Testing the optimality of a performance evaluation measure for a gainsharing contract*

RAJIV D. BANKER *University of Minnesota*

SRIKANT M. DATAR *Carnegie Mellon University and Stanford University*

MARK J. MAZUR *Carnegie Mellon University*

Abstract. Recent attention on labor productivity has resulted in many manufacturing concerns negotiating incentive contracts with the labor force. Such incentive contracts provide for management and the work force to share monetary benefits generated by productivity gains. These gain-sharing contracts require a benchmark level of labor productivity from which to assess productivity gains. This paper examines gain-sharing contracts in an agency setting, deriving conditions under which contracts using a performance evaluation measure of a ratio of total labor hours to standard direct labor hours might be optimal. These optimality conditions are characterized in terms of the fixed and variable cost components of the total labor requirement and the standard direct labor requirements. An observed gain-sharing contract based on such a measure is then evaluated using actual production data. The generalized method of moments is employed to estimate the key production parameters, indicating that the optimality conditions are violated. The characterization of optimal gain-sharing contracts thus clarifies the manner in which productivity must be measured if these programs are to provide the proper incentives to the work force.

Résumé. L'attention récemment accordée à la productivité de la main-d'oeuvre a donné lieu à la négociation de contrats de rémunération au rendement dans de nombreuses entreprises de fabrication. Ces contrats de rémunération au rendement prévoient un partage des bénéfices monétaires résultant des gains de productivité, entre la direction et la main-d'oeuvre. Les contrats de participation aux bénéfices nécessitent la détermination d'un point de repère en ce qui a trait à la productivité de la main-d'oeuvre à partir duquel on puisse évaluer les gains de productivité. Les auteurs analysent les contrats de participation aux bénéfices dans le contexte d'une relation de mandataire, en dérivant les conditions dans lesquelles le contrat prévoyant une mesure du rendement fondée sur le rapport des heures de main-d'oeuvre totales aux heures de main-d'oeuvre directe standard peut être optimal. Ces conditions d'optimalité sont définies sous forme d'éléments de coûts fixes et de coûts variables des besoins en heures de main-d'oeuvre totales et en heures de main-d'oeuvre directe standard. Les auteurs analysent un contrat de participation aux bénéfices basé sur ce genre de mesure et l'évaluent ensuite à partir des données réelles de production. Ils recourent à la méthode généralisée des moments pour estimer les principaux paramètres de production indiquant que les conditions d'optimalité sont transgressées. La définition du contrat optimal de participation aux bénéfices éclaire ainsi

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la manière dont la productivité doit être mesurée pour que de tels contrats offrent à la main-d'oeuvre les stimulants appropriés.

Introduction

Improving productivity has become an increasingly important objective of many organizations during the past decade. Productivity gains are perceived as providing long-term competitive advantages, particularly for firms operating in mature product markets. The key competitive dimension for these firms is supplying a quality product at the lowest cost. To further this objective and to motivate workers to focus on productivity improvements, some firms have begun to implement gain-sharing contracts that involve the division of productivity gains between workers and management. Such contrats are of course predicated on the ability to measure these productivity gains accurately over time. Current levels of labor productivity can then be compared to a benchmark level of labor productivity to determine the additional compensation to the work force.

There is little, if any, guidance in the extant management accounting research literature to guide the construction of productivity measures for incentive contracting. Traditional measures, such as usage variances, are clearly inadequate because of issues of direct versus indirect labor, changes in the overall quality and quantity of output, and changes in the output mix. Moreover, variance analysis is a retroactive single period analysis of performance, whereas gain-sharing contracts require a comparison of actual levels of productivity over multiple time periods. This multiperiod linkage is an integral part of productivity analysis; attempts to develop measures of productivity have only recently begun to receive some attention in the management accounting literature (e.g., Banker, Datar, and Kaplan, 1989).

Productivity measurement is an area of concern for many organizations in both the private and public sectors. This paper examines the problem of measuring productivity in the context of gain-sharing contracts. The objective is to analyze a specific performance evaluation measure used by a firm implementing a gain-sharing program. This measure is analyzed using a principal-agent model because the goal of the program was to motivate workers by providing incentives for greater effort. Our model captures the specific structure and features of the institutional setting within which a gain-sharing contract is implemented by a particular firm.[1] An estimable model of the production function is developed and restrictions on the parameters that must be satisfied for the specific performance evaluation measure to be optimal are determined. The formal principal-agent model forms the basis for the subsequent estimation and empirical tests of optimality. The basic approach adopted in the paper of developing models to reflect the particular characteristics of different settings could also be useful in exploring the optimality of other institutional arrangements.

1 This particular gain-sharing contract is based on a productivity measure that compares actual total labor hours to standard direct labor hours as detailed below.

Principal-agent models have frequently been employed in the accounting literature to examine the characteristics of optimal performance measures and incentive contracts (see for instance, Baiman and Demski, 1980; and Banker and Datar, 1989).[2] These models have typically assumed a risk-neutral principal and a risk-averse and a work-averse agent who provide a productive input a, which, with some random uncertainty θ, yields an output y. The agent's effort choice is unobservable to the principal. The optimal contract then specifies payments to the agent as a function of the output y.

This paper reports the development of a principal-agent model to evaluate the optimality of a performance measure utilized in an observed productivity-based incentive contract. Unlike the usual principal-agent models, the actual number of hours (x) worked to produce a given vector of outputs ($\mathbf{y}$) is observed by the principal. Workers are paid at some base rate for the actual number of hours worked. What is unobservable to the principal is the quality and intensity of the workers' effort (a). The objective of the productivity-based incentive contract is to motivate labor to provide more effort (a) and to reduce the actual number of hours worked (x). That is, the principal optimally trades off higher bonus or incentive payments for reduced base wages. Productivity gains may result from, for example, better coordination among work units, lower set up times, less scrap, a smaller administrative staff, or decreased absenteeism.

Two kinds of effort are modeled: the actual number of hours worked and the intensity or quality of the agent's effort. Such a formulation permits a characterization of the optimal performance evaluation measure in the gain-sharing contract as a productivity based measure. We compare this theoretically optimal productivity measure with the empirically observed performance measure (see Banker and Datar, 1987). A necessary condition for the observed gain-sharing contract to be optimal is that the performance measure used to evaluate productivity gains is itself optimally chosen. In this paper, the focus is on whether or not this necessary condition is fulfilled. Note, however, that this is not a sufficient condition for the optimality of the entire gain-sharing contract because the form of this optimal contract would also depend on correctly assessing the utility functions of the parties involved.

We employ an empirical methodology that we believe has not been previously used in the accounting literature to test the optimality of an observed performance evaluation measure. This is the generalized method of moments, a technique that does not depend on potentially restrictive distribution assumptions regarding the error term. This technique is theoretically preferred to maximum likelihood estimation when specific distribution characteristics of the situation under study cannot be inferred from economic modeling. Often theoretic analysis in accounting is used to inform the empirical specification of models but it provides no guidance to the researcher regarding distributional assumptions. In

2 These characteristics include the information content of the performance measure (e.g., sensitivity and precision) and the risk-sharing properties of the incentive contract.

these cases, the generalized method of moments could provide a useful advantage. This paper provides an example of how that might be accomplished.

The remainder of this paper is organized as follows. A general formulation of a productivity-based gain-sharing contract in an agency setting is developed in the next section, with particular attention paid to the variables observable by the parties to the contract. The optimal contract in this setting is characterized and conditions for a productivity evaluation measure to be optimal for such a gain-sharing contract are identified. Following this derivation, a model that lends itself to econometric estimation is developed. In the subsequent section, data obtained from a manufacturing plant at which such a gain-sharing contract was implemented are examined, and the empirical evidence about the optimality of this contract is evaluated. This empirical analysis of a particular firm's performance evaluation measure is provided to illustrate how the framework developed earlier can be adapted to situations actually encountered. The final section discusses the implications of this line of research.

Developing an estimable model

In the usual principal-agent model, a principal designs a contract to motivate a risk-averse agent to exert unobservable effort in a productive process that is characterized by uncertainty. The agent requires a minimum level of expected utility in order to accept the contract offered. The principal receives the residual from the production process after compensating the agent. Optimal contracts in this formulation base compensation for the agent on observed output, which the principal uses as a signal of the effort expended by the agent. The standard principal-agent formulation is modified to fit the gain-sharing environment. The aim in doing so is to identify conditions that characterize an optimal productivity measure on which the gain-sharing contract can be based.

In developing an agency theoretic model for this economic situation, it must be emphasized that the goal is not to characterize the optimal gain-sharing contract completely but to focus on the role of the performance evaluation measure employed. Moreover, this model will develop the empirical specification of the problem, leading to the estimation in the following section. As such, a number of maintained assumptions are made to conform the model to the institutional setting.

Gain-sharing contracts are prevalent in the manufacturing operations of firms in which control of the cost of producing a specified level of output is the item of interest. As in Harris, Kriebel, and Raviv (1982), the agency model is modified to abstract away from the overall profit maximizing goal of the firm. Instead, because labor costs are a major component of total cost,[3] we consider,

3 For the specific plant examined in the next section, labor costs are a large component (55–60 percent) of total manufacturing costs, and considered not substitutable for materials costs because of quality checks.

as the objective for the management, the subgoal of minimizing the labor cost to produce a given output level.[4]

For the sake of simplicity, the plant management is represented as a risk-neutral principal.[5] The principal seeks to improve labor productivity in order to reduce manufacturing costs and is willing to offer gain-sharing contracts to meet this goal. The work force is represented by a labor union bargaining with the management and, as in Hall and Lilien (1979), this bargaining unit (or union) is modeled as the risk-averse agent. Implicit in this formulation is the notion that labor hours are essentially homogeneous, and that an overall preference function for the work force exists.

Our model differs from the usual principal-agent model because, in addition to preferring higher levels of compensation[6] (ϕ), the agent displays both a disutility for working more hours (x) and for working more intensely (a) to find ways to reduce the amount of (labor) inefficiency in the production process. The agent's utility function is assumed to display a marginal rate of substitution between the compensation (ϕ) and effort (a) equal to zero (as in the usual agency model), and a marginal rate of substitution between compensation (ϕ) and hours of work (x) that is independent of the level of ϕ. Therefore, the agent's utility function can be written as $u(\phi(\cdot) - r(x)) - v(a)$ with $r'(x) > 0$ (see Keeney and Raiffa, 1976, p. 116; and Banker and Maindiratta, 1987). The agent's reservation level of expected utility is assumed to be zero, without loss of generality. Management is assumed to choose the incentive contract that minimizes the total labor cost of producing the desired output.

In contrast to production functions commonly employed in the literature in which output is a stochastic function of the levels of inputs utilized, we model labor (x) required as a function of a given vector of outputs ($\mathbf{y} = y_1, y_2, \ldots, y_n$), the effort exerted by the work force ($a \in [a_L, a_H] \subseteq IR$), and a random component ($\theta \in \Theta \subseteq IR$). Consistent with the production economics literature (Farrell 1957; and Banker and Maindiratta, 1988), it is assumed that the production function has the form

$$x = t(\mathbf{y})/\Delta(a, \theta) \tag{1}$$

where the numerator can be thought of as the standard required labor hours (direct and indirect) and the denominator as a measure of technical efficiency.[7] In this formulation we also assume $\partial\Delta/\partial a > 0$ (higher effort implies lower required labor inputs on average) and that $\Delta(\cdot)$ is monotone in θ (without loss of

4 Determining the conditions for decentralized cost minimization by a manufacturing plant (for producing a given quality and quantity of output) to be consistent with overall profit maximization by the firm is beyond the scope of this paper.

5 The extension to the case of a risk-averse principal is direct. If the principal is, indeed, risk averse, then the contracts offered (based on signals of effort) will serve also to determine the optimal risk sharing arrangement, in addition to motivating the agent to exert effort.

6 The compensation contract is based on some jointly observable information signals.

7 The optimal performance measure will depend on the production technology of the firm implementing the gain-sharing contract. The production function assumed here is not necessarily the one used by the firm examined in our subsequent empirical analysis.

generality, assume $\partial\Delta/\partial\theta > 0$). Although both management and the workers can compute $t(\mathbf{y})$, only the workers know the level of a chosen. That is, management cannot distinguish between shirking on the part of the workers and bad luck in the form of low values of θ. Management, however, will use the contracts offered to affect the level of effort chosen by the workers.

Following the tradition of the literature in both cost accounting and industrial engineering, the production function itself is specified to be

$$t(\mathbf{y}) = \beta_o + \sum_{i=1}^{n} \beta_i y_i$$

where β_o represents a fixed component of labor required, and the β_i's the variable labor requirements for producing output y_i. The notion of a fixed labor support cost (β_o) is a key element in the analysis. This can represent set-up time between production runs or any labor element that is not a function of the demanded output. The plant management and the work force must produce the exogenously set output quantities ($\mathbf{y}$) of satisfactory quality so that the work force cannot trade off quality to meet quantity targets.

Because the output vector is set exogenously, the use of $\pi(x, \mathbf{y}) = t(\mathbf{y})/x$, a ratio measure of labor productivity, as a performance evaluation measure seems reasonable. We next evaluate the information content of $\pi(x, \mathbf{y}) = t(\mathbf{y})/x$. From equation (1), it is immediate that

$$\pi(x, \mathbf{y}) = t(\mathbf{y})/x = \Delta(a, \theta) \tag{2}$$

As in Mirrlees (1976), we suppress the random term θ to yield the conditional distribution of the labor inputs, given the output vector and the level of effort chosen by the work force. It is straightforward to compute the likelihood ratio $h_a(x, \mathbf{y}, a)/h(x, \mathbf{y}, a)$, where $h(\cdot)$ is the joint density function of $(x, \mathbf{y})$ given a, and $h_a(\cdot) \equiv \partial h(\cdot)/\partial a$. The following lemma will prove useful later for our evaluation of the optimality of the gain-sharing contract and the productivity measure.

Lemma: The likelihood ratio of the joint density function $h(\cdot)$ is a function of $\pi(x, \mathbf{y}) = t(\mathbf{y})/x$ and a.

Proof: Denote the density function for the random term θ by $f(\theta)$. Assume this is continuous. From equation (2) it is seen that $t(\mathbf{y})/x = \Delta(a, \theta)$. the assumption that $\Delta(\cdot)$ is monotone in θ implies the implicit function theorem can be used to write θ as a function of the other variables, as in $\theta = \psi(t(\mathbf{y})/x, a)$. It is clear that

$$\partial\psi/\partial x = -\psi_\pi(t(\mathbf{y})/x, a) \cdot t(\mathbf{y})/x^2$$

where the subscript π refers to the partial derivative with respect to the first argument. From the relation for θ given above, it follows that an increase in θ is associated with an increase in $\psi(\pi(x, \mathbf{y}), a)$ for any chosen effort a. Using the assumption $\Delta_\theta > 0$ in conjuction with equation (2) leads to the conclusion that $\psi_\pi(\cdot)$ is positive. This, in turn, implies $\partial\psi/\partial x < 0$, from the equation above.

As in DeGroot (1975, pp 131–139), we obtain the joint density function

$$\begin{aligned} h(x,y,a) &= f(\psi(t(y)/x),a)) \cdot |d\psi/dx| \\ &= f(\psi(t(y)/x),a)) \cdot \psi_\pi(\cdot) \cdot t(y)/x^2 \\ &= R(t(y)/x,a) \cdot t(y)/x^2 \end{aligned}$$

where $R(\cdot) = f(\psi(t(y)/x,a)) \cdot \psi_\pi(\cdot)$ is clearly a function of only $t(y)/x$ and a. Then the likelihood ratio is given by

$$\frac{h_a(x,y,a)}{h(x,y,a)} = \frac{[\partial R(\cdot)/\partial a] \cdot [t(y)/x^2]}{[R(\cdot)] \cdot [t(y)/x^2]} = \frac{\partial R(\cdot)/\partial a}{R(\cdot)}$$

Therefore, the likelihood ratio, $h_a(\cdot) / h(\cdot)$, is a function of $\pi(x,y) = t(y)/x$ and a. Q.E.D.

In general, one would expect that the likelihood ratio would be a function of x, a, and the entire vector y. However, for the particular production function analyzed here, the values $\pi(x,y) = t(y)/x$ and a are sufficient to determine the likelihood ratio.

Assuming that the work force has a unique choice of a for the optimal incentive contract offered by the firm, the probability distribution of the labor input and outputs for this effort level a^* is easily computed.[8] Of particular interest is the behavior of this joint distribution for small deviations in the effort level from a^*, because the optimal compensation contract is based, in part, on the likelihood ratio $h_a(\cdot)/h(\cdot)$. To study this behavior, further restrictions on the conditional distribution are useful.

A plausible assumption on this conditional distribution is that it exhibits the monotone likelihood ratio property (MLRP) as in Milgrom (1981) with respect to the summary performance measure $\pi(x,y)$. That is,

$$\frac{\partial}{\partial \pi}\left(\frac{h_a(x,y,a)}{h(x,y,a)}\right) > 0.$$

where $\pi = t(y)/x$.

Intuitively, this property means that from higher values of the labor productivity measure π we can infer (in a probabilistic sense) a higher level of work-force effort. For the remainder of the paper, it is assumed that the production process is characterized by the MLRP.

Management seeks to minimize the total cost of compensating the work force, subject to producing the required output. Management offers the appropriate gain-sharing contract to meet this goal. The work-force accepts the contract offered if it meets or surpasses a reservation utility level for each output level y, and chooses the effort level to maximize expected utility.[9]

8 The interested reader is referred to Rogerson (1985) for a discussion of the implications of this assumption.

9 For analytical convenience we assume that there are a finite number of potential values for the output level y

Writing this problem out formally, management desires to

$\min_{a,x,\phi} E[\phi(x, y)]$

subject to

$E[u(\phi(x, y) - r(x))] - v(a) \geq 0 \ldots$ for each y

$\frac{\partial}{\partial a}(E[u(\phi(x, y) - r(x))] - v(a)) = 0 \ldots$ for each y

The constraints in this problem are simply the usual individual rationality and self-selection constraints, respectively. Assuming an interior solution and the usual regularity conditions, and proceeding as in Mirrlees (1976), it can be shown that for each output level y, the optimal gain-sharing contract $\phi^*(x, y)$ is characterized by

$$\frac{1}{u'(\phi^*(x, y) - r(x))} = \lambda_y + \mu_y \frac{h_a(x, y, a_y^*)}{h(x, y, a_y^*)} \tag{3}$$

where λ_y, μ_y are Lagrange multipliers and $h_a(\cdot)/h(\cdot)$ is as defined above, and evaluated at $a = a_y^*$. Equation (3) implicitly describes the optimal compensation function in the production setting characterized above, and this result is not restricted to gain-sharing contracts that provide only non-negative bonuses. By inverting the utility function, this point can be seen clearly

$$\phi^*(x, y) = r(x) + u'^{-1}[1/(\lambda_y + \mu_y h_a(\cdot)/h(\cdot))] \tag{4}$$

where the left side is total compensation. One can think of $G(x, y) = u'^{-1}[1/(\lambda_y + \mu_y h_a(\cdot)/h(\cdot))]$ as the gain-sharing (or incentive) component of the optimal compensation contract.[10]

As mentioned earlier, the prime concern of this paper is with the accounting aspects of constructing an optimal evaluation measure on which the gain-sharing contract can be based. Equation (4) indicates that the optimal gain-sharing compensation scheme will depend on the utility function of the agent, the values of the Lagrangian multipliers, and the likelihood ratio evaluated at a_y^*. Thus, the implementation of an optimal gain-sharing plan requires a performance measure that will allow the principal to infer the value of the likelihood ratio. Proposition 1 provides a concise characterization of this measure.

Proposition 1: In the context of the production environment described, $\rho(x, y)$ is an optimal performance measure if and only if there is some function $K(\cdot)$ that maps $\rho(x, y)$ to $t(y)/x$.

Proof: Suppose there is some function $K(\rho)$ that maps the performance measure $\rho(x, y)$ into $t(y)/x$. Then, one can go from $\rho(\cdot)$ to $h_a(\cdot)/h(\cdot)$ via this function $K(\rho)$

10 Note that $G(x, y)$ is simply total compensation $\phi(x, y)$ less the utility cost to the agent (in dollar terms) for working x hours. The latter term can be thought of as analogous to the ordinary wage bill.

because the earlier lemma showed that $h_a(\cdot)/h(\cdot)$ evaluated at a_y^*, the optimal action choice for the work force, was a function solely of $t(y)/x$. In this manner, the optimal compensation payment derived in the principal/agent framework can be computed from $\rho(\cdot)$, meaning $\rho(\cdot)$ is an optimal performance measure.

To prove the converse, start with $\rho(\cdot)$ being an optimal performance measure. Therefore, there must exist some function $J(\cdot)$ that maps $\rho(\cdot)$ into $h_a(\cdot)/h(\cdot)$ in order to compute the contracted compensation payment. But $h_a(\cdot)/h(\cdot)$ is strictly monotone increasing in $t(y)/x$ by the MLRP. So $h_a(\cdot)/h(\cdot)$ can be inverted to obtain the value of $t(y)/x$ (recall this function is evaluated at $a = a_y^*$). Call this inverse function $H^{-1}(\cdot)$. Then, because $J(\rho) = h_a(\cdot)/h(\cdot)$. there is some function $K(\rho) = H^{-1}(J(\rho)) = t(y)/x$. So from any optimal performance measure ρ, the quantity $t(y)/x$ can be derived. Q.E.D.

From this proposition, it is seen that for the production process considered, an optimal performance measure must be a function of $\pi(x, y) = t(y)/x$. Such a performance measure can then be used to compensate the work force efficiently through the optimal incentive contract. If a gain-sharing contract uses a performance measure that cannot be mapped onto $t(y)/x$, then this contract could be improved upon by choosing a performance measure that can be used in this mapping. Although the form of the optimal incentive contract will depend on the particular utility function for the work force, the optimal performance evaluation measure used must lead to $t(y)/x$.

To analyze gain-sharing contracts, some attention must be paid to the institutional setting in which these agreements are utilized. Quite often gain-sharing contracts attempt to use information that is already collected for engineering and/or cost accounting purposes rather than instituting a new set of measures that might pertain only to the gain-sharing agreement. In this way, the administrative costs of the gain-sharing contract can be kept relatively low. One measure of productivity is the ratio of standard direct labor hours to the actually observed total labor hours (direct plus indirect) required for the level of output (see Banker and Datar, 1987). The amount of gain-sharing bonus paid (if any) is a predetermined function of this ratio. The use of prior years' standard direct labor hours in the performance evaluation measure implicitly posits that the underlying production function is unchanged by the introduction of the gain-sharing contract.

In other words, the gainsharing element $G(\cdot)$ is a function of s/x alone, where $s = \sum_{i=1}^{n} s_i y_i$ is the total number of standard direct labor hours, s_i is the number of standard direct labor hours per unit of output $y_i (i = 1, \ldots n)$, and x is the total observed number of direct *and* indirect labor hours. Thus, s/x is a particular aggregation of the basic information signals in the input and output quantities. The question of interest is whether this aggregate can serve as the optimal performance evaluation measure. If so, then s/x can optimally be used as a basis for gain-sharing compensation.

Proposition 2: If $t(\mathbf{y}) = \beta_0 + \sum_{i=1}^{n} \beta_i y_i$, then a gain-sharing contract based only on the performance evaluation measure $s/x = \sum_{i=1}^{n} s_i y_i / x$ is optimal if and only if (i) $\beta_0 = 0$ and (ii) $\beta_i / s_i = \beta_j / s_j$ for all $i, j = 1, \ldots, n$.

Proof: The proof is straightforward. The optimal incentive contract is a function of $(\beta_0 + \sum_{i=1}^{n} \beta_i y_i)/x$, from proposition 1. Therefore, if s/x is optimal, there must exist a function $\xi(\cdot)$ mapping s/x to $\beta_0 + \sum_{i=1}^{n} \beta_i y_i / x$ for all realized values of y_i and x; that is, $\xi(s/x) = (\beta_0 + \sum_{i=1}^{n} \beta_i y_i)/x$ for all y_i, x. This means $\xi(\cdot)$ must be affine; there exists a constant ξ_0 such that $\xi_0 s = \beta_0 + \sum_{i=1}^{n} \beta_i y_i$ for all y_i. It follows then that $\beta_0 = 0$ and $\xi_0 s_i = \beta_i$ for all $i = 1, \ldots, n$, and thus, $\beta_i / s_i = \beta_j / s_j = \xi_0$ for all $i \neq j$. Q.E.D.

The above result follows directly from proposition 1, but it is important in providing empirically testable hypotheses. In our estimable model, therefore, we have two maintained assumptions:

1 There exists a production function $t(\mathbf{y})$ impacted multiplicatively by the agent's effort a and random factor θ via some function $\Delta(a, \theta)$.
2 The production function $t(\mathbf{y})$ is linear, as in the standard cost accounting models; that is, $t(\mathbf{y}) = \beta_0 + \sum_{i=1}^{n} \beta_i y_i$.

The test of optimality of a gain-sharing bonus contract based on only the aggregate measure s/x is then equivalent to the test of the following two joint hypotheses about the characteristics of the production function:

$H_1 : \beta_0 = 0$, and

$H_2 : \beta_i / \beta_j = s_i / s_j$ for all $i, j = 1, \ldots, n$.

That is, under our maintained assumptions, an observed gain-sharing bonus contract is not optimal if either of the two hypotheses, H_1 or H_2, is rejected.

The model is now in a form suitable for econometric estimation. Because the production function is not directly observable, it must be estimated from the observed data on past realizations of the required output quantities y_{it} and labor hours x_t for periods $t = 1, 2, \ldots, T$. The effort level chosen by the work force is also unobservable. But if we use data obtained *prior* to the implementation of the incentive contract, we can infer that for periods $t = 1, 2, \ldots, T$, the work force must have chosen the *same* effort level (a^o) optimal under the pre–gain-sharing institutional set up and constraints (such as the monitoring system that was in place). Therefore, $\Delta(a_t, \theta_t) = \Delta(a^o, \theta_t)$ for all $t = 1, 2, \ldots, T$. The estimation model for periods $t = 1, 2, \ldots, T$ can now be written as

$$x_t = \left(\beta_0 + \sum_{i=1}^{n} \beta_i y_{it} \right) / \Delta(a^o, \theta_t)$$

The values $\Delta(a^o, \theta_t)$ represent the error term, and $\beta_0, \beta_i, i = 1, 2, \ldots, n$, are the parameters to be estimated from the data for $t = 1, 2, \ldots, T$ periods. The model, however, is nonlinear due to the multiplicative relation between $t(\mathbf{y})$ and $1/\Delta(\cdot)$, and the ordinary least squares (OLS) methods for estimation will not be adequate. Estimation of the model is undertaken in the next section with reference to data obtained from a manufacturing plant.

Empirical estimation

Econometric methods for estimating the nonlinear model specified and the results of such estimation for a particular data set are reported in this section. First a brief introduction to the gain-sharing contract itself and the data is provided.

The gain-sharing contract under consideration was implemented at one manufacturing plant of a multiplant firm. This particular plant produced two mature products via a time-tested process. Each product was subject to stringent quality inspections by the purchaser, meaning the manufacturer was unable to vary output quality. Output mix and quantity, however, could vary substantially, depending on the orders received by the firm. The output quantities demanded, however, are exogenous to the plant level because the plant management did not have any responsibility for pricing decisions or order solicitation.

The gain-sharing contract negotiated between the union and the plant management and implemented at the plant had a structure similar to that described in the previous section. Productivity gains were measured by the ratio s/x, and the standard direct labor hours $s = 16.2y_1 + 4.2y_2$ were based on industrial engineering studies.[11] The ratio s_t/x_t for a period t after the implementation of the gain-sharing contract is compared with a benchmark value s_0/x_0 determined as a ratio of average standard direct labor hours and average actual total labor hours for a base period preceding the implementation of the gain-sharing program. A bonus is paid to the workers if s_t/x_t is greater than s_0/x_0 in a period t, but no bonus payments are made if s_t/x_t is less than or equal to s_0/x_0.[12]

Data on the production process were collected for the 33 months before the gain-sharing agreement was implemented. The data on standard direct labor hours and actual total labor hours were obtained from the monthly accounting and manufacturing reports. Actual total labor hours included both direct and indirect labor hours. For this pre–gain-sharing period, compensation for the work force did not include any bonus or other incentive payments, and we assume the work force chose the lowest effort level possible, given existing institutional constraints.

In the estimable model presented in the previous section, the term $1/\Delta(a^o, \theta_t)$ represents the deviations from the structural specification $\beta_0 + \sum_{i=1}^{2} \beta_i y_{it}$ for the manufacturing plant under consideration. Writing $-\ln \Delta(a^0, \theta_t) = \epsilon$ as the error term, we can express the model as

11 All input and output data are multiplied by the same constant to maintain confidentiality.

12 Bonuses paid under this contract are based on the ratio of s_t/x_t to s_0/x_0 and the size of this bonus is a weakly increasing function of the ratio. The workers are not penalized (paid a negative bonus) as part of the gain-sharing agreement if s_t/x_t is less than s_0/x_0.

$$\ln x_t = \ln(\beta_0 + \beta_1 y_{1t} + \beta_2 y_{2t}) - \ln \Delta(a^0, \theta_t)$$

The standard maximum likelihood estimation procedure would require the assumption of a specific distribution (lognormal) for the stochastic error term $1/\Delta(a^0, \theta_t)$, along with the assumption that the error term is homoskedastic. As noted by Hansen and Singleton (1982), maximum likelihood estimates will generally be (asymptotically) more efficient than nonlinear instrumental variable estimation methods when the distributional assumptions made reflect the economic reality of the situation under study. However, if the distributional assumptions made for the maximum likelihood estimation are in error, the estimates obtained may fail to be consistent. Given this concern, we employ a more robust estimation procedure, the generalized method of moments (GMM).

The GMM method is described in detail in Hansen (1982) and Hansen and Singleton (1982). To our knowledge, it has not been utilized in the accounting literature. The method is computationally somewhat more complex than standard econometric techniques. However, the additional effort required to use GMM has a payoff in terms of estimates that are asymptotically efficient and yet do not rely on specific assumptions on the distribution of the error term, or the nature of serial correlation or heteroskedasticity. Instead, the more fundamental properties of stationarity and ergodicity are assumed to hold. The estimators obtained using GMM will be consistent even when the error terms are serially correlated and/or when the error terms are conditionally heteroskedastic; that is, the variance can depend on the observed values (see Hansen and Singleton, 1982). These can be major advantages to the researcher conducting empirical work.

Under the GMM technique, the estimators are defined in terms of the following orthogonality conditions, involving the error term ϵ_t, which are referred to as the instruments of the estimation.[13] These conditions are derived from the distribution of the error term

$$\epsilon_t = \ln(\beta_0 + \beta_1 y_{1t} + \beta_2 y_{2t}) - \ln x_t$$

The GMM estimates are of course dependent on the particular instrumental variables chosen and orthogonality conditions imposed. For instance, if we believed that the data reflected autocorrelation of the first degree, the instrumental variables could have been chosen in a way to implement the Cochran-Orcutt procedure. In this manner, the various corrections to least squares estimation can be considered as special cases of GMM estimation. The GMM estimates are consistent and robust to the exact distributional assumptions on the error term, making them superior along this dimension.

13 The stationary and ergodicity of the chosen instruments follow from the fact that they are measurable functions of the y_{it} values. From the choice of instruments and the form of the disturbance terms, it can be verified that the orthogonal vector function satisfies its measurability and supercontinuity requirements. The other conditions for assuring consistency and asymptotic normality of the GMM estimates are assumed to be satisfied (see Hansen, 1982).

TABLE 1
Results of GMM Estimation

$x_t = 15.1336 + 30.7301\, y_{1t} + 9.3981\, y_{2t}$
(2.6471) (2.6104) (1.2358)

Variance—Covariance Matrix

	β_0	β_1	β_2
β_0	7.0073		
β_1	−5.9519	6.8140	
β_2	−1.6245	0.0409	1.5272

Standard errors are in parentheses.

At the current stage of development, use of GMM is still somewhat an art form. The researcher is able to incorporate prior knowledge regarding the error structure directly into the estimation procedure. In this particular case, we chose to utilize GMM instruments consistent with the orthogonality conditions $E[\epsilon \cdot \partial\epsilon/\partial\beta_i] = 0$ for $i = 0, 1, 2$. These conditions can be written as:

$$\sum_{t=1}^{T} \frac{\epsilon_t}{\beta_0 + \beta_1 y_{1t} + \beta_2 y_{2t}} = 0$$

$$\sum_{t=1}^{T} \frac{\epsilon_t y_{1t}}{\beta_0 + \beta_1 y_{1t} + \beta_2 y_{2t}} = 0$$

$$\sum_{t=1}^{T} \frac{\epsilon_t y_{2t}}{\beta_0 + \beta_1 y_{1t} + \beta_2 y_{2t}} = 0$$

The similarity of these conditions to those that determine the maximum likelihood estimator should be apparent. One can think intuitively of the maximum likelihood conditions as being the GMM instruments that are orthogonal to the error term.

The paricular instrumental variables we have chosen yield asymptotically efficient maximum likelihood estimates. The significance tests take into account the particular structure of the error term (in terms of, for example, serial correlation or heteroskedasticity). These are therefore valid tests and yield consistent estimates even though the error terms have not been explicitly corrected. Contrasted with least squares estimation where the estimates and hypothesis tests are conditional on the assumed form being correct, the GMM methodology imposes very few restictions on the error term.

The GMM estimation results are presented in Table 1, where the parenthetical figures below the coefficient estimates are the asymptotic standard errors.

Table 1 reports the estimates of the coefficients, asymptotic standard errors, and the variance-covariance matrix. The intercept term β_0 is significantly different from zero (at the 0.01 level), and hypothesis H_1 can be rejected.[14] Because H_1 is one of the maintained assumptions of the model, this result indicates that the observed performance measure is inconsistent with the necessary optimality conditions generated in the previous section. Hypothesis H_2 requires β_1/β_2 to be equal to the ratio $s_1/s_2 = 3.83$. This hypothesis can be tested by evaluating whether $\beta_1 - 3.83\beta_2$ is significantly different from zero. The variance of $(\beta_1 - 3.83\beta_2)$ is given by $\text{Var}(\beta_1) - 2(3.83)\text{Cov}(\beta_1, \beta_2) + (3.83)^2\text{Var}(\beta_2) = 28.878$. So the standard deviation of this expression is 5.374, and the test statistic is given by

$$\frac{30.7301 - (3.83)(9.3981)}{5.374} = -0.9758$$

Thus, the hypothesis H_2 cannot be rejected at conventional levels of significance.[15]

The hypothesis tests led to the conclusion that the ratio form of the performace evaluation measure utilized by this plant is not consistent with the optimality conditions described by proposition 2 because H_1: $\beta_0 = 0$ was rejected by the data under our maintained assumptions.[16]

One final point should be noted regarding the empirical results that indicate the observed performance evaluation measure is not optimal with respect to the model specified here. Although our assumptions are very plausible and employ a fairly general structure, the principal-agent model we have specified is a single-period model. It is possible that a different model that specifies a different production relationship and incorporates multiperiod considerations might be consistent with the performance evaluation measure used in the observed gain-sharing contract. Given the current state of principal-agent analysis, it is not apparent which features such an alternative model would have, nor is the intuition clear as to how these features would make the utilized performance evaluation measure optimal. Further research on multiperiod agency models is required before this desirable extension of our model can be made.

Conclusion

This paper contributes to the accounting literature in four ways. First, it brings

14 This constant term represents approximately 30 percent of total labor costs.

15 We also performed a nonlinear regression using the NLIN procedure described in the SAS User's Guide (1985) and obtained similar results. Under the added assumption of a lognormally distributed error term, this approach yields maximum likelihood estimates. The coefficients from this nonlinear regression procedure were $\beta_0 = 15.0747, \beta_1 = 30.7526$, and $\beta_2 = 9.5142$, with asymptotic standard errors of 2.1772, 2.2113, and 1.1726, respectively.

16 Although the gain-sharing contract is not consistent with the optimality conditions, the introduction of an incentive plan may affect the productivity of the labor force. The extent and direction of such productivity changes in the period following the introduction of the plan are examined in Banker, Datar and Rajan (1987).

ideas from the productivity measurement literature to bear on the issue of optimal design of gain-sharing contracts. Second, the standard agency model is modified to incorporate two distinct measures of effort by the agent: hours worked and intensity. This modification allows us to evaluate an actual gain-sharing contract. The third contribution, then, is in formally developing a model that could be used in an empirical examination of an agency situation. Because the framework constructed for the empirical analysis is relatively general, we expect that it could be adopted for use with a variety of production technologies. Fourth, the empirical results suggest the gain-sharing contract utilized by this firm is suboptimal. Our model also suggests an alternative productivity measure $(t(y)/x)$ that will be an optimal performance evaluation measure for *any* risk-averse utility function for the agent.

Even though the simple ratio of actual total labor hours to standard direct labor hours can be an optimal performance evaluation measure for some classes of production functions, the production correspondence for the manufacturing plant we examimed does not have the required properties. In this regard, management must use care in selecting the forms of productivity contracts offered; otherwise, suboptimal contracting may occur.

The empirical analysis reported in this paper illustrates the problems encountered in using the principal-agent framework as a *predictive* model of incentive contracts. This is especially true in settings in which the two parties are initiating a change in their contractual relationship (such as the introduction of gain-sharing and incentive bonuses at our site) and where the relevant management accounting methodology to support performance evaluation measurement (such as productivity gain evaluation at our site) is still in its infancy. It may take a considerable length of time before the contracting parties converge (by trial and error) on the optimal arrangements. But, just as single-person decision theory has been employed in a *prescriptive* mode, our analysis indicates how agency theoretic models may be employed to determine the optimal method for measuring productivity gains for gain-sharing contracts.

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