6.2

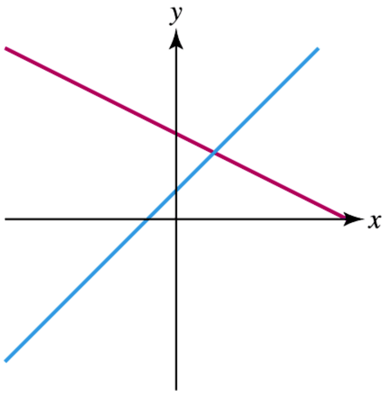
Systems of Equations in Two Variables

- Recognize different types of linear systems
- Apply the elimination method
- Solve systems using elimination and substitution
- Solve systems with no solution
- Solve systems with infinitely many solutions

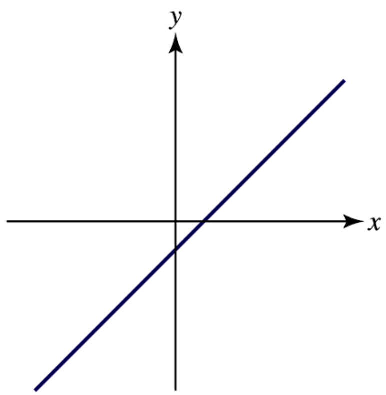
PEARSON
Addison Wesley

We saw previously that a system of two equations in two unknowns has only 3 possible solution sets.

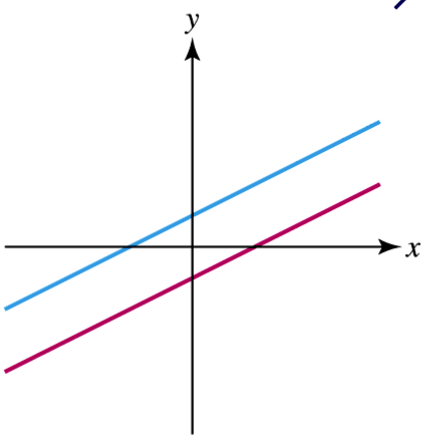
A unique solution; exactly one ordered pair (r, s) that satisfies both equations. The graphs of the corresponding lines intersect *once*.



Infinitely many solutions; the graphs of the corresponding lines are identical.



No solutions; no ordered pair (r, s) satisfies both equations. The graphs of the corresponding lines do not intersect. The two lines are parallel.



Some terminology:

We say the system is CONSISTENT if there is at least one solution.

1. In the case of a unique solution, the system is consistent and we say the equations are independent.

2. In the case of infinitely many solutions, we say the system is consistent, but the equations are dependent.

3. In the case of no solutions, we say the system is INCONSISTENT.

In Section 6.1 we used the Method of Substitution. This was a symbolic method that had us solve for one variable from an equation and substitute it into the second equation. A lot of algebra involved.

Here we introduce a second procedure called the Method of Elimination. This method is also symbolic, but involves less algebra. This method can also be used on systems of linear equations with more variables and equations.

We show the Method of Elimination using examples.

Example: **Use elimination to determine the solution set of the system and then state whether the system is consistent or inconsistent. If the system is consistent state whether the equations are independent or dependent.**

$$3x - y = 7$$
$$5x + y = 9$$

Observe that the coefficients of y have the same absolute value, but opposite signs; -1 & 1.

To eliminate y we will add the two equations.

$$\begin{array}{l} 3x - y = 7 \\ \underline{5x + y = 9} \\ 8x \qquad = 16 \end{array}$$

← **This gives us one equation in the single variable x.**

Solve for x; we get x = 2.

For our system $\begin{aligned} 3x - y &= 7 \\ 5x + y &= 9 \end{aligned}$ **we have x = 2.**

How can we find a value for y?

Substituting x = 2 into either equation gives y = -1.

What is the solution set of the system?

The solution set is the unique point (2, -1).

Is the system consistent?

Are the equations independent or dependent?

Example: **Use elimination to determine the solution set of the system and then state whether the system is consistent or inconsistent. If the system is consistent state whether the equations are independent or dependent.**

$$-4x + 2y = 8$$
$$2x - y = -4$$

Lets try to eliminate y.

To do that we need to get the coefficients of y to have the same absolute value but opposite signs.

One way to do that is to multiply the second equation by 2.

The new second equation is

$$2(2x) - 2y = 2(-4) \rightarrow 4x - 2y = -8$$

Now the system of equations is

$$-4x + 2y = 8$$
$$4x - 2y = -8$$

This system will have the same solution set as the original system.

Now add the two equations.

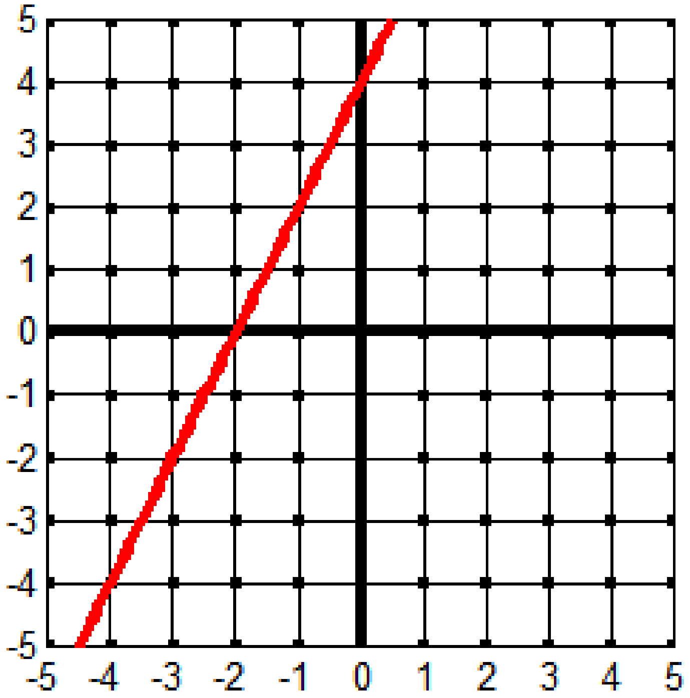
$$\begin{array}{r} -4x + 2y = 8 \\ \underline{4x - 2y = -8} \\ 0 \;+\; 0 \;=\; 0 \end{array}$$

The result is equation 0 = 0, which is always true. This means the two equations represent the same line.

So the solution set is all the points that lie on the line -4x + 2y = 8.

So there are infinitely many solutions to the system.

The graph of the solution set is all points along the line. ➔



We found that using elimination that the system

$$-4x + 2y = 8$$
$$2x - y = -4$$

has infinitely many solutions.

Is the system consistent?

Are the two equations independent?

Find 4 different solutions.

Example: **Use elimination to determine the solution set of the system and then state whether the system is consistent or inconsistent. If the system is consistent state whether the equations are independent or dependent.**

$$x - y = 5$$

$$x - y = -2$$

Using elimination, if we subtract the second equation from the first, we obtain the following result.

$$\begin{array}{r} x - y = 5 \\ x - y = -2 \\ \hline 0 = 7 \end{array}$$

The equation 0 = 7 is a contradiction that is never true.

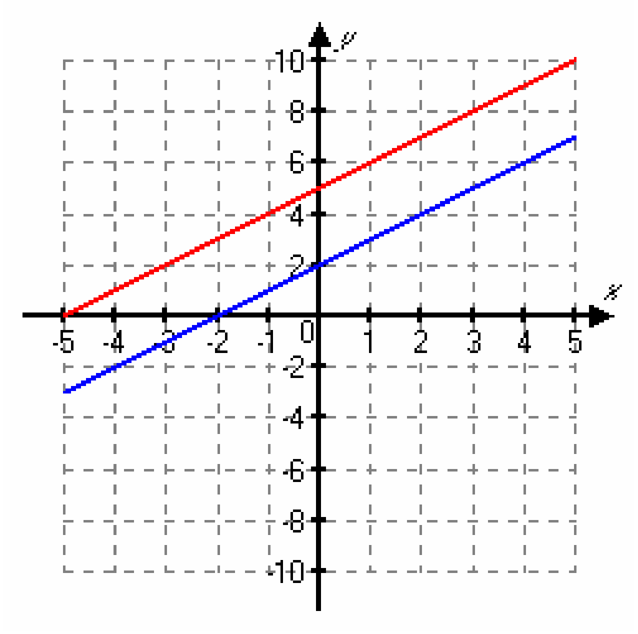
Therefore there are no solutions, and the system is inconsistent.

From the graphs of the two equations

$$x - y = 5$$

$$x - y = -2$$

We see that they are parallel lines which never intersect, hence there are no solutions to the system.



Example: **Solve the system by using elimination.**

$$3x - 4y = 1$$

$$2x + 3y = 12$$

To apply elimination we need to get the coefficients of one of the variables to have the same absolute value but have different signs.

Observe that the coefficients of y have opposite signs.

One way to get the coefficients of y have the same absolute value is multiply equation #1 by 3 and multiply equation #2 by 4.

3(3x) -3(4y) = 3(1) **This gives ➔** **9x - 12y = 3**

4(2x) + 4(3y) = 4(12) **8x + 12y = 48**

Using these two "new equations" eliminate variable y. Add the two equations

9x - 12y = 3

8x + 12y = 48

$$\begin{aligned} 9x - 12y &= 3 \\ \underline{8x + 12y} &\underline{= 48} \\ 17x \quad\quad &= 51 \end{aligned}$$

Solving for x we get x = 3

To find y go back to either of t he original equations and substitute 3 for x and solve for y.

$$3x - 4y = 1$$

$$2x + 3y = 12$$

← 3(3) – 4y = 1

9 – 4y = 1 → -4y = -8 → y = 2

So the solution set is the unique solution x = 3, y = 2.

Verbal Problem

A student took out two loans which totaled \$3000. One loan was had 8% interest attached and the other 10% interest. The total annual interest for both loans was compounded annually and for the first year totaled \$264.

a) Determine a system of equations that models this system of equations.

b) Determine the value of the each of the loans.

P. 505 #71.

Verbal Problem

Americans spent $40.4 billion on Internet shopping in 2002 for travel and computer equipment. The amount spent for travel was triple the amount spent for computer equipment.

a) Determine a system of equations that models this system of equations.

b) Determine the amount spent on travel and the amount spent on computer equipment.

P. 505 #78.

Some in-class problems.

1. $x + 3y = 10$
   $-x + 2y = 5$

2. $2x - 3y = 10$
   $5x + 2y = 7$

3. $5x + 10y = 10$
   $x + 2y = 2$

4. $0.5x - y = 5$
   $x - 0.5y = 4$