Model with Categorical Parameter Space

- The frequency (number of claims) is given by die A_1 or A_2, where a marked face indicates an insurance claim.

  $\Pr\{A_1\} = \Pr\{A_2\} = \dfrac{1}{2}$, and

  $\Pr\{\text{Claim} | A_1\} = \dfrac{1}{6}, \quad \Pr\{\text{Claim} | A_2\} = \dfrac{1}{2}.$

- The severity (claim amount) is given by spinner B_1 or B_2.

  $\Pr\{B_1\} = \Pr\{B_2\} = \dfrac{1}{2}$, and

  $\Pr\{\text{Amount} = 2 | B_1\} = \dfrac{5}{6}, \ \Pr\{\text{Amount} = 14 | B_1\} = \dfrac{1}{6},$

  $\Pr\{\text{Amount} = 2 | B_2\} = \dfrac{1}{2}, \ \Pr\{\text{Amount} = 14 | B_2\} = \dfrac{1}{2}.$

- Let X_k = total claim costs in year k.

  Then $X_k = 0$ if there is no claim,

  $= 2$ if there is a claim, and its amount is 2, and

  $= 14$ if there is a claim, and its amount is 14.

- Note that in this case, $\theta = [A_i, B_j]$ for some $i \quad 1,2$ and $j \quad 1,2$;

  thus, θ is a categorical, as opposed to real-valued, state parameter.

Some Bayesian Calculations

- $$E[X_1] = (0)\Pr\{X_1 = 0\} + (2)\Pr\{X_1 = 2\} + (14)\Pr\{X_1 = 14\}$$
$$= (0)\sum_{i=1}^{2}\sum_{j=1}^{2}\Pr\{X_1 = 0 | A_i, B_j\}\Pr\{A_i, B_j\}$$
$$+(2)\sum_{i=1}^{2}\sum_{j=1}^{2}\Pr\{X_1 = 2 | A_i, B_j\}\Pr\{A_i, B_j\}$$
$$+(14)\sum_{i=1}^{2}\sum_{j=1}^{2}\Pr\{X_1 = 14 | A_i, B_j\}\Pr\{A_i, B_j\},$$ <u>or</u>

$$E[X_1] = \sum_{i=1}^{2}\sum_{j=1}^{2}E[X_1 | A_i, B_j]\Pr\{A_i, B_j\}.$$

- $$E[X_2 | X_1] = (0)\Pr\{X_2 = 0 | X_1\} + (2)\Pr\{X_1 = 2 | X_1\} + (14)\Pr\{X_2 = 14 | X_1\}$$
$$= (0)\sum_{i=1}^{2}\sum_{j=1}^{2}\Pr\{X_2 = 0 | X_1, A_i, B_j\}\Pr\{A_i, B_j | X_1\}$$
$$+(2)\sum_{i=1}^{2}\sum_{j=1}^{2}\Pr\{X_2 = 2 | X_1, A_i, B_j\}\Pr\{A_i, B_j | X_1\}$$
$$+(14)\sum_{i=1}^{2}\sum_{j=1}^{2}\Pr\{X_2 = 14 | X_1, A_i, B_j\}\Pr\{A_i, B_j | X_1\}$$

$$= (0)\sum_{i=1}^{2}\sum_{j=1}^{2}\Pr\{X_2 = 0 | A_i, B_j\}\Pr\{A_i, B_j | X_1\}$$
$$+(2)\sum_{i=1}^{2}\sum_{j=1}^{2}\Pr\{X_2 = 2 | A_i, B_j\}\Pr\{A_i, B_j | X_1\}$$
$$+(14)\sum_{i=1}^{2}\sum_{j=1}^{2}\Pr\{X_2 = 14 | A_i, B_j\}\Pr\{A_i, B_j | X_1\},$$

where $\Pr\{A_i, B_j | X_1\} = \dfrac{\Pr\{X_1 | A_i, B_j\}\Pr\{A_i, B_j\}}{\sum_{i'=1}^{2}\sum_{j'=1}^{2}\Pr\{X_1 | A_{i'}, B_{j'}\}\Pr\{A_{i'}, B_{j'}\}}$ <u>or</u>

$$E[X_2 | X_1] = \sum_{i=1}^{2}\sum_{j=1}^{2}E[X_2 | X_1, A_i, B_j]\Pr\{A_i, B_j | X_1\}$$
$$= \sum_{i=1}^{2}\sum_{j=1}^{2}E[X_2 | A_i, B_j]\Pr\{A_i, B_j | X_1\},$$

where $\Pr\{A_i, B_j | X_1\}$ is as above.

Some Bühlmann Calculations

- $C = zX_1 + (1 - z)H$,

  where $H = E[X_1]$ and $z = \dfrac{n}{n + \dfrac{EVPV}{VHM}}$.

- From above, we know that

$$H = E[X_1] = \sum_{i=1}^{2} \sum_{j=1}^{2} E[X_1 | A_i, B_j] \Pr\{A_i, B_j\}.$$

- Similarly,

$$VHM = \sum_{i=1}^{2} \sum_{j=1}^{2} \left(E[X_1 | A_i, B_j]\right)^2 \Pr\{A_i, B_j\} - \left(E[X_1]\right)^2.$$

- Finally,

$$EVPV = \sum_{i=1}^{2} \sum_{j=1}^{2} Var[X_1 | A_i, B_j] \Pr\{A_i, B_j\}$$

$$= \sum_{i=1}^{2} \sum_{j=1}^{2} \left\{ E[X_1^2 | A_i, B_j] - \left(E[X_1 | A_i, B_j]\right)^2 \right\} \Pr\{A_i, B_j\}.$$