1.3

Functions and Their Representations

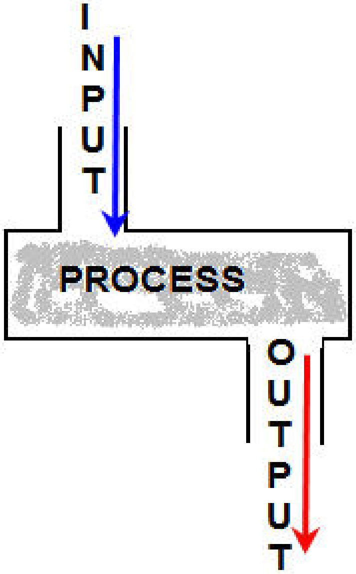
- Learn function notation
- Represent a function four different ways
- Define a function formally
- Identify the domain and range of a function
- Use calculators to represent functions (individual reading)
- Identify functions

Idea Behind a Function

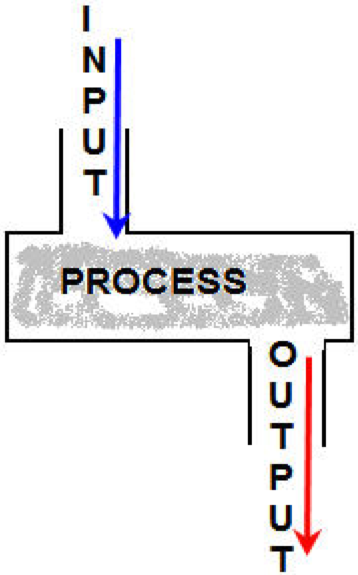
- Recall that a relation is a set of ordered pairs (x,y) .
- If we think of values of *x* as being inputs and values of *y* as being outputs, a function is a *relation* such that for each input there is exactly one output.

This is symbolized by *output* = *f*(input) or *y* = *f*(*x*)

So a function is a "manufacturing" process:



Anatomy of a Function

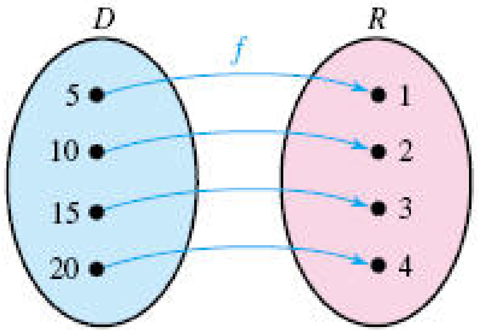


The set of valid inputs x is called the DOMAIN of the function and the set of corresponding outputs y is called the RANGE.

In some cases we will explicitly state the domain, in other cases we will take the domain to be all inputs that "make sense" for the way the function is defined. (This latter case is sometimes called the "implied domain".)

Representations of Functions

- **Verbal representation ➔ WORDS**
- **Numerical representation ➔ TABLE of values**
- **Diagrammatic Representation ➔ DIAGRAM**



- **Symbolic representation ➔ FORMULA**
- **Graphical representation ➔ GRAPH**

EXAMPLES:

Verbal: **To find the discount as a percent when an item is on sale, compute the percent of change in price as (original – sale) divided by the original price and multiply by 100.**

If an SUV was originally $22,000 but is on sale for $17,000, what is the discount as a percent?

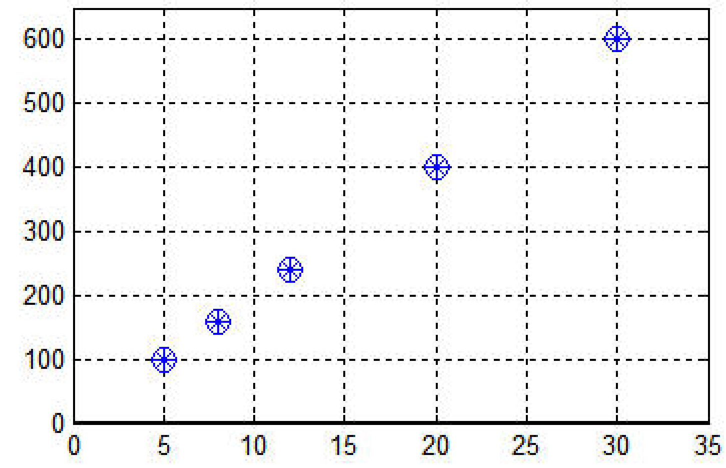
Table:

Distance traveled in t seconds by an object traveling 20 ft/sec

t (seconds)	5	8	12	20	30
d (feet)	100	160	240	400	600

Formula: **Distance traveled in t seconds by an object traveling 20 ft/sec ➔ d = 20 t**

Graph: **Corresponds to the table.**



Function Notation

$y = f(x)$

$y = f(x)$ ← symbol

- Is pronounced "y is a function of x."
- Means that given a value of x (input), there is exactly one corresponding value of y (output).
- x is called the independent variable as it represents inputs, and y is called the dependent variable as it represents outputs.
- Note that: $f(x)$ is NOT f multiplied by x. f is NOT a variable, but the name of a function (the name of a relationship between variables).

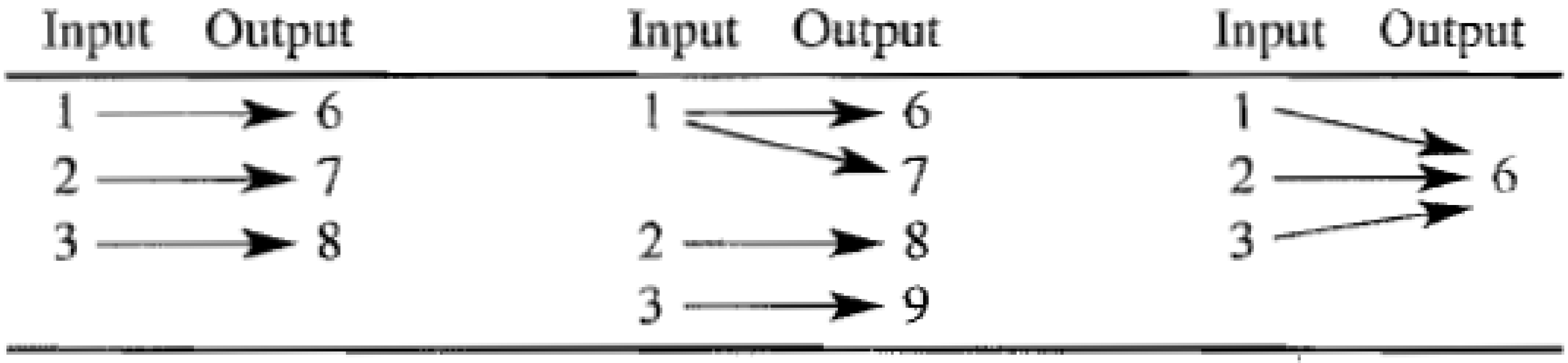
Formal Definition of a Function

- **A function is a relation in which each element of the domain corresponds to exactly one element in the range.**

Reminder about Domain and Range of a Function

- **The set of all meaningful inputs is called the DOMAIN of the function.**
- **The set of corresponding outputs is called the RANGE of the function.**

Example: **Which of the following are relations and which are functions.**



Give a reason for your answer.

Example

Given the following data, is *y* a function of *x*?

Input x	3	4	8
Output *y*	6	6	– 5

Note: The data in the table can be written as the set of ordered pairs {(3,6), (4,6), (8, –5)}.

Yes, y **is a function of *x*, because for each value of *x*, there is just one corresponding value of *y*. Using function notation we write *f*(3) = 6; *f*(4) = 6; *f*(8) = –5.**

Example The table shows the Olympic year and the winning distance for the 16 pound shot put.

1. Is D a function of T?
2. Is T a function of D?

Olympic Shot Put

Year, T	Winning Distance in Feet Thrown, D
1960	65
1964	67
1968	67
1972	70
1976	70
1980	70
1984	70
1988	74
1992	71
1996	71
2000	70
2004	69

Source: The World Almanac and Book of Facts, 2006.

Example

- Suppose a car travels at 70 miles per hour. Let y be the distance the car travels in x hours. Then $y = 70x$. Does this formula represent a function? Explain.
- Since for each value of x (that is, the time in hours the car travels) there is just one corresponding value of y (that is, the distance traveled), y is a function of x and we write

$$y = f(x) = 70x$$

- What is f(3) and interpret the output.

  $f(3) = 70(3) = 210$. This means that the car travels 210 miles in 3 hours.

Example

- Undergraduate Classification at Study-Hard University (SHU) is a function of Hours Earned. We can write this in function notation as $C = f(H)$.

$C = f(H)$

- **Classification of Students at SHU**

From the Catalogue

```
No student may be classified
as a sophomore until after
earning at least 30 semester
hours.

No student may be classified
as a junior until after
earning at least 60 hours.

No student may be classified
as a senior until after
earning at least 90 hours.
```

- **Evaluate $f(20)$**
  - $f(20)$ = **Freshman**
- **Evaluate $f(30)$**
  - $f(30)$ = **Sophomore**
- **Evaluate $f(0)$**
  - $f(0)$ = **Freshman**
- **Evaluate $f(61)$**
  - $f(61)$ = **Junior**

Is *C* = *f*(*H*), a function of *H*?

To answer YES, you must be sure that for each value of *H* there is exactly one corresponding value of C.

Is it a function?

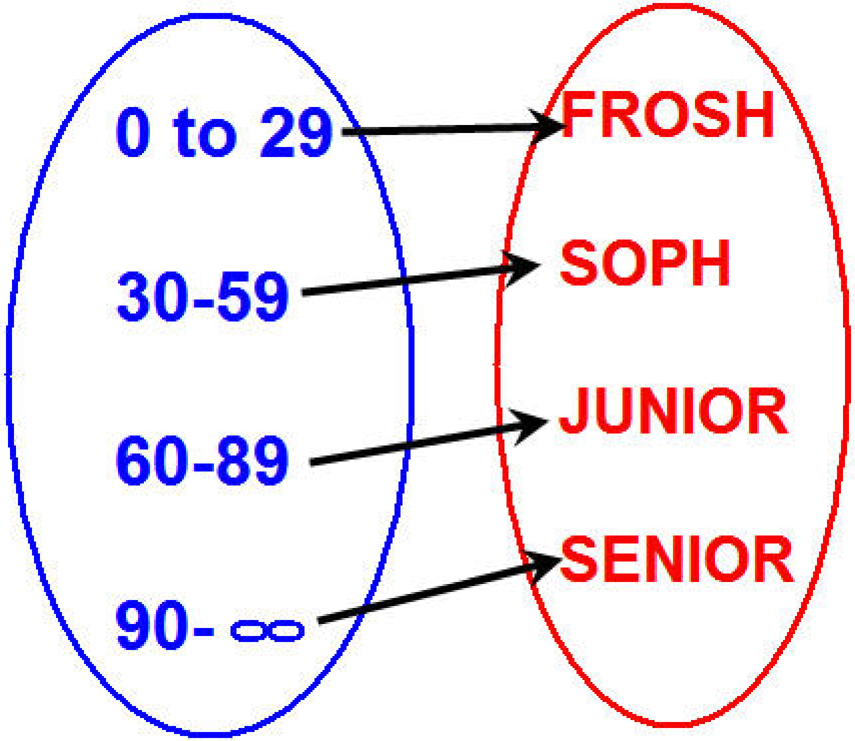
$C = f(H)$ **What is the domain of f ?**

$C = f(H)$ What is the domain of *f* ?

- **The domain of f is the set of <u>non-negative integers</u>** $\{0,1,2,3,4...\}$
  - **Alternatively, some individuals say the domain is the set of <u>positive rational numbers</u>, since technically one could earn a fractional number of hours if they transferred in some quarter hours. For example, 4 quarter hours = 2 2/3 semester hours.**
  - **Some might say the domain is the set of <u>non-negative real numbers</u> [0, ∞), but this set includes irrational numbers. It is impossible to earn an irrational number of credit hours. For example, one could not earn π hours.**

What is the range of *f*? Range of *f* is {Fr, Soph, Jr, Sr}

Diagram of $C = f(H)$



<u>Question:</u> If a student was classified a junior could you tell exactly how many credits she had? That is, is $H = g(C)$ a function? EXPLAIN!

$C = f(H)$

Symbolic Representation

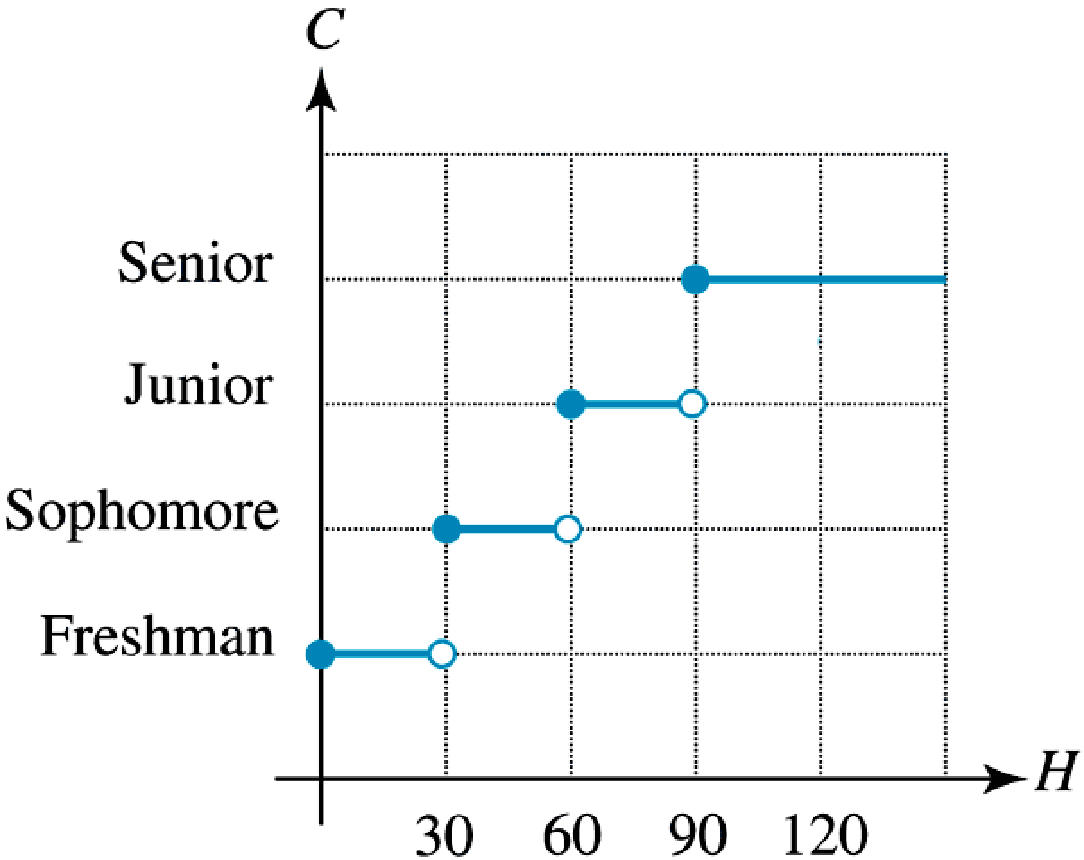
$$C = f(H) = \begin{cases} \text{Freshman if } 0 \le H < 30 \\ \text{Sopho if } 30 \le H < 60 \\ \text{Junior if } 60 \le H < 90 \\ \text{Senior if } H \ge 90 \end{cases}$$

$C = f(H)$

Graphical Representation

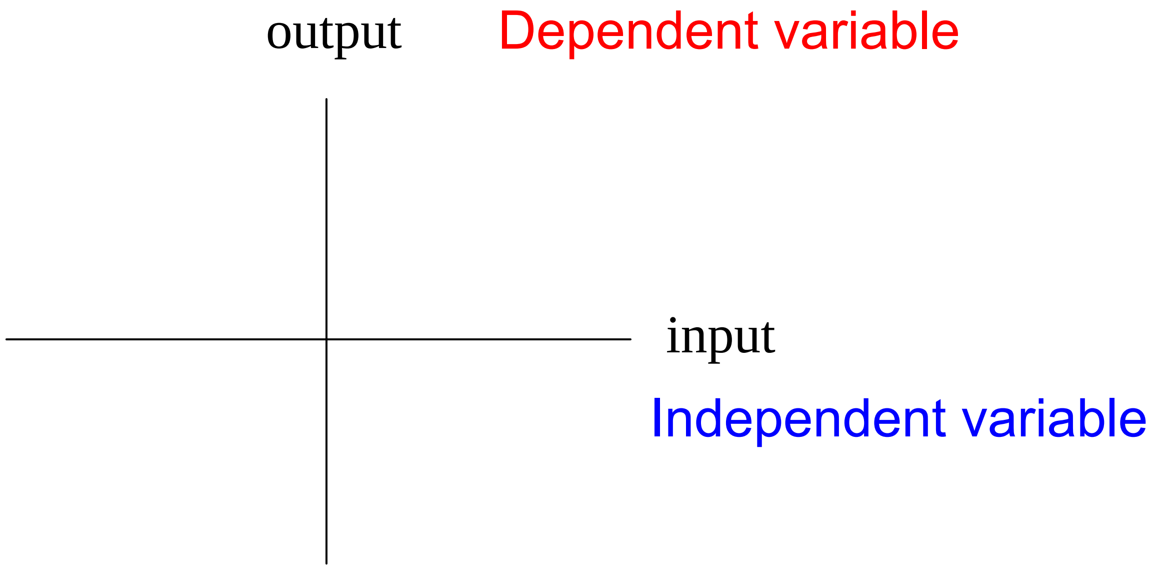
Note that in this graph the domain is considered to be [0, ∞)

instead of {0,1,2,3…}



Some Notes on Graphical Representation

Inputs are typically graphed on the horizontal axis and outputs are typically graphed on the vertical axis.



Example: **How would the axes be labeled in order to graph the following functions?**

1. **Density of water is a function of temperature.**

   Write a expression for this function using contextual names.

2. **Sales tax is a function of cost.**

   Write a expression for this function using contextual names.

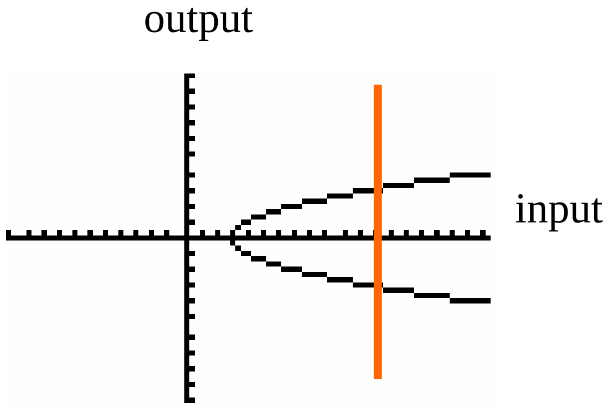
3. **A quantity Q is a function of time t.**

   Write a expression for this function using contextual names.

Notes on Graphical Representation Continued

- **Vertical line test (p. 39). To determine if a graph represents a function, simply visualize vertical lines in the *xy*-plane. If each vertical line intersects the graph at no more than one point, then it is the graph of a function.**

Why does this work?



MORE Examples

Given $f(x)=\frac{3}{5-2x}$

Determine the domain.

Compute f(2).

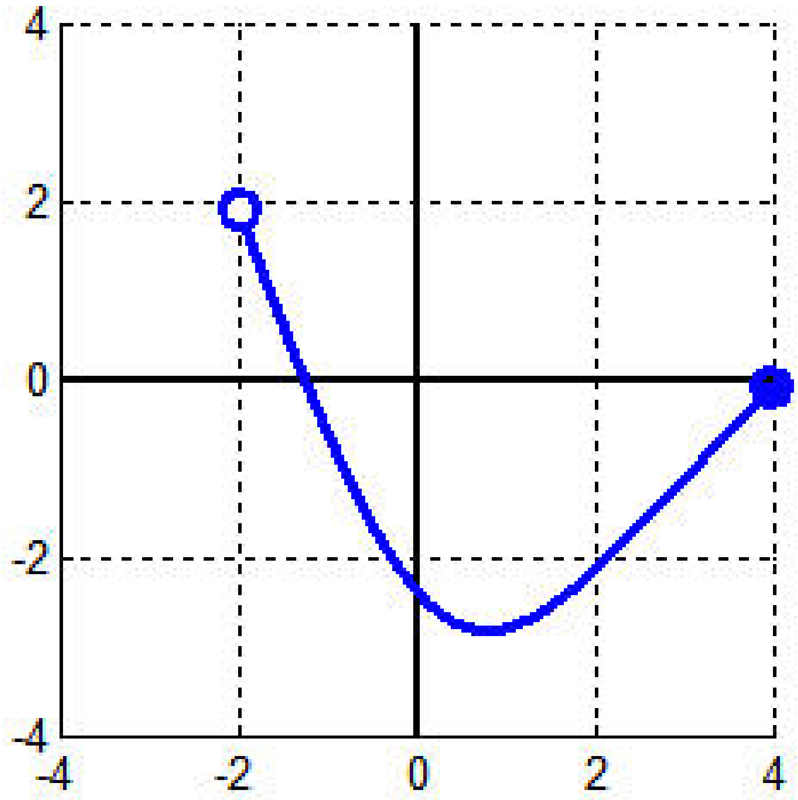
Compute f(-3)

Compute f(a+2).

Compute f(Δ).

Compute f(½).

Determine the domain of the function shown below.



Which of the following are functions? Explain!

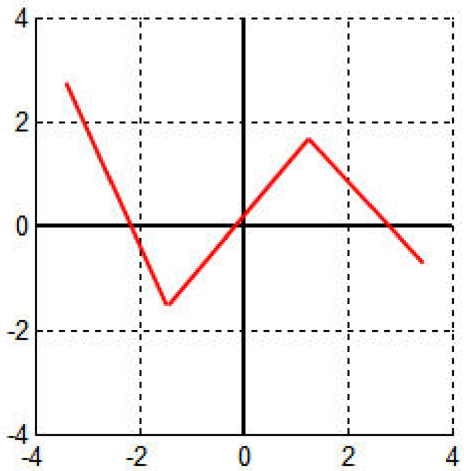


Figure 1.

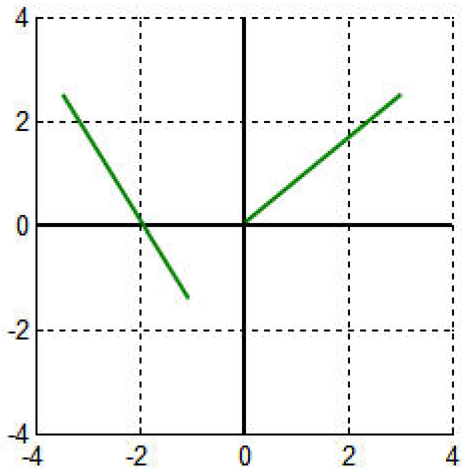


Figure 2.

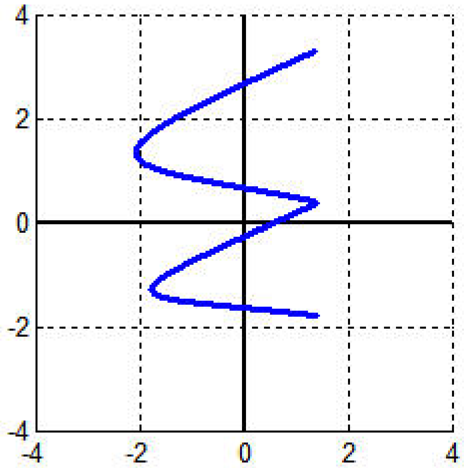
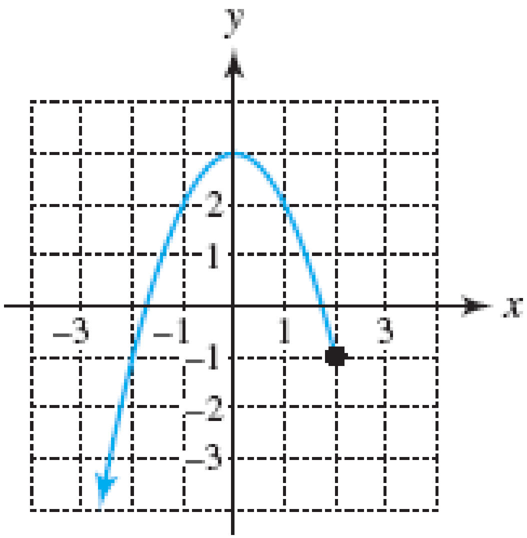
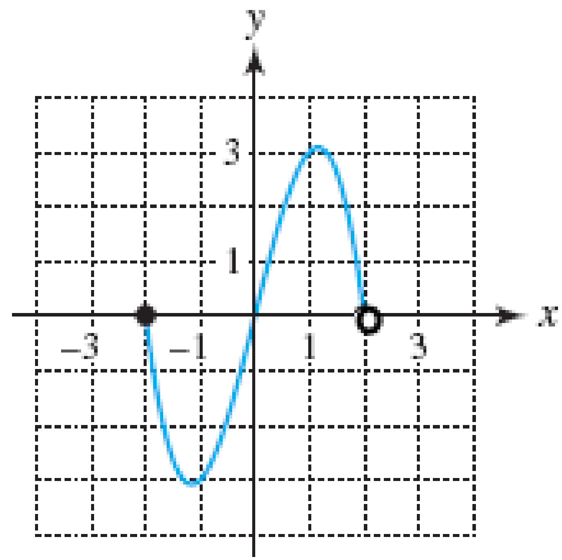
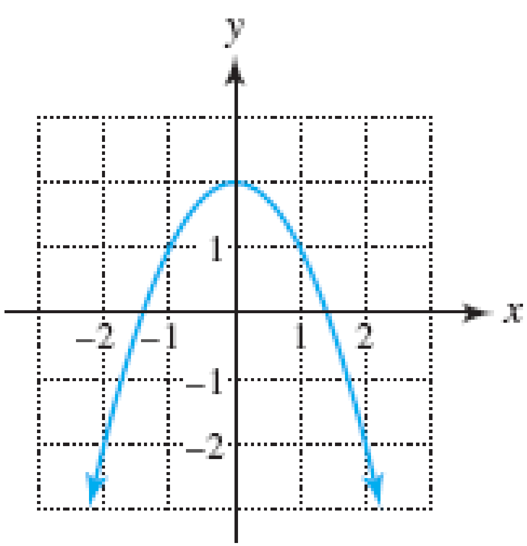


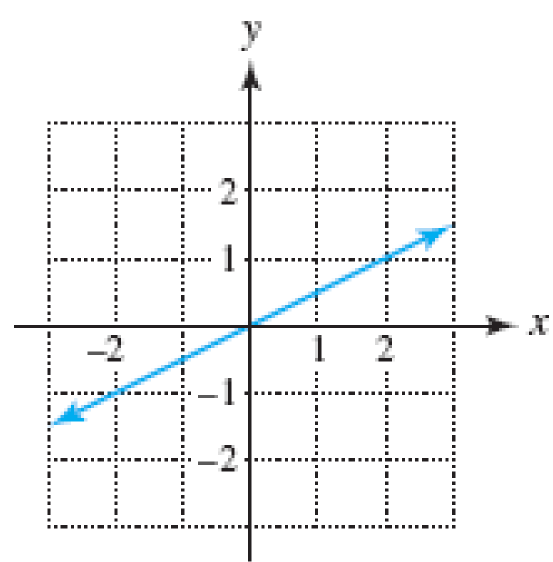
Figure 3.

Example: **Determine the domain and range of each of the functions pictured below.**









Questions: Identifying Functions

- Given $x = y^2$, is y a function of x? Why or why not? That is, given a value of x, is there just one corresponding value of y ?

  No, if $x = 4$, then $y = 2$ or $y = -2$.

- Given $x = y^2$, is x a function of y? Why or why not? That is, given a value of y, is there just one corresponding value of x ?

  Yes, given a value of y, there is just one corresponding value of x, namely y^2.

- Given $y = x^2 - 2$, is y a function of x ? Why or why not? That is, given a value of x, is there just one corresponding value of y?

  Yes, given a value of x, there is just one corresponding value of y, namely $x^2 - 2$.