Exploring the Coinsurance Clause

Notation:

For a property insurance policy, let:

L denote the loss amount,
F denote the policy face value,
V denote the full value of the total property insured, and
c denote the coinsurance factor.

Case I—"Coinsurance Clause":

Under the "coinsurance clause," the insurance payment is given by

$$I_c = Min\left\{L, F, L\frac{F}{cV}\right\}.$$

Now note that

$$I_c < L \iff A \equiv \left\{F < L \cap F < L\frac{F}{cV}\right\} \text{ or } B \equiv \left\{L\frac{F}{cV} < L \cap L\frac{F}{cV} < F\right\}$$
$$\iff A \equiv \{F < L \cap cV < L\} \text{ or } B \equiv \{F < cV \cap L < cV\}.$$

In other words, the coinsurance clause reduces the insurance payment if and only if either case A or case B obtains.

Examples:

Consider the six possible orderings of L, F, and cV (with $V = 100{,}000$, $c = .80$):

$L < F < cV$
$F = 60{,}000$
$L = 50{,}000$
$I = 37{,}500$ (reduced, case B)

$F < L < cV$
$F = 60{,}000$
$L = 70{,}000$
$I = 52{,}500$ (reduced, case B)

$F < cV < L$
$F = 60,000$
$L = 90,000$
$I = 60,000 < 67,500$ (reduced, case A)

$L < cV < F$
$F = 90,000$
$L = 70,000$
$I = 70,000 < 78,750$ (not reduced)

$cV < L < F$
$F = 90,000$
$L = 85,000$
$I = 85,000 < 95,625$ (not reduced)

$cV < F < L$
$F = 90,000$
$L = 100,000$
$I = 90,000 < 112,500$ (reduced, case A)

Pure Premium:

If $F < cV$, then

$$
\begin{aligned}
E[I_c] &= p\left[\int_0^F L\frac{F}{cV}s(L)dL + \int_F^{cV} L\frac{F}{cV}s(L)dL + F\int_{cV}^{V} s(L)dL\right] \\
&= p\left[\int_0^{cV} L\frac{F}{cV}s(L)dL + F\int_{cV}^{V} s(L)dL\right],
\end{aligned}
$$

where p denotes the probability of a loss, and $s(L)$ denotes the probability density function describing the relative likelihoods of different loss amounts, L. It follows that the pure premium (i.e., rate per dollars of property insured) is given by

$$
R_c = \frac{E[I_c]}{F} = p\left[\frac{1}{cV}\int_0^{cV} Ls(L)dL + \int_{cV}^{V} s(L)dL\right].
$$

Consequently,

$$
\frac{dR_c}{dF} = 0.
$$

If $F > cV$, then

$$E[I_c] = p\left[\int_0^{cV} Ls(L)dL + \int_{cV}^{F} Ls(L)dL + F\int_F^V s(L)dL\right]$$
$$= p\left[\int_0^F Ls(L)dL + F\int_F^V s(L)dL\right].$$

It follows that the pure premium is given by

$$R_c = \frac{E[I_c]}{F} = p\left[\frac{1}{F}\int_0^F Ls(L)dL + \int_F^V s(L)dL\right].$$

Consequently,

$$\frac{dR_c}{dF} = -p\left[\frac{1}{F^2}\int_0^F Ls(L)dL\right] < 0.$$

<u>Case II—No "Coinsurance"</u>:

Now consider a simple property insurance policy with a policy limit equal to the face value, F, and no coinsurance clause; i.e., the insurance payment is given by

$$I_0 = Min\{L, F\}.$$

Pure Premium:

In this case,

$$E[I_0] = p\left[\int_0^F Ls(L)dL + F\int_F^V s(L)dL\right],$$

and so the pure premium is given by

$$R_0 = \frac{E[I_0]}{F} = p\left[\frac{1}{F}\int_0^F Ls(L)dL + \int_F^V s(L)dL\right].$$

Consequently,

$$\frac{dR_0}{dF} = -p\left[\frac{1}{F^2}\int_0^F Ls(L)dL\right] < 0.$$