5.1

Combining Functions

- Perform arithmetic operations on functions
- Perform composition of functions

Five Ways of Combining Two Functions f and g

• ***Addition***	• $f+g$
• ***Subtraction***	• $f-g$
• ***Multiplication***	• fg
• ***Division***	• $\dfrac{f}{g}$
• ***Composition***	• $f \circ g$

Definitions

If $f(x)$ and $g(x)$ both exist, the sum, difference, product, quotient and composition of two functions f and g are defined by

- ***Addition*** $$(f+g)(x)=f(x)+g(x)$$
- ***Subtraction*** $$(f-g)(x)=f(x)-g(x)$$
- ***Multiplication*** $$(fg)(x)=f(x)\times g(x)$$
- ***Division*** $$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)} \text{ where } g(x)\neq 0$$
- ***Composition*** $$(f\circ g)(x)=f(g(x))$$

Examples of Evaluating Combinations of Functions – Using Symbolic Representations

Example of Addition of Functions:

Let $f(x) = x^2 + 2x$ and $g(x) = 3x - 1$

Find the symbolic representation for the function $f + g$ and use this to evaluate $(f + g)(2)$

$(f + g)(x) = f(x) + g(x) = (x^2 + 2x) + (3x – 1)$ ← Use definitons

$(f + g)(x) = x^2 + 5x – 1$ ← Combine terms

$(f + g)(2) = 2^2 + 5(2) – 1 = 13$ ← Set x = 2 (evaluate)

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or Evaluate each function, then combine

$(f + g)(2) = f(2) + g(2)$ ← evaluate each function

$= (2^2 + 2(2)) + (3(2) – 1)$ ← show the terms

$= 13$ ← combine

Graphical Example of Addition of Functions

Compute

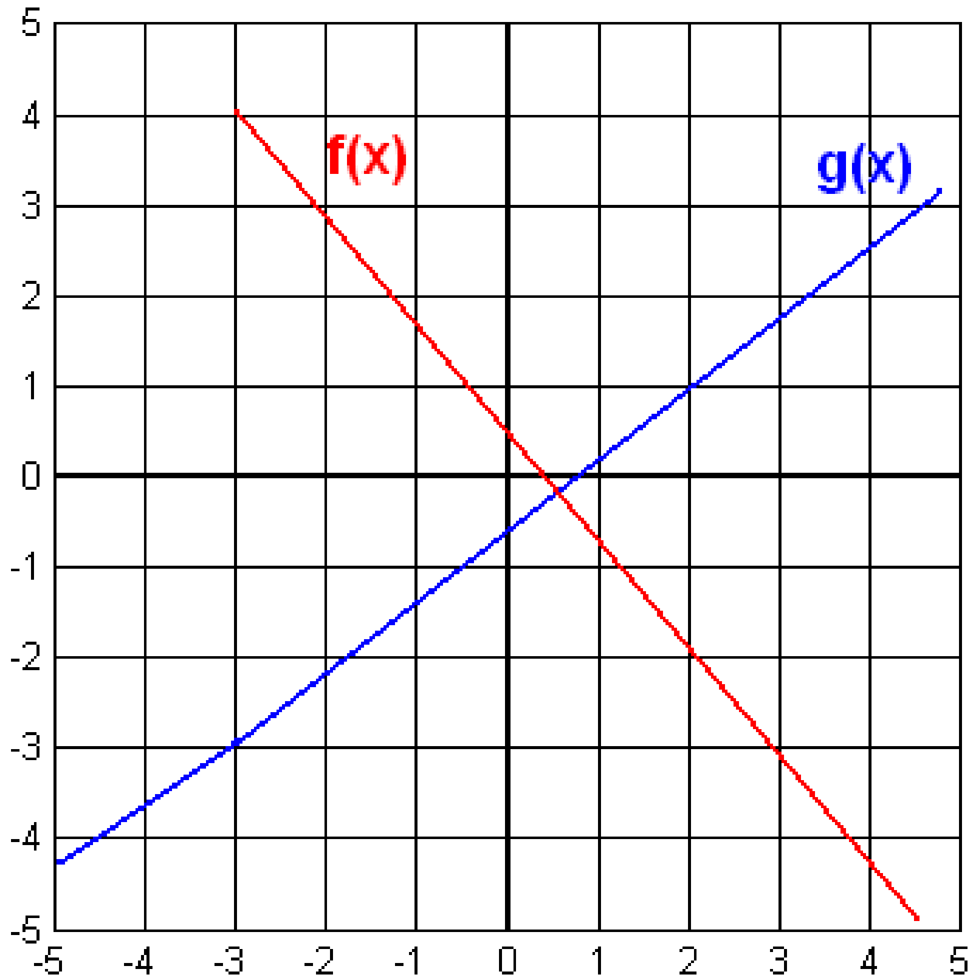
f(2) + g(2)

-2 + 1 = -1

Compute

f(-3) + g(-3)

4 + -3 = 1



Example of Multiplication of Functions:

Let $f(x) = x^2 + 2x$ and $g(x) = 3x - 1$

Find the symbolic representation for the function *fg* and use this to evaluate $(fg)(2)$

$(fg)(x) = (x^2 + 2x)(3x - 1)$ ← use the formulas

$(fg)(x) = 3x^3 + 6x^2 - x^2 - 2x$ ← multiply the terms

$(fg)(x) = 3x^3 + 5x^2 - 2x$ ← combine terms

So $(fg)(2) = 3(2)^3 + 5(2)^2 - 2(2) = 40$ ← set x = 2

Or

$(fg)(2) = f(2) \times g(2)$ ← use the individual formulas

$= (2^2 + 2(2)) \times (3(2) - 1)$ ← set x = 2 in the formulas

$= 8 \times 5 = 40$ ← evaluate & multiply

Example of Composition of Functions:

Let $f(x) = x^2 + 2x$ and $g(x) = 3x - 1$

Find the symbolic representation for the function $f \circ g$ and use this to evaluate $(f \circ g)(2)$

Example of Composition of Functions:

Let $f(x) = x^2 + 2x$ and $g(x) = 3x - 1$

Find the symbolic representation for the function $f \circ g$ and use this to evaluate $(f \circ g)(2)$

$(f \circ g)(x) = f(g(x)) = f(3x - 1)$ ← **use formula for g(x)**

$= (3x - 1)^2 + 2(3x - 1)$ ← **evaluate f at g(x)**

$(f \circ g)(x) = (3x - 1)(3x - 1) + 6x - 2$ ← **expand terms**

$(f \circ g)(x) = 9x^2 - 3x - 3x + 1 + 6x - 2$ ← **multiply**

$(f \circ g)(x) = 9x^2 - 1$ ← **combine terms**

So $(f \circ g)(2) = 9(2)^2 - 1 = 35$ ← **set x = 2**

Example: Let $f(x) = 1-5x$ and $g(x) = \sqrt{x^2 + 6}$

Find $(f \circ g)(x)$.

Find $(g \circ f)(x)$.

Example: Composition from tabular displays for functions.

x	1	3	4	6
$f(x)$	2	6	5	7

x	2	3	5	7
$g(x)$	4	2	6	0

Find $(g \circ f)(1)$

Find $(f \circ g)(4)$

Find $(f \circ f)(3)$

BB

Modeling the urban heat island phenomenon

Cities are made up of large amounts of concrete and asphalt that heat up in the daytime from sunlight but do not cool off completely at night. As a result, urban areas tend to be warmer than the surrounding rural areas. This effect is called the *urban heat island* and has been documented in cities throughout the world. In Figure 5.11 function f computes the average increase in nighttime summer temperatures in degrees Celsius at Sky Harbor Airport in Phoenix from 1948 to 1990. In this graph 1948 is the base year with a zero temperature increase. This rise in urban temperature increased peak demand for electricity. In Figure 5.12 function g computes the percent increase in electrical demand for an average nighttime temperature increase of x degrees Celsius. **(Source:** W. Cotton and R. Pielke, *Human Impacts on Weather and Climate*.)

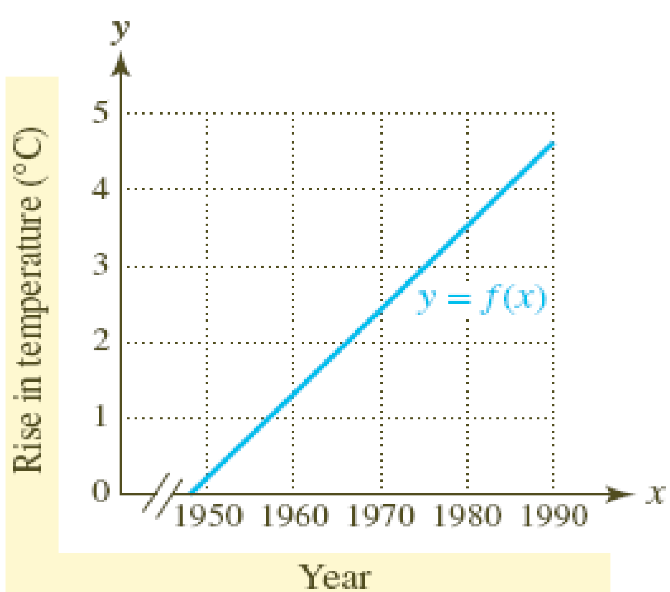


FIGURE 5.11 Nighttime Temperature Increase

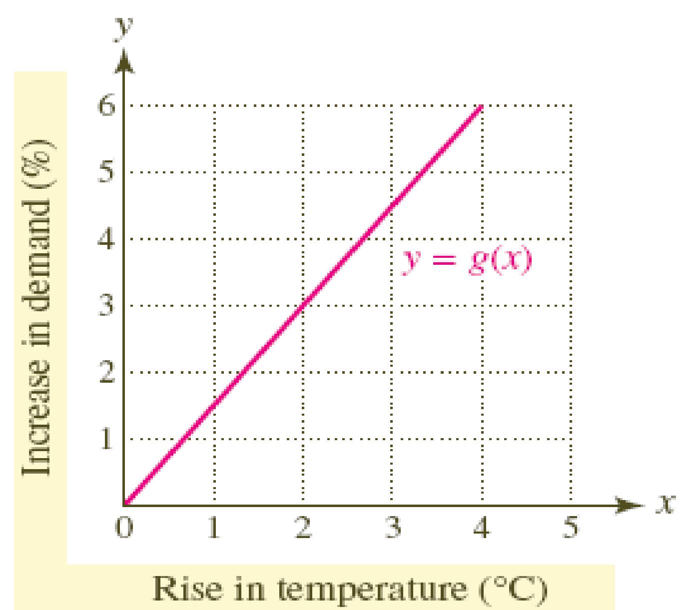


FIGURE 5.12 Percent Increase in Electrical Demand

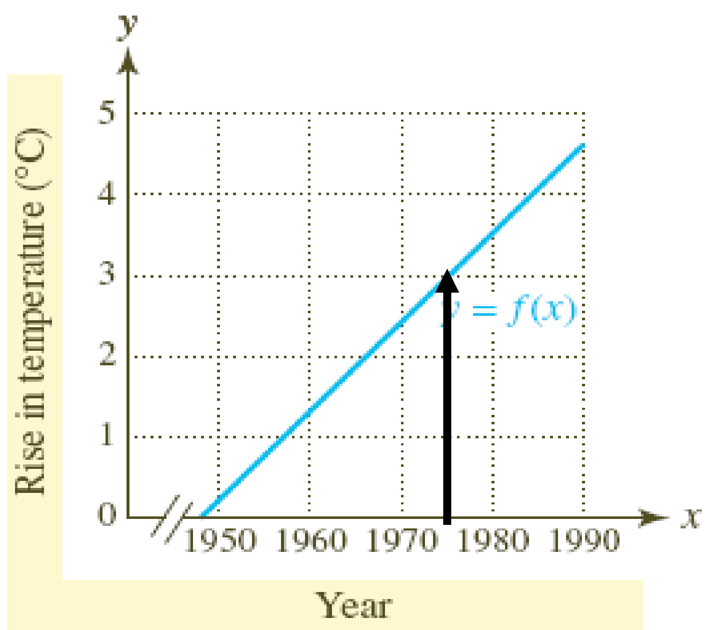


FIGURE 5.11 Nighttime Temperature Increase

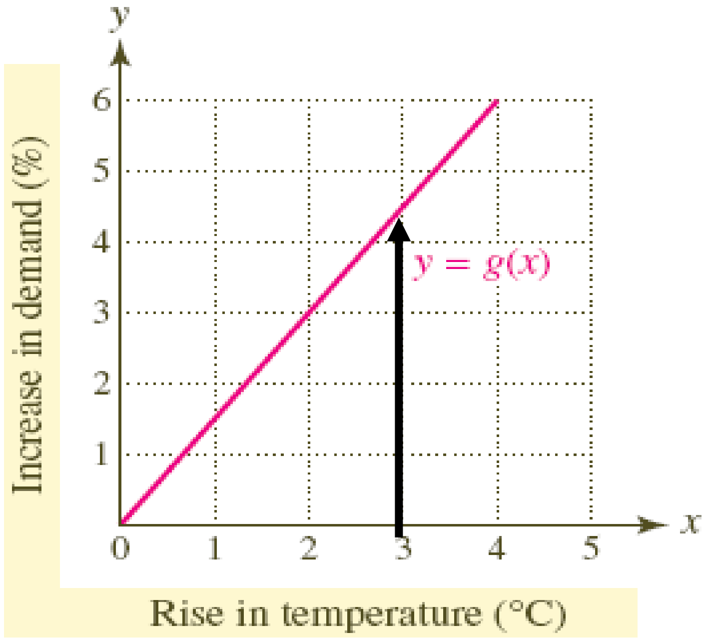


FIGURE 5.12 Percent Increase in Electrical Demand

(a) Evaluate $(g \circ f)(1975)$ graphically.

f(1975) = ______

g(f(1975)) = ______

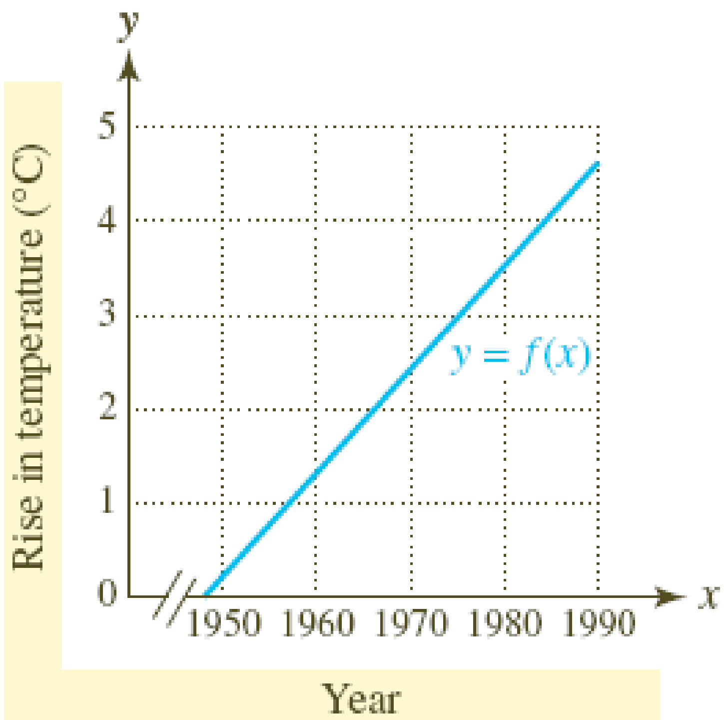


FIGURE 5.11 Nighttime Temperature Increase

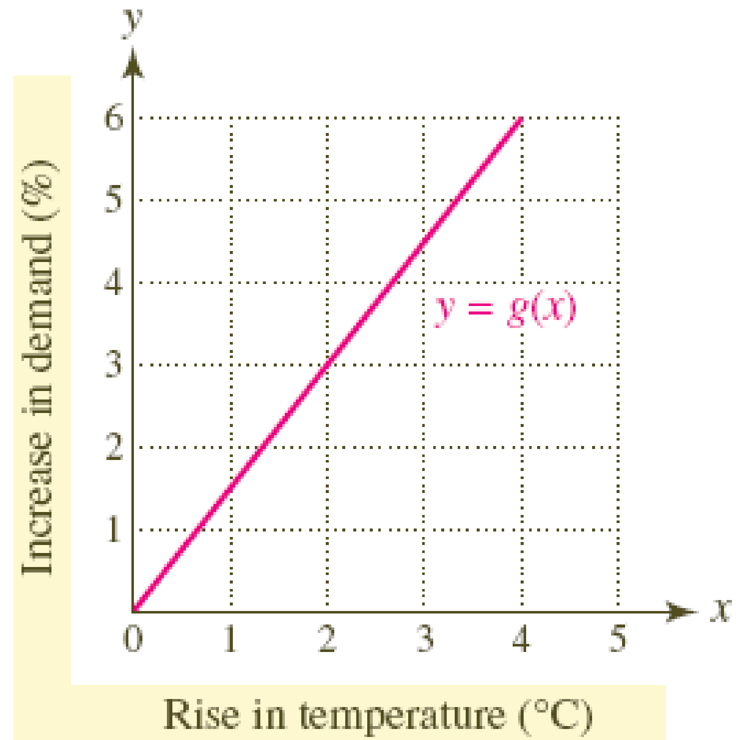


FIGURE 5.12 Percent Increase in Electrical Demand

Interpret $(g \circ f)(x)$.

$(g \circ f)(x)$ computes the percent increase in peak electrical demand during year x.

Evaluating Combinations of Functions Numerically

- Given numerical representations for *f* and *g* in the table

x	**5**	**6**	**7**	**8**
$f(x)$	8	7	6	5
$g(x)$	6	5	8	7

- Evaluate combinations of *f* and *g* as specified.

x	**5**	**6**	**7**	**8**
$(f+g)(x)$				
$(fg)(x)$				
$(f \circ g)(x)$				

Examples:

$(f + g)(5) = f(5) + g(5) = 8 + 6 = 14$

$(fg)(5) = f(5) \cdot g(5) = 8 \cdot 6 = 48$

$(f \circ g)(5) = f(g(5)) = f(6) = 7$

x	5	6	7	8
$f(x)$	8	7	6	5
$g(x)$	6	5	8	7

x	5	6	7	8
$(f + g)(x)$	14			
$(fg)(x)$	48			
$(f \circ g)(x)$	7			

Answers:

Given

x	5	6	7	8
$f(x)$	8	7	6	5
$g(x)$	6	5	8	7

x	5	6	7	8
$(f + g)(x)$	14	12	14	12
$(fg)(x)$	48	35	48	35
$(f \circ g)(x)$	7	8	5	6