments.

Extreme Risks (Bernoulli Model)

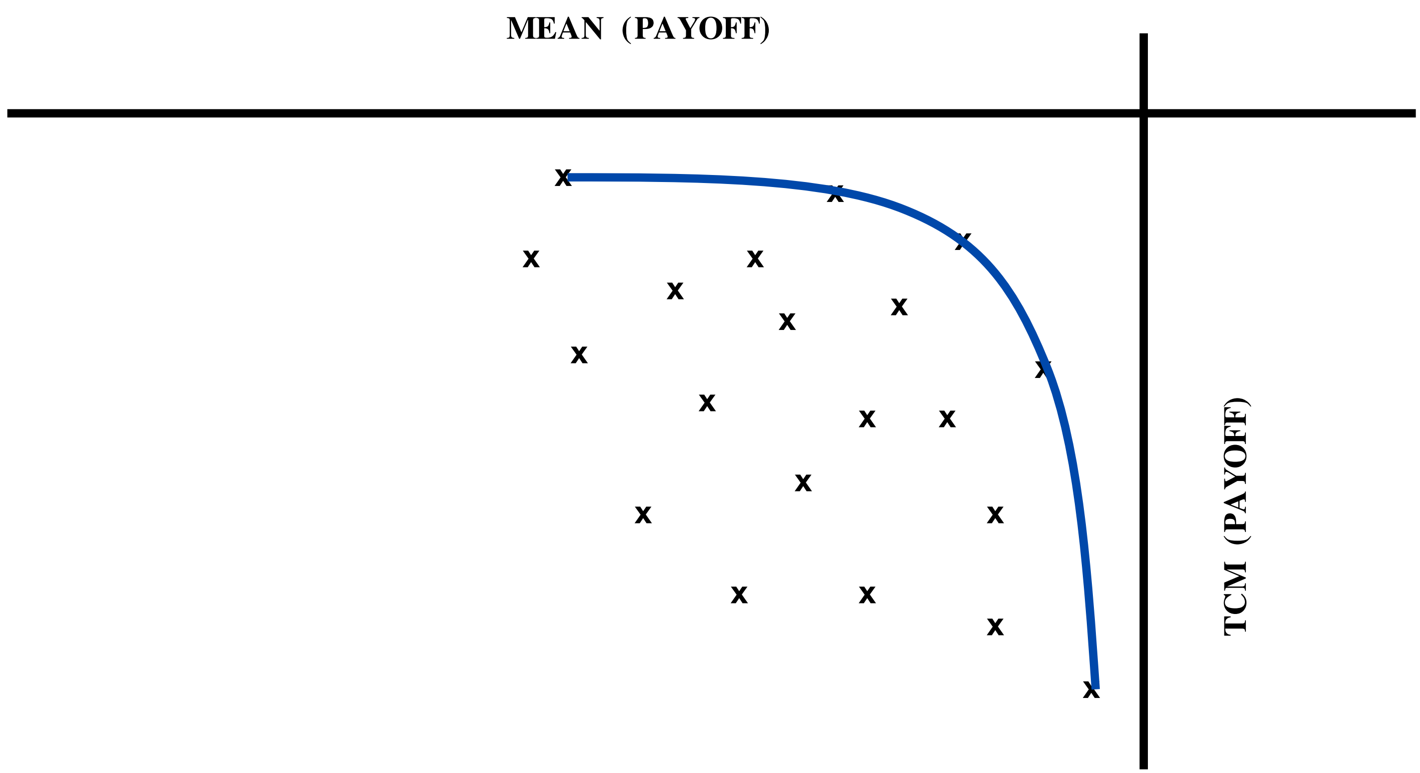
- If extreme-event loss is modeled as

  $X = I \times L$,

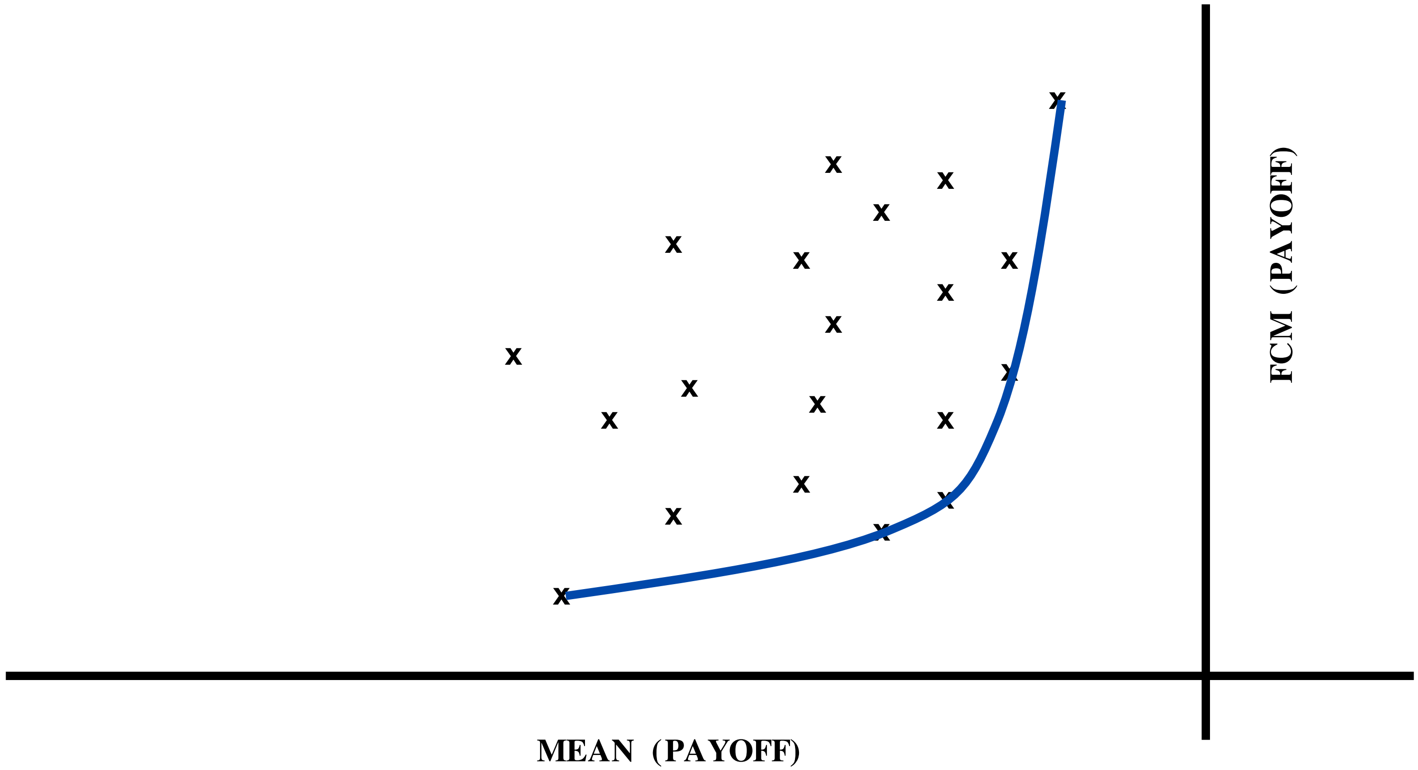
  where I ~ Bernoulli(p) and L is large negative constant, then as p goes to zero,

  $lim\ (Sk_f\,[X])^2 / Ku_f\,[X] = 1 > 3/4.$

Mean-TCM Optimization



Mean-FCM Optimization



Managing TCM/FCM Risk

- Ordinary Diversification

  |TCM| and |FCM| of sample mean shrink to 0.

$$\lim_{n \to \infty} \left| TCM\left[\overline{X}_n\right] \right| = 0$$

$$\lim_{n \to \infty} \left| FCM\left[\overline{X}_n\right] \right| = 0$$

Managing TCM/FCM Risk

- Generalized Hedging

  |TCM| and |FCM| can be diminished by aggregating +/- "correlated" risks.

$$TCM[X+Y] = TCM[X] + TCM[Y] + 3E\left[(X-\mu_X)^2(Y-\mu_Y)\right] + 3E\left[(X-\mu_X)(Y-\mu_Y)^2\right]$$

$$FCM[X+Y] = FCM[X] + FCM[Y] + 4E\left[(X-\mu_X)^3(Y-\mu_Y)\right] + 6E\left[(X-\mu_X)^2(Y-\mu_Y)^2\right] + 4E\left[(X-\mu_X)(Y-\mu_Y)^3\right]$$

Managing TCM/FCM Risk

- Balanced Speculation

  |Skewness| can be diminished by aggregating independent risks of opposite skew (?!)

$$TCM[X+Y] = TCM[X] + TCM[Y]$$

$$+3E\left[(X-\mu_X)^2(Y-\mu_Y)\right] + 3E\left[(X-\mu_X)(Y-\mu_Y)^2\right]$$

Public Policy Implications

- How can we encourage balanced speculation?
  - ✓ Deregulate insurer/reinsurer underwriting activity.
  - ✓ Deregulate insurer/reinsurer investment activity.
- How can we prevent owners/managers from externalizing risk?
  - ✓ Resurrect corporate unlimited liability (?!)
  - ✓ Mandate diffuse ownership of insurer/reinsurer stocks.
  - ✓ Regulate owner/manager compensation.

Reference

- Powers, M. R., 2003, "'Leapfrogging' the Variance: The Financial Management of Extreme-Event Risk," *Journal of Risk Finance*, 4, 4, 26-39.