The Total Claim Cost Model

- For a given policy period, let:

  N = total number of claims,

  X_i = amount of claim i, for $i = 1,2,\ldots,N$,

  where the X_i are i.i.d. random variables independent of N for all i, and

  $L = \sum_{i=1}^{N} X_i$ = total claim costs.

- Then:

  $$E[L] = E_N\left[E_{L|N}[L|N]\right] = E_N\left[NE[X_i]\right] = E[N]E[X_i] \text{ and}$$

  $$Var[L] = E_N\left[Var_{L|N}[L|N]\right] + Var_N\left[E_{L|N}[L|N]\right]$$

  $$= E_N\left[NVar[X_i]\right] + Var_N\left[NE[X_i]\right]$$

  $$= E[N]Var[X_i] + \left(E[X_i]\right)^2 Var[N]$$

  (see the total variance formula discussed previously).

Heterogeneous Risks (Between and Within Rate Classifications)

- Let $N_{ij} | \lambda_{ij} \sim \text{Poisson}(\lambda_{ij})$ denote the number of claims from insured j in rate classification i for a given policy period.

  For fixed i, let $\lambda_{i,j} \sim \text{gamma}(a_i, \beta_i)$.

  Results:

  (1) For insured j selected at random from rate classification i,

  $$\Pr\{N_{i,j} = n\} = \binom{a_i + n - 1}{a_i - 1} \left(\frac{\beta_i}{\beta_i + 1}\right)^{a_i} \left(\frac{1}{\beta_i + 1}\right)^{n};$$

  i.e., for fixed i, $N_{i,j} \sim \text{negative binomial}\left(r = a_i, p = \frac{\beta_i}{\beta_i + 1}\right)$.

  (2) For an insured selected at random from the entire system (i.e., over both i, j), $N_{i,j}$ is generally <u>not</u> ~ negative binomial, because $\lambda_{i,j}$ is not ~ gamma when i is not fixed.

- Given a vector of observations of m insureds from rate classification i, $N_{i,1}, N_{i,2}, \ldots, N_{i,m}$, what is the conditional distribution of $\lambda_{i,j}$?

  Answer: For fixed i, it is known that $\lambda_{i,j} \sim \text{gamma}(a_i, \beta_i)$. Thus, the distribution of $\lambda_{i,j}$ is not subject to updating, and always remains $\text{gamma}(a_i, \beta_i)$.

- Given a vector of m observations of insured j from rate classification i, $N_{i,j,1}, N_{i,j,2}, \ldots, N_{i,j,m}$ what is the conditional distribution of $\lambda_{i,j}$?

  Answer: In this case, we have the Bayesian updating model using the gamma-Poisson conjugate family; therefore,

  $$\lambda_{i,j} | N_{i,j,1}, N_{i,j,2}, \ldots, N_{i,j,m} \sim \text{gamma}\left(\alpha_i + \sum_{k=1}^{m} N_{i,j,k}, \beta_i + m\right).$$

- Here, the gamma distribution is used to model the statistician's uncertainty about the Poisson parameter λ, rather than to model the distribution of λ over a fixed population (i.e., rate classification i).