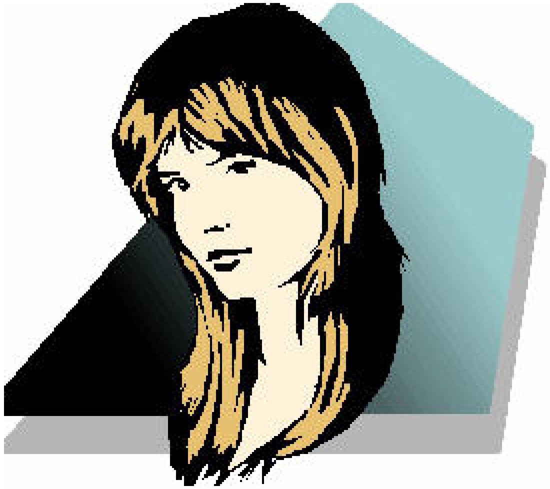
2.1

Linear Functions and Models

- Recognize exact and approximate models
- Identify the graph of a linear function
- Identify a table of values for a linear function
- Model data with a linear function

Not this kind of model!

But a mathematical model!

What is mathematical modeling?

Mathematical modeling is the process of using various mathematical structures - graphs, equations, verbal expressions, diagrams, scatterplots, tree diagrams, and so forth - to represent real world situations.

The model provides an *abstraction* that reduces a problem to its essential characteristics.

When a mathematical structure such as a graph is used to describe and study a real world problem we call such a structure a mathematical model for the original problem.

If a large amount of data is involved requiring *number crunching* the phrase computational model is often used.

Where are math models used?

Math models are used in diverse areas such as business, economics, physics, chemistry, astronomy, psychology, engineering, population dynamics and (of course) mathematics.

Regardless of where it is used, a model is an *abstraction* that has two basic characteristics:

1. A model is able to explain present phenomena. It should not contradict data and information known to be correct.

2. A model is able to make predictions about data or results. It should be able to use current information to forecast phenomena or create new information.

Examples of what models can do.

Forecast business trends, design shapes of cars, estimate ecological trends, control highway traffic, describe epidemics, predict weather.

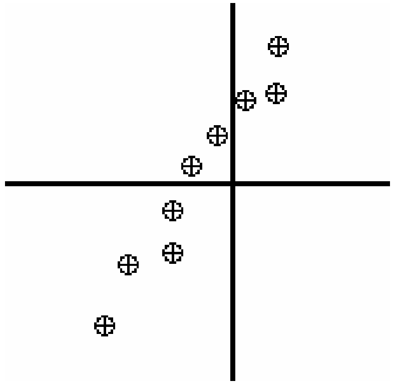
Mathematics is the language of technology. It is the "Invisible Glue" that permeates many of the advances in medicine, science and business.

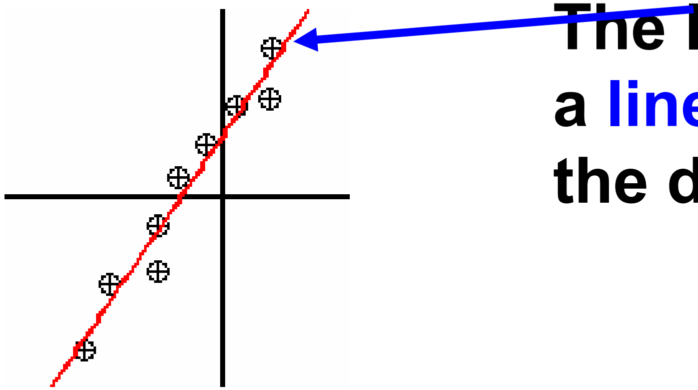
In this chapter we focus on linear models and study ways to build such models.

Exact and Approximate Models

Most mathematical models are not exact representations of data or information. For example, a scatter plot of data may appear to be nearly linear, but not exactly linear.

Example:





The line provides a linear model for the data.

It is an approximate model since not all the data points lie on the line.

If function y = f(t) is a model for a set of data,

$$\{(t_1, s_1), (t_2, s_2), (t_3, s_3), (t_4, s_4)\}$$

then it is an exact model provided

$$f(t_j) = s_j, \; j = 1, 2, 3, 4$$

This says that every data point lies on the graph of f(t).

If any data point fails to lie on the graph of y = f(t), then the function is an approximate model.

Example of An Exact Model

Show that function $f(x) = 2.1x - 7$ models

The data in table exactly.

x	–1	0	1	2
y	– 9.1	– 7	– 4.9	– 2.8

Note that:

- $f(-1) = 2.1(-1) - 7 = -9.1$ (Agrees with value in table)
- $f(0) = 2.1(0) - 7 = -7$ (Agrees with value in table)
- $f(1) = 2.1(1) - 7 = -4.9$ (Agrees with value in table)
- $f(2) = 2.1(2) - 7 = -2.8$ (Agrees with value in table)

Example of An Approximate Model

Show that function $f(x) = 5x + 2.1$ models the data in the table approximately.

x	–1	0	1
y	– 2.9	2.1	7

Note that:

- $f(-1) = 5(-1) + 2.1 = -2.9$ (Agrees with value in table)
- $f(0) = 5(0) + 2.1 = 2.1$ (Agrees with value in table)
- $f(1) = 5(1) + 2.1 = 7.1 \neq 7$ (Value is approximately the value in the table, but not exactly.)

Linear Functions

The general equation of a linear function is

$$y = f(x) = mx + b$$

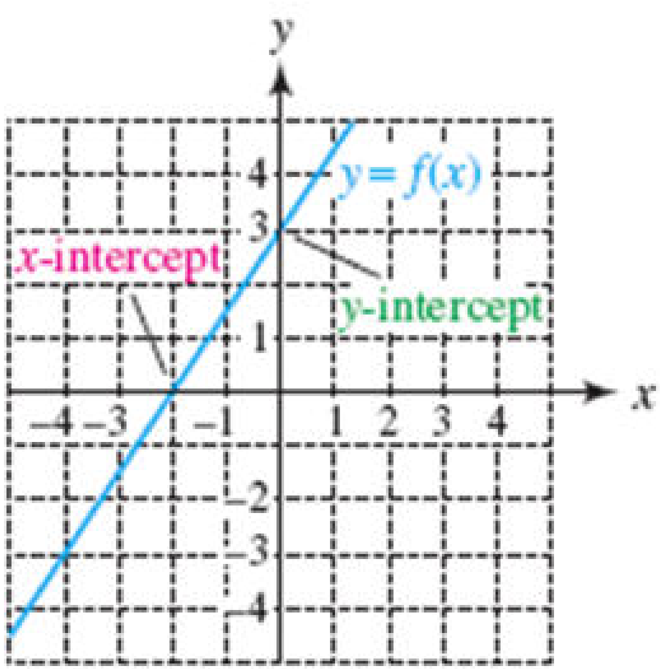
Properties of a linear function:

- Its graph is a nonvertical line.
- The rate of change of a linear function is the slope of the line and its value is m.
- The domain of a linear function is all real numbers (unless we specify a particular restriction).

More properties of linear function y = f(x) = mx + b.

• The y-intercept of the graph of a linear function is the value of the y-coordinate of the point where it crosses the y-axis. To find the y-intercept, compute f(0); the result is the value of b. So the point (0, b) is on the graph of the linear function.

• The x-intercept of the graph of a linear function is the value of the x-coordinate of the point where it crosses the x-axis. To find the x-intercept, set f(x) = 0, giving equation 0 = mx + b and solve for x. We get x = -b/m (assuming m is not zero).



The x-intercept is also called the "zero" of the linear function.

The expression for a linear function $y = mx + b$ is called the slope-intercept form for the equation of a line.

Special Case:

Linear function $y = f(x) = b$. (constant function)

Line has slope $m = 0$ ➔ graph is a horizontal line.

The y-intercept is b.

What about an x-intercept?

Determining a Linear Function from Data or a Graph

Case: Given a data table find the equation $y = f(x) = mx + b$

x	-3	-1	0	1	2
y=f(x)	-1.5	1.5	3	4.5	6

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Find the slope m.

Equation: ________________

Find the y-intercept b.

Find the x-intercept.

Determining a Linear Function from Data or a Graph

Case: Given a data table find the equation y = f(x) = mx + b

x	-4	-2	1	2	4
y=f(x)	7	5.5	3.25	2.5	1

Find the slope m.

Equation: ______________

Find the y-intercept.b.

Find the x-intercept.

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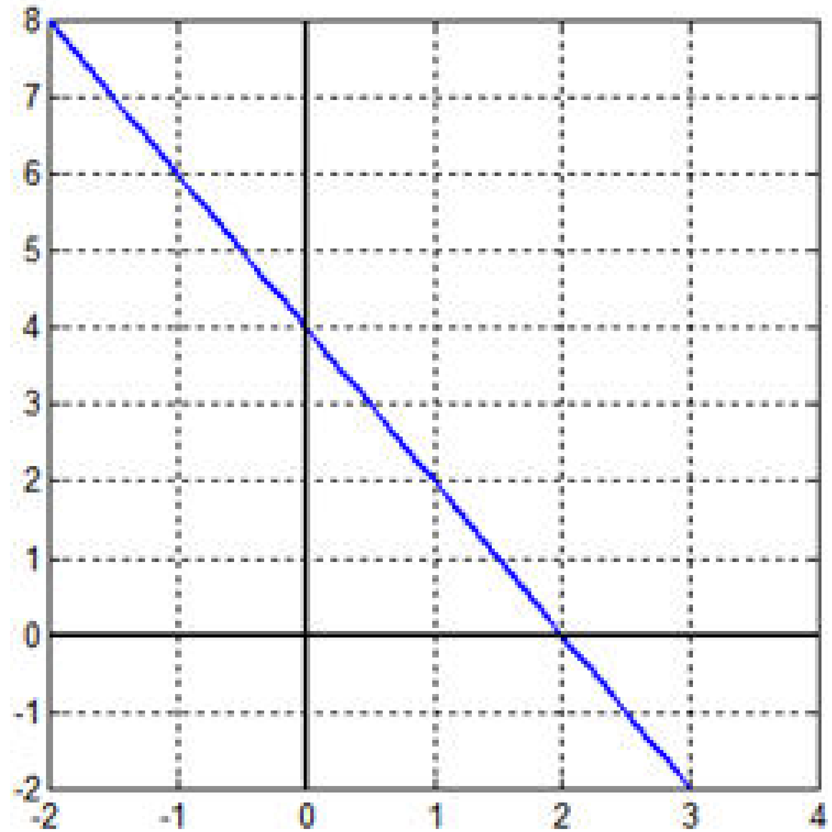
Case: **Given a graph find the equation y = f(x) = mx + b**

Find the slope m.

Find the y-intercept b.

Find the x-intercept.



Equation: _______________

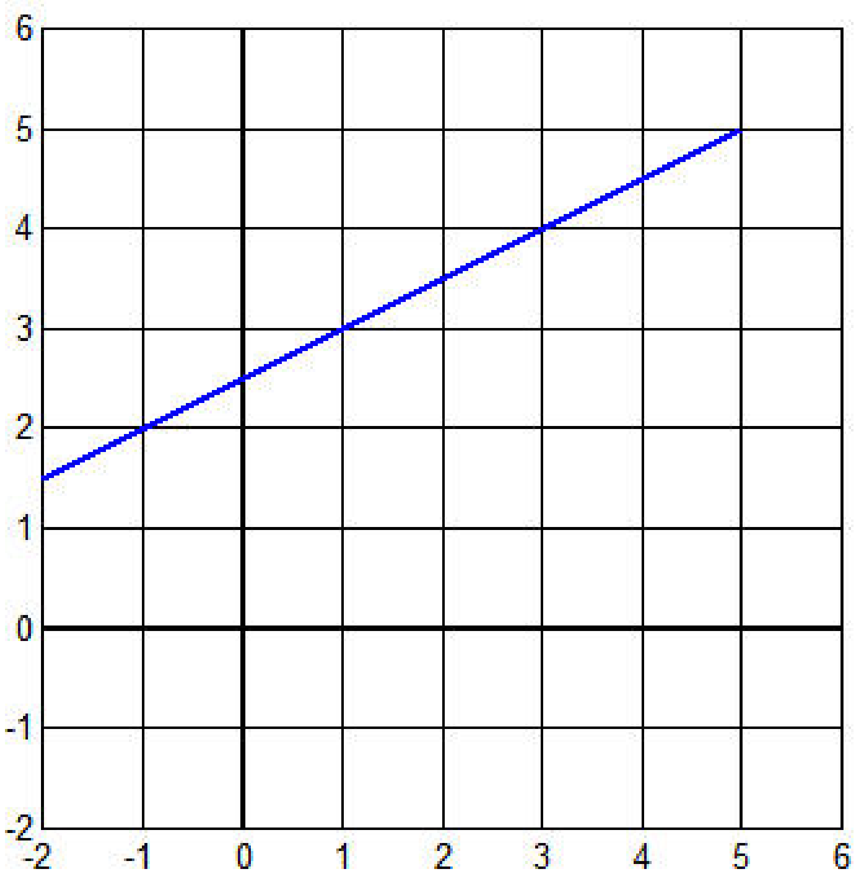
BB

Case: Given a graph find the equation y = f(x) = mx + b

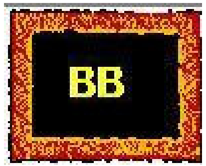
Find the slope m.

Find the y-intercept b.

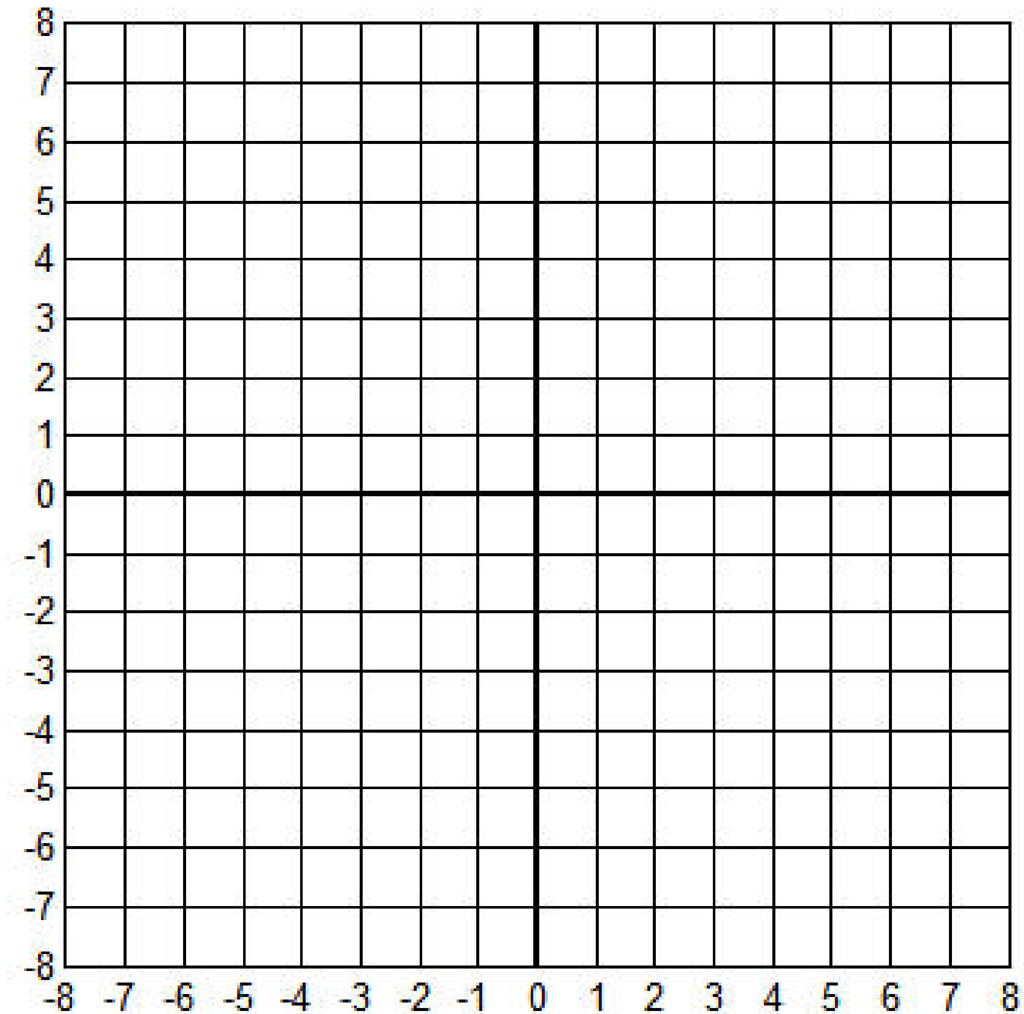
Find the x-intercept.



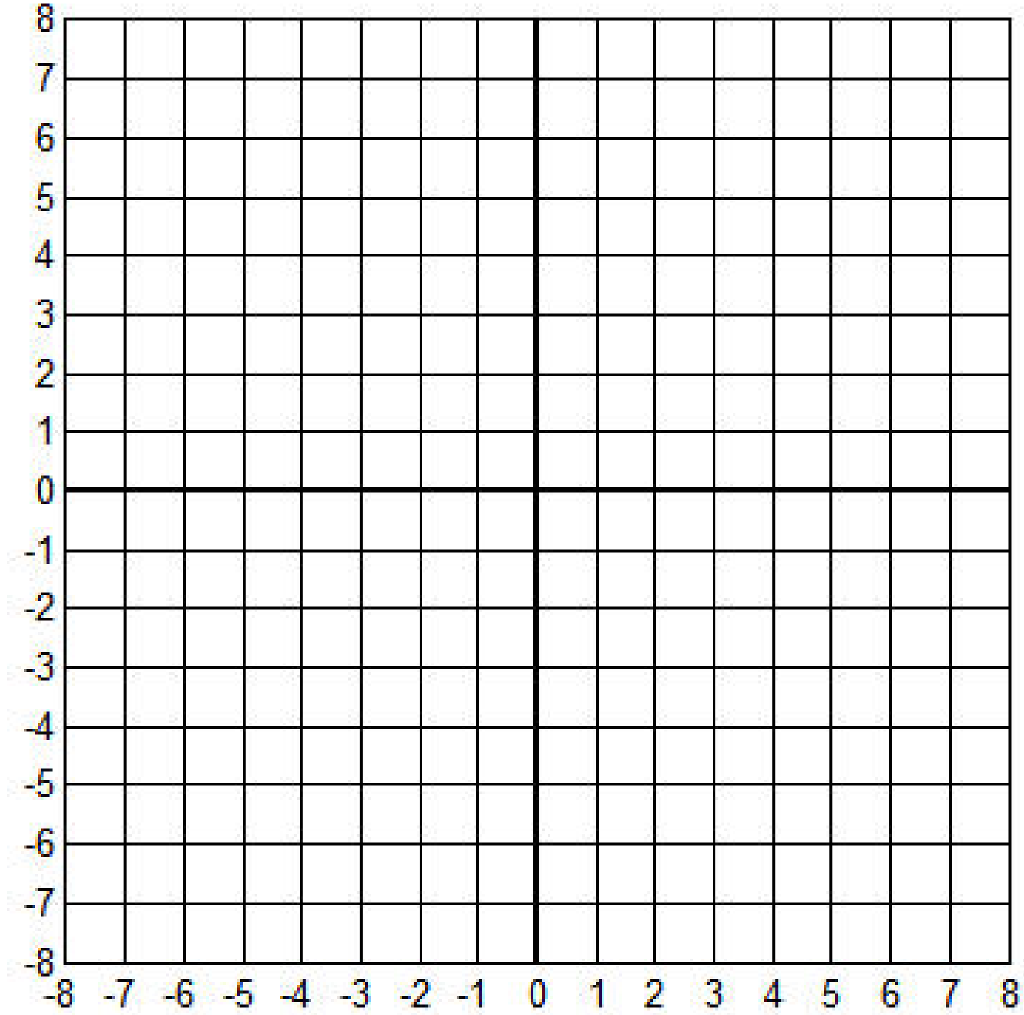
Equation: ________________



Graph the linear function y = f(x) = 4x – 6.

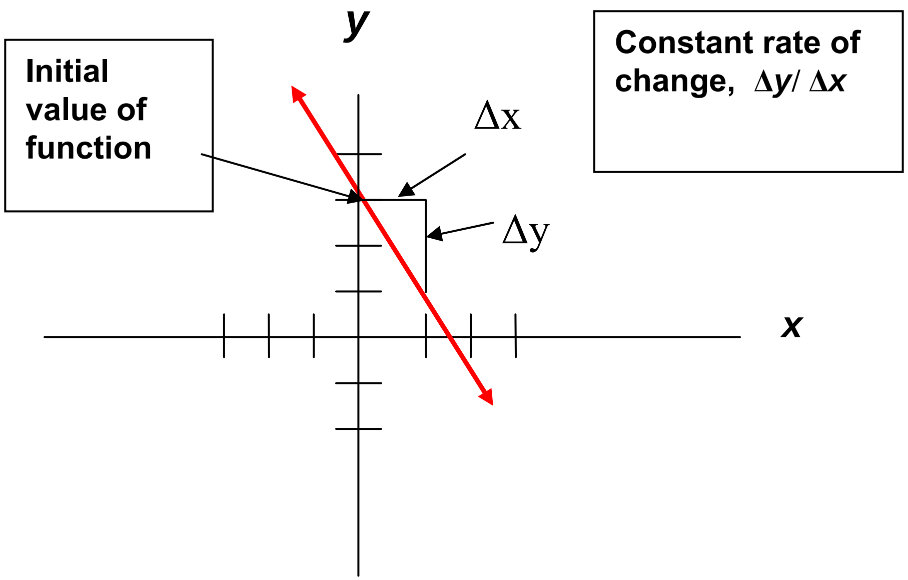


Graph the linear function $y = f(x) = 3 - x$.



Key Ideas for Modeling with Linear Functions $f(x) = mx + b$

- A linear function has a constant rate of change, that is a constant slope.
- $f(0) = m(0) + b = b$. When the input of the function is 0, the output is b. So the y – intercept b is sometimes called the initial value of the function.



Linear Function Model

- To model a quantity that is changing at a constant rate, the following may be used.

$f(x)$ = (constant rate of change)x + initial value

- Because
  - constant rate of change corresponds to the slope
  - initial amount corresponds to the y – intercept

this is simply $\boldsymbol{f(x) = mx + b}$

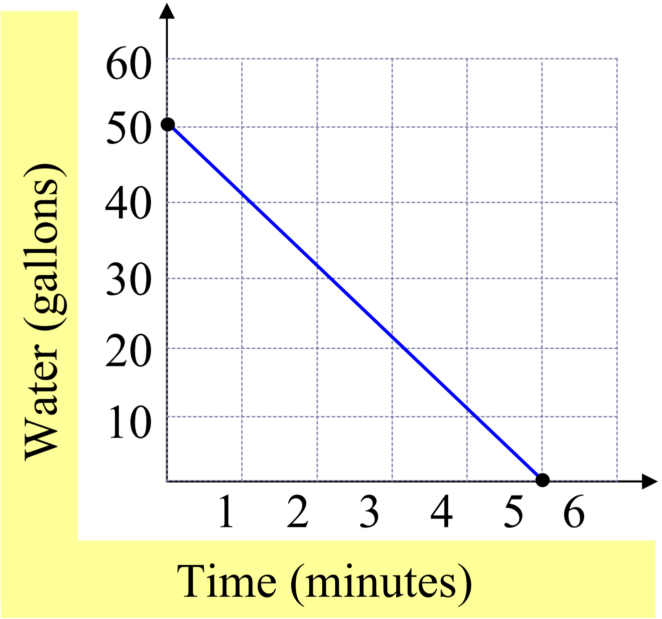
Example of Modeling with a Linear Function

- A 50-gallon tank is initially full of water and being drained at a constant rate of 10 gallons per minute. Write a formula that models the number of gallons of water in the tank after x minutes.

- The water in the tank is changing at a constant rate, so the linear function model $f(x)$ = (constant rate of change)x + initial amount applies.

- So $f(x) = (-10 \text{ gal/min})(x \text{ min}) + 50 \text{ gal}$. Without specifically writing the units, this is $f(x) = -10x + 50$.

Graph the model.



Example: In Oklahoma in 2004 the state income tax as published in the form of a table which listed taxable income in column 1 and the tax owed in column 2. A portion of the table appears below.

Taxable Income	State Tax Owed
$15,000	$780
$15,500	$825

Find the tax rate for income between $15,000 and $15,500. Write your answer as a rational number and a percent.

Construct a linear function that computes the tax owed if someone makes x dollars more than $15,000.

	x	y
Taxable Income	Income more than $15,000	State Tax Owed
$15,000	$0	$780
$15,500	$500	$825

How much tax is owed

on an income of $15,350?

Crauder, 3rd edition P.202

Example: **A party is being catered in a hall that rents for $275 and the cost per guest is $45.**

Write a linear function for the Cost of the party if there are x guests.

Example: **You rent space at a flea market to sell some necklaces you made. It costs $40 to rent the space and you sell the necklaces at $20 each.**

Write a linear function that determines your net income at the end of the day.

Example: **Two common ways to measure temperature are °Fahrenheit and °Celsius. We know the following discrete data.**

	°Celsius	**°Fahrenheit**
Freezing	0°	32°
Boiling	100°	212°

ASSUMING that the temperature scales are related by a linear function, compute the rate of change so that Fahrenheit is a function of Celsius.

Construct a linear function to compute Fahrenheit given Celsius.

Let x = Celsius and y = Fahrenheit, then $y = mx + b$.

What is the value of m?

How do we find the value for b?