Actuarial Science 2101 – Sample Quiz 3

1. The age (in years) at which a certain tunnel will collapse is a continuous random variable X whose p.d.f. is symmetric about the value 125, and whose raw moments, $E_X\left[X^k\right]$, are well-defined for $k \in \{1,2,3,\ldots\}$.

(a) Which of the following must equal $E_X[X]$?

A. $\sqrt{E_X\left[X^2\right]}$ B. $\sqrt{Var_X[X]}$ C. $CV_X[X]$ D. $CS_X[X]$ E. None of above

(b) Which of the following must equal 0?

A. $E_X\left[X^2\right]-(125)^2$

B. $E_X\left[(X-125)^2\right]$

C. $E_X\left[X^3\right]-(125)^3$

D. $E_X\left[(X-125)^3\right]$

E. None of above

2. Let X be a continuous liability loss amount with p.d.f. $f_X(x)>0$ on the sample space $x>0$. Furthermore, let X be subject to an ordinary deductible of size $d>0$, and define the random variable $W = X|X>d$ on the sample space $w>d$.

(a) Which of the following expressions equals $E_W\left[W^2\right]$?

A. $\int_d^\infty (x-d)^2 f_X(x)dx$

B. $d^2F_X(d)+\int_d^\infty (x-d)^2 f_X(x)dx$

C. $\int_d^\infty x^2 \frac{f_X(x)}{1-F_X(d)}dx$

D. $d^2F_X(d)+\int_d^\infty x^2 \frac{f_X(x)}{1-F_X(d)}dx$

E. None of above

(b) Which of the following expressions must be negative?

A. $E_X[X]-E_W[W]$ B. $SD_X[X]-SD_W[W]$ C. $Corr[X,W]$ D. $Cov[X,W]$ E. None of above

3. Let X be a discrete random variable with m.g.f. $M_X(t)=\exp\left(Y\left(e^t-1\right)\right)$, where $Y \in (0,\infty)$ is a parameter.

(a) Express $E_X[X]$ in terms of Y.

A. $1/Y^2$ B. $1/Y$ C. Y D. Y^2 E. None of above

(b) Now assume that Y itself is a continuous random variable with m.g.f. $M_Y(t)=\lambda/(\lambda-t)$, where $\lambda \in (0,\infty)$ is a parameter. Express $E_X[X]$ in terms of λ.

A. $1/\lambda^2$ B. $1/\lambda$ C. λ D. λ^2 E. None of above