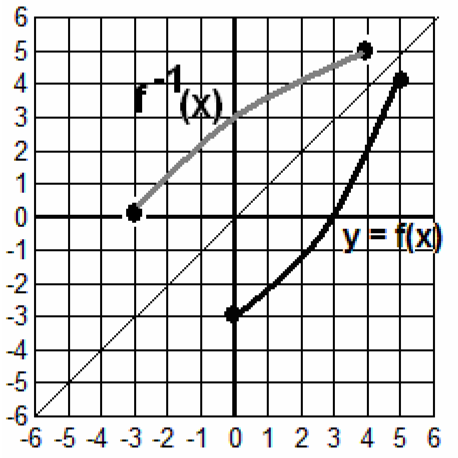
Solutions to Review for FINAL Spring 2009

1. Slope between the points is m = -11; **y – 6 = -11(x – 2)** is one way to give the equation.
2. m = -3; (a) **y -5 = -3(x-1)** (b) **y – 3 = 1/3 (x+2)**
3. (a) As x increase so does y, so there will be a positive correlation.
   (b) Table of information. Final entries are the sums.

x	y	xy	x^2
-4	1.2	-4.8	16
-2	2.8	-5.6	4
0	5.3	0	0
2	6.7	13.4	4
4	9.1	36.4	16
0	25.1	39.4	40

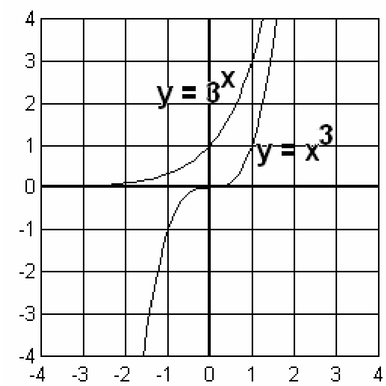
   Slope = 0.985 Y-intercept = 5.02
4. Let x = amount of fine sand, then 50 – x = amount of coarse sand.
   Equation: 0.26x + 0.15(50 – x) = 8.95 ➔ solve for x ➔ 0.11x = 1.45 ➔ x = 13
   So each bag contains 13lbs of fine sand and 37 lbs of coarse sand.
5. System #1: Infinitely many solutions. System #2: Unique solution x = 2, y = -3.
   System #3: No solutions.
6. Vertex $\mathbf{h} = \frac{-\mathbf{b}}{\mathbf{2a}} = \frac{\mathbf{1}}{\mathbf{4}}$, $\mathbf{k} = \frac{-\mathbf{25}}{\mathbf{8}}$; x-intercepts, factor or use quadratic formula; x = 3/2, -1
7. L = 18 + w; Perimeter = 180 = 2W + 2L = 2W + 2(18 + W) ➔ 180 = 36 + 4W ➔ 4W = 144; W = 36, so then L = 54.
8. Let x = amount in the account paying 5%, then 5000 – x = amount in the account paying 7%.
   Equation: 0.05x + 0.07(5000 – x) = 325 ➔ 350 – 0.02x = 325 ➔ -0.02 = -25 ➔ x = 1250 so invest 3750 in the account paying 7%.
9. (a) $(f \circ g)(x) = f(g(x)) = 3(2x^2 + 1) - 2 = \mathbf{6x^2 + 1}$
   $(g \circ f)(x) = g(f(x)) = 2(3x - 2)^2 + 1 = 2(9x^2 - 12x + 4) + 1 = \mathbf{18x^2 - 24x + 9}$
   (b) $(f \circ g)(x) = f(g(x)) = \sqrt{\mathbf{x^2 - 5 + 1}} = \sqrt{\mathbf{x^2 - 4}}$
   $(g \circ f)(x) = g(f(x)) = \left(\sqrt{\mathbf{x+1}}\right)^{\mathbf{2}} - \mathbf{5} = \mathbf{x + 1 - 5} = \mathbf{x - 4}$
10. (a) $(f \circ g)(3) = f(g(3)) = f(5) = -1$
    (b) $(g \circ f)(4) = g(f(4)) = g(2) = 0$
    (c) $(g \circ f)(3) = g(f(3)) = g(0)$ undefined since no value of g is given for x = 0
11. Apply the horizontal line test.
12. (a) YES (b) NO (c) YES (d) NO
13. $y = x^2 + 1$ where $x \geq 0$; solve for x, then interchange the names x and y.
    $y - 1 = x^2$ ➔ $\mathbf{x} = \sqrt{\mathbf{y - 1}}$ (we choose the positive square root since $x \geq 0$)
    Now interchange names: $\mathbf{y} = \sqrt{\mathbf{x - 1}}$ (Note that for the inverse function $x \geq 1$.)

14.



15. (a) Observe that as x increase by 1 unit y decreases in each case by 4 units ➔ linear equation ➔ m = -4, $y = 16 - 4x$.
(b) Observe that as x increases by 1 unit that the value of y is ¼ the previous value ➔ exponential function ➔ $y = Ca^x$; here C = 16 and a = ¼.

16.



17. Isolate 10^{3x} on one side, then take the log of both sides.

$$4(10^{3x}) = 244 \to 10^{3x} = 61$$
$$\to \log(10^{3x}) = \log(61) \to 3x \approx 1.785 \to x \approx 0.595$$

18. Isolate log(2x) on one side, then use both sides as exponents on 10.

$$5\log(2x) = 16 \to \log(2x) = \frac{16}{5} \to 10^{\log(2x)} = \log\left(\frac{16}{5}\right)$$
$$\to 2x = \log\left(\frac{16}{5}\right) \to x = 0.5\log\left(\frac{16}{5}\right) \to x \approx 792.4$$

19. Using (1,3) ➔ 3 = Ca ➔ C = 3/a and using (2, 7) ➔ $7 = Ca^2$ ➔ replace C by 3/a and solve for a. We get a = 7/3. Then since C = 3/a we get C = 9/7. So $y = 9/7\ (7/3)^x$ or if you approximate the fractions $y \approx 1.29\ (2.33)^x$.

20. $A = 5000\left(1 + \frac{0.035}{4}\right)^{4\times 5} \approx \$5951.70$

21. $f(2) = 1 - e^{-1} \approx 0.63$ This means that there is about a 63% chance that at least one car will enter the intersection within a 2 minute period.

22. (a) f(0) = 0.48 ln(1) + 27 = 27 ➔ this says the pressure at the eye is 27.
f(100) = 0.48 ln(101) + 27 ≈ 29.2 ➔ this says a 100 miles from the eye the pressure is 29.2
(b) Solve 28 = 0.48 ln(x+1) + 27 for x; isolate ln(x + 1) on one side and then use both sides as exponent on e. ➔

$$28 = 0.48\ \ln(x+1) + 27 \to 1 = 0.48\ln(x+1) \to \ln(x+1) = \frac{1}{0.48} \approx 2.08$$
$$\text{Then } e^{\ln(x+1)} \approx e^{2.08} \to x + 1 \approx e^{2.08} \to x \approx e^{2.08} - 1 \to x \approx 7$$

23. (a) $\log(2x^4) = \log(2) + \log(x^4) = \log(2) + 4\log(x)$

(b) $\ln\left(\frac{7x^3}{k}\right) = \ln(7x^3) - \ln(k) = \ln(7) + \ln(x^3) - \ln(k) = \ln(7) + 3\ln(x) - \ln(k)$

(c)

$$\log\left(\frac{\sqrt{x+1}}{(x-2)^3}\right) = \log\left(\sqrt{x+1}\right) - \log(x-2)^3 = \log(x+1)^{1/2} - 3\log(x-2) = \tfrac{1}{2}\log(x+1) - 3\log(x-2)$$

24. (a) $\ln(2e)+\ln\left(\frac{1}{e}\right)=\ln\left(2e\frac{1}{e}\right)=\ln(2)$

(b) $\log(27)+\log\left(x^3\right)=\log\left(27x^3\right)$

(c) $\log(x^3)-\log(x^2)=\log\left(\frac{x^3}{x^2}\right)=\log(x)$

25. (a) Take the log of both sides: $\log 10^{x+2}=\log 10^{3x}\to x+2=3x\to x=1$

(b) Isolate $(1.5)^{2x}$ on one side then take the log of both sides.

$7(1.5)^{2x}-3=12\to 7(1.5)^{2x}=15\to(1.5)^{2x}=\frac{15}{7}$

$\to\log(1.5)^{2x}=\log\left(\frac{15}{7}\right)\to 2x\log(1.5)=\log\left(\frac{15}{7}\right)\to 2x=\frac{\log\left(\frac{15}{7}\right)}{\log(1.5)}\to x\approx 0.9398$

(c) Take the log of both sides; $\log\left(\frac{1}{6}\right)^{x+2}=\log(3)\to(x+2)\log\left(\frac{1}{6}\right)=\log(3)$

$\to x+2=\frac{\log(3)}{\log\left(\frac{1}{6}\right)}\to x\approx -0.613-2\to x\approx -2.613$