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A COMPARATIVE APPLICATION OF DATA ENVELOPMENT ANALYSIS AND TRANSLOG METHODS: AN ILLUSTRATIVE STUDY OF HOSPITAL PRODUCTION*

RAJIV D. BANKER, ROBERT F. CONRAD AND ROBERT P. STRAUSS
School of Urban and Public Affairs, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
Department of Economics, Emory University, Atlanta, Georgia 30322
School of Urban and Public Affairs, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

This paper compares inferences about hospital cost and production correspondences from two different estimation models: (1) the econometric modeling of the translog cost function, and (2) the application of Data Envelopment Analysis (DEA). While there are numerous examples of the application of each approach to empirical data, this paper provides insights into the relative strengths of the estimation methods by applying both models to the same data.

The translog results suggest that constant reruns are operant, whereas the DEA results suggest that both increasing and decreasing returns to scale may be observed in different segments of the production correspondence, in turn suggesting that the translog model may be 'averaging' diametrically opposite behavior. On the other hand, by examining the rate of output transformation, both models agree that patient days devoted to care of children are more resource intensive than those devoted to adults or to the elderly. In addition, we compare estimates of technical efficiencies of individual hospitals obtained from the two methods. The DEA estimates are found to be highly related to the capacity utilization, but no such relationship was found for the translog estimates.

This comparative application of different estimation models to the same data to obtain inferences about the nature of underlying cost and production correspondences sheds interesting light on the strengths of each approach, and suggests the need for additional research comparing estimation models using real as well as simulated data.
(ORGANIZATIONAL STUDIES; DATA ENVELOPMENT ANALYSIS; HOSPITAL COSTS; EFFICIENCY EVALUATION; PRODUCTION FUNCTIONS; TRANSLOG ESTIMATION; RETURNS TO SCALE)

1. Introduction

A. Charnes, W. W. Cooper and E. Rhodes (1979) suggested a mathematical programming approach, referred to as Data Envelopment Analysis (DEA), to estimate the efficiencies of decision making units. More recent developments described by Banker, Charnes and Cooper (1984), Banker (1983, 1984, 1985), Banker and Maindiratta (1983, 1986) and Banker and Morey (1986a, b) have extended DEA to the estimation of cost and production correspondences. Unlike the classical econometric approaches that require a pre-specification of a parametric functional form and several implicit or explicit assumptions about the production correspondences,[1] DEA requires only an assumption of convexity of the production possibility set, and employs a

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[1]See also Hildenbrand (1981) for an alternative nonparametric approach to the estimation of short-run production functions.

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postulate of minimum extrapolation from observed data to estimate production correspondences. Furthermore, DEA also permits examination of particular production characteristics, such as efficiencies, returns to scale and rates of transformation, prevailing in specific segments of the production possibility set. In addition, DEA is useful for applications in many nonprofit organizations and complex production situations because it readily models multiple-output multiple-input technologies.

To date, results of DEA have not been compared to those from more traditional econometric techniques used for estimation of production functions. One of the most common parametric methods employed for estimating multiple output-multiple input technologies has been the translog cost function proposed by Christensen, Jorgensen and Lau (1973) and Brown, Caves and Christensen (1979). It should be noted, however, that this method yields estimates of "average" production functions. This is in contrast to the application of DEA and also other more recent econometric methods for estimating frontier production functions developed by Aigner, Lovell and Schmidt (1977) and Jondrow, Lovell, Materov and Schmidt (1982). These recent parametric methods have been generally applied only to single output-multiple input situations.[2] The extension of parametric methods for frontier estimation to the multiple output case raises several additional theoretical and computational problems. By imposing the assumption of no allocative inefficiencies, we can estimate a *frontier* translog cost function and technical inefficiencies, and compare the indirect estimates of *frontier* production characteristics with the direct estimates obtained from DEA. Because the estimates of different characteristics of the production correspondence provided by these two methods are commonly employed for policy inferences, their comparison is useful and interesting.

In a recent study of North Carolina hospitals, Conrad and Strauss (1983) specify a four-input and three-output production technology, and using the translog cost function method, estimate the parameters of the production correspondence. In this paper, our objective is to estimate the production correspondence employing DEA with the same data-set and the same four-input, three-output specification of the production technology and compare the DEA results to those from the translog analysis. Of interest are the similarities or differences between the two approaches in ascertaining whether there are increasing, constant or decreasing returns to scale, and estimating marginal rates of output transformation and technical inefficiencies of individual hospitals. For this purpose, we shall extend Richmond's (1974) Corrected Ordinary Least Squares approach for estimating frontier production functions to multiple-output situations. Employing Conrad and Strauss' estimation of a multivariate system with factor share equations, we shall estimate frontier translog cost function and compare

TABLE 1

Areas of Comparison between DEA and Translog Estimates

DEA	Translog
1. Most productive scale size and returns to scale	Returns to scale
2. Marginal rates of output transformation	Marginal rates of output transformation
3. Technical efficiency, and technical and scale efficiency	Technical efficiency assuming zero allocative inefficiency

[2]See Lovell and Sickles (1983) for recent applications to multi-output situations.

the translog estimates of the production characteristics specified in Table 1 with the corresponding DEA estimates.

2. Translog Joint Cost Function Estimation

Conrad and Strauss (1983) based their study on the cost and production data available from the audited Medicare cost reports submitted for fiscal year 1978 by 114 North Carolina hospitals to North Carolina Blue Cross-Blue Shield, and similar reports submitted to the Duke Endowment. In order to facilitate estimation, they considered four major aggregated inputs: (1) nursing services, (2) ancillary services (including operating room, anesthesiology, laboratory and x-ray labor), (3) administrative and general services (including dietary and housekeeping labor), and (4) capital. Furthermore, to examine the impact of the utilization of hospital services by patients of different age groups, they considered three outputs: (1) patient days for inpatients below age 14, (2) patient days for inpatients aged between 14 and 65, and (3) patient days for inpatients aged above 65. Clearly, with the availability of more detailed DRG-based case-mix data, this simple production model could be made increasingly realistic in future research. The price per unit hour of nursing, ancillary and general services were derived as average costs, including fringe benefits, from total costs and hours as defined in the Medicare cost reports. The cost share of capital was calculated as the sum of depreciation and interest charges.[3]

Under DEA, production correspondences are estimated directly. Econometric estimation of production correspondences may proceed directly or may proceed indirectly and relate costs to output quantities and input prices in conjunction with additional structural information provided by, for instance, Shephard's Lemma. For a variety of reasons, including computational considerations, the latter indirect approach has typically been employed in obtaining inferences about production correspondences, and will be the approach developed below. Following Brown, Caves and Christensen (1979), the following frontier transcendental logarithmic joint cost function is employed to estimate the hospital cost relationships:

$$\ln c^* = \alpha_o + \sum_{r=1}^{3} \alpha_r \ln y_r + \sum_{i=1}^{4} \beta_i \ln w_i + \frac{1}{2} \sum_{r=1}^{3} \sum_{t=1}^{3} \delta_{rt} \ln y_r \ln y_t + \frac{1}{2} \sum_{i=1}^{4} \sum_{k=1}^{4} \gamma_{ik} \ln w_i \ln w_k + \sum_{r=1}^{3} \sum_{i=1}^{4} \rho_{ri} \ln y_r \ln w_i \qquad (1)$$

where y_r represent the $r = 1,2,3$ output quantities, and w_i represent the $i = 1,2,3,4$ input prices. Also, $\delta_{rt} = \delta_{tr}$ and $\gamma_{ik} = \gamma_{ki}$.[4] Furthermore, the assumption that the cost function is linearly homogeneous in input prices, implies the following restrictions[5] on the above joint cost function:

$$\sum_{i=1}^{4} \beta_i = 1, \qquad \sum_{i=1}^{4} \rho_{ri} = 0 \quad \text{for each } r, \quad \text{and} \quad \sum_{i=1}^{4} \gamma_{ik} = 0 \quad \text{for each } k. \qquad (2)$$

As a direct extension of the Corrected Ordinary Least Square approach of Richmond (1974), we write in $\ln c^* \leqslant \ln c$, where c^* is the estimated efficient cost and c is

[3]However, capital as a resource is difficult to assess due to timing of expenditures and charges for depreciation. Therefore, any inferences from this study must be considered with this limitation in mind.

[4]Note that the use of the translog function is valid because all of the outputs for all of the hospitals were positive.

[5]See Berndt and Christensen (1973) for a detailed analysis.

the observed cost for each hospital. Therefore, in (1) we adjust the intercept term α_o sufficiently downward so that the estimate of the efficient cost c^* in (1) is less than or equal to the observed cost c for each hospital. Greene (1983) has shown that, under minor assumptions, the estimates of the other parameters in (1) will be consistent.

The translog function contains a large number of parameters even for a relatively small number of inputs and outputs. As a result, its estimation via ordinary least squares is likely to result in imprecise parameter estimates due to multicollinearity. This problem is alleviated by the employment of Shephard's Lemma to derive 4 cost share equations (of which 3 are independent) in the following form:

$$\frac{w_i x_i^*}{c^*} = \beta_i + \sum_{k=1}^{4} \gamma_{ik} \ln w_k + \sum_{r=1}^{3} \rho_{ri} \ln y_r \qquad \text{for each} \quad i = 1,2,3,4 \tag{3}$$

where x_i^*, $i = 1,2,3,4$, represent the cost-minimizing input quantities for each hospital.

We impose an additional assumption that there is no allocative inefficiency, and write the (radial) technical efficiency as θ. Therefore, the observed cost share for input i is given by $w_i\theta x_i^*/\sum_{k=1}^{4} w_k\theta x_k^*$, which is clearly equal to the efficient cost share $w_i x_i^*/c^*$. Thus, we can replace the quantities $w_i x_i^*/c^*$ on the left-hand side of (3) by the observed cost shares for input i. The above multivariate system of equations in (1) and (3), with correlated disturbance terms, can then be iteratively estimated by the procedure outlined by Zellner (1962, 1963).

In order to test for constant returns to scale, the following restrictions are required to impose homogeneity of degree one on the translog cost function:

$$\sum_{r=1}^{3} \alpha_r = 1, \qquad \sum_{r=1}^{3} \delta_{rt} = 0, \qquad \text{for each} \quad t = 1,2,3, \quad \text{and}$$

$$\sum_{r=1}^{3} \rho_{rt} = 0 \qquad \text{for each} \quad t = 1,2,3. \tag{4}$$

The F-criterion may then be used to test this hypothesis.

Having estimated the parameters of the joint cost function, the marginal costs for each output may be estimated using the relationship defined by:

$$\frac{\partial c^*}{\partial y_r} = \left(\alpha_r + \sum_{t=1}^{3} \delta_{rt} \ln y_r + \sum_{t=1}^{4} \rho_{rt} \ln w_t \right) \frac{\hat{c}^*}{y_r}\,. \tag{5}$$

3. The DEA Model

Assuming convexity of production possibility sets, and employing a postulate of minimum extrapolation from observed data, Banker, Charnes and Cooper (1984) provide a linear programming model for estimating productive efficiencies and other production characteristics of technology specified by an efficient correspondence between inputs and outputs. Let y_{rj} and x_{ij} be the observed output ($r = 1,2,3$) and input[6] ($i = 1,2,3,4$) values for the 114 hospitals ($j = 1, \ldots, 114$). To estimate the

[6]The capital input is measured in terms of number of beds, which may leave out other dimensions of capital such as equipment. Any inferences from this study must be considered with this limitation in mind. Banker and Morey (1986a) describe a modification to the linear program to account for fixed inputs, such as capital, or the number of beds in our study of hospitals. However, we shall not explore this model here to maintain consistency with the long-term cost function estimated with the translog approach.

technical efficiency of any of these hospitals, referenced now by the subscript 0, the following programming problem may be formulated.

$$h_0^* = \min f_0 + \epsilon\left[\sum_{r=1}^{3} s_{r0}^+ + \sum_{i=1}^{4} s_{i0}^-\right] \quad \text{subject to} \tag{6}$$

$$\sum_{j=1}^{114} \lambda_j x_{ij} + s_{i0}^- = f_0 x_{i0}, \qquad i = 1, \dots, 4,$$

$$\sum_{j=1}^{114} \lambda_j y_{rj} - s_{r0}^+ = y_{r0}, \qquad r = 1, \dots, 3,$$

$$\sum_{k=1}^{114} \lambda_j = 1,$$

$$f_0, \lambda_j, s_{r0}^+, s_{i0}^- \geqslant 0,$$

and ϵ is a small positive non-Archimedean quantity.

In actual implementations, one may specify a sufficiently small value for ϵ to solve (6) in one step. We employ an alternative two-stage approach to first identify the minimum radial efficiency, f_0^*, and then setting f_0 equal to this minimum f_0^* we identify the maximum possible slacks s_{r0}^+ and s_{i0}^- in the constraints. This ensures consistency with the desired prioritized optimization in the non-Archimedean specification. It may be noted that the computed h_0^* will depend on the specified (small) value of ϵ. However, the objective of the analysis is to distinguish between the efficient and inefficient hospitals, and with our treatment of efficiency as a categorical variable in §6, the actual value of ϵ does not have any practical significance.

The dual of the above program in (6) may then be represented as:

$$h_0^* = \max \sum_{r=1}^{3} u_r y_{r0} - u_0 \quad \text{subject to} \tag{7}$$

$$\sum_{i=1}^{4} v_i x_{i0} = 1,$$

$$\sum_{r=1}^{3} u_r y_{rj} - \sum_{i=1}^{4} v_i x_{ij} - u_0 \geqslant 0, \qquad j = 1, \dots, 114,$$

$u_r, v_i \geqslant \epsilon > 0$, and u_0 is unconstrained in sign.

Banker, Charnes and Cooper (1984) show that the ratios of the variables u_r and v_i provide estimates of the marginal rates of transformation of outputs, marginal rates of substitution of inputs and marginal productivities. For instance, the ratio $u_3 : u_1$ measures the marginal rate of transformation of output 3 for output 1(MRT 3 : 1). In other words, it measures the number of units by which production of output 1 could be increased if the production of output 3 were reduced by one unit. These computations, of course, reflect production characteristics measured at the margin on a specific segment of the efficient production surface.

In addition, Banker, Charnes and Cooper (1984) also show that the returns to scale at the referent efficient point are estimated by the sign of the variable u_0. Increasing returns to scale are indicated for $u_0^* < 0$, constant returns for $u_0^* = 0$, and decreasing returns for $u_0^* > 0$. However, for most applications, it is more meaningful to work with the related notion of the most productive scale size (mpss), introduced by Banker (1984). A production possibility (X, Y) represents a mpss if and only if for any production possibility given by $(\beta X, \alpha Y)$, where α and β are positive scalars, the ratio of α/β is less than or equal to one. Thus, a mpss represents the greatest productivity of resources for any given mix of inputs and outputs—the scale size at which decreasing returns to scale have not yet set in, but all productivity gains due to increasing returns to scale have been exploited. For the estimation of the mpss, the original CCR formulation is employed:

$$t_0^* = \min f_0 + \epsilon\left[\sum_{r=1}^{3} s_{r0}^+ + \sum_{i=1}^{4} s_{i0}^-\right] \quad \text{subject to} \tag{8}$$

$$\sum_{j=1}^{114} \lambda_j x_{ij} + s_{i0}^- = f_0 x_{i0}, \qquad i = 1, \ldots, 4,$$

$$\sum_{j=1}^{114} \lambda_j y_{rj} - s_{r0}^+ = y_{r0}, \qquad r = 1, \ldots, 3,$$

$$f_0, \lambda_j, s_{r0}^+, s_{i0}^- \geqslant 0,$$

and ϵ is small positive non-Archimedean quantity.

Writing $k_0^* = \sum_{j=1}^{114} \lambda_j^*$ Banker (1984) showed that for a mix of inputs and outputs given by (X_0, Y_0), the point

$$\left(\frac{f_0^*}{k_0^*} X_0, \frac{1}{k_0^*} Y_0\right)$$

represents a production possibility, which is also a mpss.

4. Returns to Scale and Most Productive Scale Size

In this and subsequent sections, we compare results of the application of the translog and DEA models to the data used by Conrad and Strauss (1983). We begin here with a comparison of estimates relating to returns to scale possibilities.

As noted above, satisfaction of (4) implies constant returns to scale (or linear homogeneity) in the translog case. This hypothesis cannot be rejected at the one percent significance level. See Exhibit 1.

The returns to scale for a particular observed input-output mix may be examined using DEA by estimating the corresponding most productive scale size (mpss). A relatively low mpss indicates that decreasing returns to scale set in early. On the other hand, a relatively high mpss indicates that increasing returns to scale prevail even at medium or larger scale sizes.

In particular, we compared the mpss for different output mixes, that is, different proportions of patient days for the three age groups—below 14 years, between 14 and 65 years, and above 65 years. See Exhibit 2. The population of 114 hospitals was divided into four quarters based on the proportion of patient days for patients aged below 14 years. The mean mpss for the 29 hospitals with a high proportion of patient days below 14 is 160 beds, while for the 29 hospitals with a low proportion of such patient days, the mean mpss is only 110 beds. The difference is found to be statistically significant[7] at .0001 level using Welch's two-samples-means one-tailed test. The 114 hospitals were again divided into four quarters based on the proportion of patient days for patients aged between 14 and 65 years. The mean mpss of 223 beds for the 29 hospitals with a high proportion of patient days between 14 and 65 years is found to be

[7]We intend that these statistics be regarded as descriptive statistics. They cannot be interpreted in their customary inferential mode because the usual assumptions about the independence of the samples may be violated in these cases.

EXHIBIT 1A

*Parameter Estimates for the Translog Cost Function**

Parameter	Unrestricted	Linear Homogeneous	Parameter	Unrestricted	Linear Homogeneous
α_0	8.2648***	6.6680***	γ_{22}	0.2182	0.2411
	(0.9240)	(0.5161)		(0.0152)	(0.0142)
α_1	0.3217	0.4598	γ_{23}	− 0.1124	− 0.1382
	(0.1552)	(0.0933)		(0.0147)	(0.0127)
α_2	0 3461	0.6667	γ_{24}	− 0.0587	0.0990
	(0.3228)	(0.1825)		(0.0224)	(0.0194)
α_3	− 0.1204	− 0.1264	γ_{33}	0.1010	0.0266
	(0.3678)	(0.1357)		(0.0226)	(0.0197)
β_1	− 0.2940	− 0.2459	**γ_{34}	0.0751	0.0737
	(0.0634)	(0.0513)		(.1026)	(0.0238)
β_2	0 1648	0.0630	**γ_{44}	.0006	− 0.0242
	(0 0784)	(0.0742)		(.1060)	(.0313)
β_3	1.0508	0.8240	ρ_{11}	− 0.0220	− 0.0249
	(0.0970)	(0.0909)		(0.0046)	(0.0044)
**β_4	0.0784	0.3589	ρ_{12}	− 0.0039	− 0.0045
	(.1399)	(.1281)		(0.0061)	(0.0057)
δ_{11}	0.0185	0.0531	ρ_{13}	0.0288	0.0343
	(0.0241)	(0.0238)		(0.0069)	(0.0066)
δ_{12}	− 0.0064	0.0332	**ρ_{14}	− 0.0003	− 0.0049
	(0.0515)	(0.0441)		(.0103)	(0.0098)
δ_{13}	− 0.0058	− 0.0863	ρ_{21}	− 0.0704	− 0.0092
	(0.0490)	(0.0272)		(0.0111)	(0.0092)
δ_{22}	0 2513	0 4592	ρ_{22}	0.0291	0.0418
	(0.1406)	(0.1071)		(0.0141)	(0.0066)
δ_{23}	− 0.1402	− 0.4924	ρ_{23}	0.0076	− 0.0760
	(0.1347)	(0.0755)		(0.0161)	(0.0089)
δ_{33}	0.2177	0.5787	ρ_{24}	0.0337	0.0434
	(0.1446)	(0.0589)		(0.2411)	(0.0171)
γ_{11}	0.1311	0.0155	ρ_{31}	− 0.0305	0.0341
	(0.0078)	(0.0029)		(0.0100)	(0.0071)
γ_{12}	− 0.0471	− 0.0039	ρ_{32}	(0.0380)	− 0.0373
	(0.0073)	(0.0036)		(0.0131)	(0.0052)
γ_{13}	− 0.0637	0.0379	ρ_{33}	0.0054	− 0.0168
	(0.0100)	(0.0042)		(0.0149)	(0.0060)
**γ_{14}	− 0.0223	0.0495	ρ_{34}	− 0.0129	0.0200
	(0.0146)	(0.0062)		(0.0222)	(0.0107)

*Standard errors in parentheses.
**Coefficient and standard error are derived.
***Before adjusting to obtain frontier estimates.
Source· Conrad and Strauss (1983, Table 1).

EXHIBIT 1B

Translog Test Results for Constant Returns to Scale

Number of Restrictions in the Constrained Model	7
Value of the F-statistic	0.01
Critical level of F-statistic at 0.01 significance level	2.64

Source: Conrad and Strauss (1983, Table 2).

EXHIBIT 2

*Most Productive Scale Size****

	Proportion of Patient Days below 14 Years		Proportion of Patient Days between 14 and 65 Years		Proportion of Patient Days above 65 Years	
	29 Hospitals with Highest Proportion (> 11.08%)	29 Hospitals with Lowest Proportion (< 5.20%)	29 Hospitals with Highest Proportion (> 57.6%)	29 Hospitals with Lowest Proportion (< 49.2%)	29 Hospitals with Highest Proportion (> 44.2%)	29 Hospitals with Lowest Proportion (< 31.8%)
Mean	260	110	223	108	99	245
Standard Deviation	173	65.4	183	52.8	47.7	178
Welch's Mean Test:						
t*		4.365		3.226		− 4.268
$d \cdot f.$**		35		32		31
p		< 0.0001		0.0029		0.0002
Median	201	90	157	97	91	201
Mann–Whitney Test:						
p		< 0.0001		0.0005		< 0.0001

$$*t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}}$$

$$**d \cdot f. = \frac{\left[(s_1^2/n_1) + (s_2^2/n_2)\right]^2}{\frac{(s_1^2/n_1)^2}{(n_1 - 1)} + \frac{(s_2^2/n_2)^2}{(n_2 - 1)}}$$

$$***\text{mpss} = \left(\frac{\theta_0^*}{\sum_{j=1}^{114} \lambda_j^*}\right) \times (\text{number of beds})$$

significantly greater (at the 0.0029 level) than the mean mpss of 108 beds for the 29 hospitals with a high proportion of such patient days. Finally, the mean mpss of 99 beds for the 29 hospitals with a high proportion of patient days for patients aged over 65 years is found to be significantly smaller (at the 0.002 level) than the mean mpss of 245 beds for the 29 hospitals with a low proportion of such older patients. Similar results were obtained when these differences in mpss were tested using the nonparametric Mann–Whitney test. Our tests, therefore, reveal that decreasing returns to scale set in early when there is a high proportion of older (Medicare) patients. On the other hand, when the proportion of patient days below 65 years is high, it is still possible to exploit increasing returns to scale when the capacity of the hospital is less than 200 beds.

This illustrates how DEA can be employed to examine returns to scale in *specific* regions of the production possibility set. It is interesting to recall at this stage that when a translog function was fitted to the *aggregate* data, a constant return to scale hypothesis could not be rejected. It appears, therefore, that the existence of increasing returns to scale in some regions is compensated by decreasing returns to scale prevailing elsewhere; and on the basis of the aggregate data for the entire production possibility set, it is not possible to reject a constant returns to scale hypothesis. But, employing DEA we are able to examine the possibility of increasing or decreasing returns to scale prevailing in specific segments of the production possibility set.

5. Marginal Rates of Output Transformation

The estimates of the marginal rate of output transformation were obtained from the translog model[8] by calculating the ratios of the various marginal cost functions:

$$\mathrm{MRT}(C:A)=\frac{\partial c^*}{\partial y_C}\Big/\frac{\partial c^*}{\partial y_A},\qquad \mathrm{MRT}(B:A)=\frac{\partial c^*}{\partial y_B}\Big/\frac{\partial c^*}{\partial y_A},$$

$$\mathrm{MRT}(C:B)=\frac{\partial c^*}{\partial y_C}\Big/\frac{\partial c^*}{\partial y_B}.$$

Exhibit 3 contains the medians of these MRT values for hospitals within the same groupings as those used in Exhibits 2 and 4, namely on the basis of the proportion of patient days for different age-groups of patients. It is evident from Exhibit 3 that one patient day of care for those under age 14 can be transformed into more than one day of either adult care or care for the elderly. With regard to the relationship between the rate of output transformation of elderly care for adult care, it is equally clear that one day of adult care can be substituted for more than a day of elderly care. Alternatively, this may be viewed as suggesting that less than a day of elderly care may be traded off for a day of adult care. These findings from the translog model are consistent with the findings of the relative magnitude of marginal costs reported by Conrad and Strauss (1983, p. 348). While the magnitude of these rates of transformation is consistent in general, substantial differences exist for hospitals with different proportions of patient days for the three categories of patients.

EXHIBIT 3

Translog Estimates of Marginal Rates of Output Transformation:
(entry is median MRT within group)

	Proportion of Patient Days below 14 Years		Proportion of Patient Days between 14 and 65 Years		Proportion of Patient Days above 65 Years	
	Highest 29 Hospitals	Lowest 29 Hospitals	Highest 29 Hospitals	Lowest 29 Hospitals	Highest 29 Hospitals	Lowest 29 Hospitals
MRT(C : A)	3.83	2.99	5.12	2.99	3.22	4.83
MRT(A : B)	0.669	0.610	0.245	0.411	0.160	0.331
MRT(C : B)	2.21	2.28	1.70	1.82	2.07	2.12

Note: MRT(C : A) ≡ Marginal Rate of Transformation of C for A, etc.
A ≡ Patient Days above 65 years.
B ≡ Patient Days between 14 and 65 years.
C ≡ Patient Days below 14 years.

[8] In particular, we utilize the restricted estimation results reflecting linear homogeneity since they could not be rejected at any reasonable significance level.

EXHIBIT 4

DEA estimates of Marginal Rates of Output Transformation

	Proportion of Patient Days below 14 Years		Proportion of Patient Days between 14 and 65 Years		Proportion of Patient Days above 65 Years	
	Highest 29 Hospitals	Lowest 29 Hospitals	Highest 29 Hospitals	Lowest 29 Hospitals	Highest 29 Hospitals	Lowest 29 Hospitals
MRT(C . A)						
Median	4.48	3.25	19.90	4.56	4.56	5.08
Mann–Whitney Test	$p = 0.0093$		$p = 0.0131$		$p = 0.0050$	
MRT(A : B)						
Median	0.731	0.332	0.093	0.645	0.584	0.349
Mann–Whitney Test	$p = 0.0201$		$p = 0.0386$		$p = 0.9010$	
MRT(C : B)						
Median	1 87	0.50	0 50	2.11	1.87	1.23
Mann–Whitney Test	$p = 0.0401$		$p = 0.8581$		$p = 0.1968$	

Note: MRT$(C:A) \equiv$ Marginal Rate of Transformation of C for $A = u_C/u_A$

$A \equiv$ Patient Days above 65 years

$B \equiv$ Patient Days between 14 and 65 years.

$C \equiv$ Patient Days below 14.

The DEA results are generally similar to the pattern exhibited by the translog estimates. See Exhibit 4. For instance, the median for MRT$(C:A)$ where output C represents patient days below 14 years and output A represents patient days above 65 years, was estimated by DEA to be 4.48 for the 29 hospitals with a high proportion of such patients, and 3.25 for the 29 hospitals with a low proportion of such patients. The translog estimates were 3.83 and 2.99 respectively for these two categories of hospitals. Although the estimates of MRT's by the two methods are not identical, it is interesting to note that the DEA results also indicate that one day of child care may be traded off for more than one day of either adult or elderly care. The resource intensity of child care is well known in the hospital cost literature and is generally attributed to the special services provided to infants and children, i.e. nurseries, and children's wards, and the high expense of neo-natal care.

6. Efficiency Evaluation

In DEA, the technical efficiency of individual observation is estimated as in (6) to reflect its radial distance from the directly estimated production frontier. As discussed earlier, the translog method involves the estimation of the frontier cost function, and the production characteristics are derived indirectly from the estimated frontier cost function. Our assumptions of no allocative efficiency and radial technical efficiency ($x_i^* = \theta x_i$, $i = 1, \ldots, 4$) imply

$$\frac{c^*}{c} = \frac{\sum w_i x_i^*}{\sum w_i x_i} = \frac{\sum w_i \theta x_i}{\sum w_i x_i} = \theta \quad \text{or, equivalently,} \quad \ln c^* - \ln c = \ln\theta.$$

Thus, our estimates of individual technical efficiencies in the translog method reflect their distances from the estimated frontier cost function.

In DEA, 45 observations were estimated to be technically efficient, 37 had technical efficiency estimates between 0.9 and 1.0, and the remaining 32 were evaluated to have efficiency ratings below 0.9. We also categorized the observations into three classes on the basis of their technical efficiency ratings with the translog method using the linear homogeneous model. The 45 observations with the highest efficiency ratings were put in the first class, the next 37 in the second, and the lowest 32 in the third class. Exhibit 5A provides a comparative tabulation of the DEA and translog technical efficiency ratings, categorized into 3 corresponding groups. The χ^2 statistic is found to be 6.14, which is not significant at the 10 percent level.

EXHIBIT 5

Comparison of Different Efficiency Ratings

(A) Translog Ratings	DEA Technical Efficiency Ratings: Highest $h_0^* = 1$	Medium $0.9 \leq h_0^* < 1$	Lowest $h_0^* < 0.9$	Total
Highest	23 (17.8)	12 (14.6)	10 (12.6)	45
Medium	9 (14.6)	15 (12.0)	13 (10.4)	37
Lowest	13 (12.6)	10 (10.4)	9 (9.0)	32
	45	37	32	114

$\chi^2 = 6.14$ with 4 d.f., $p > 0.10$.

(B) Translog Ratings	DEA Technical and Scale Efficiency Ratings: Highest	Medium	Lowest	Total
Highest	24 (17.8)	12 (14.6)	9 (12.6)	45
Medium	9 (14.6)	18 (12.0)	10 (10.4)	37
Lowest	12 (12.6)	7 (10.4)	13 (9.0)	32
	45	37	32	114

$\chi^2 = 11.79$ with 4 d.f., $p < 0.05$.

(C) DEA Technical Efficiency Ratings	DEA Technical and Scale Efficiency Ratings: Highest	Medium	Lowest	Total
Highest	37 (17.8)	5 (14.6)	3 (12.6)	45
Medium	8 (14.6)	26 (12.0)	3 (10.4)	37
Lowest	0 (12.6)	6 (10.4)	26 (9.0)	32
	45	37	32	114

$\chi^2 = 105.76$ with 4 d.f., $p < 0.001$.

The numbers in parentheses indicate expected frequencies.

It may be noted, however, that the translog estimates are obtained from the constrained linear homogeneous model because the constant returns to scale hypothesis was not rejected. Therefore, we compare the translog estimates next with the comparable (linear homogeneous) DEA model in (8), which provides the estimates of combined technical and scale efficiencies. Categorizing the observations into three groups as above, we report the comparison in Exhibit 5B. The χ^2 statistic is now 11.79 which is significant at the 5 percent level, suggesting that the efficiency ratings from

EXHIBIT 6

Comparison of Efficiency Ratings and Capacity Utilization

(A) Capacity Utilization	DEA Technical Efficiency Ratings			
	Highest	Medium	Lowest	Total
Highest > 82.6%	22 (15.0)	16 (12.3)	0 (10.7)	38
Medium 71.3–82.6%	12 (15.0)	13 (12.3)	13 (10.7)	38
Lowest < 71.3%	11 (15.0)	8 (12.3)	19 (10.7)	38
	45	37	32	114

$\chi^2 = 25.27$ with 4 f.d., $p < 0.001$.

(B) Capacity Utilization	DEA Technical and Scale Efficiency Ratings			
	Highest	Medium	Lowest	Total
Highest > 82.6%	25 (15 0)	13 (12.3)	0 (10.7)	38
Medium 71.3–82.6%	10 (15 0)	21 (12.3)	7 (10.7)	38
Lowest < 71.3%	10 (15.0)	3 (12.3)	25 (10.7)	38
	45	37	32	114

$\chi^2 = 54.38$ with 4 d.f., $p < 0.001$.

(C) Capacity Utilization	Translog Technical Efficiency Ratings			
	Highest	Medium	Lowest	Total
Highest > 82.6%	15 (15.0)	15 (12.3)	8 (10.7)	38
Medium 71.3–82.6%	16 (15.0)	14 (12.3)	8 (10.7)	38
Lowest < 71.3%	14 (15.0)	8 (12.3)	16 (10.7)	38
	45	37	32	114

$\chi^2 = 6.46$ with 4 d.f., $p > 0.10$.

The numbers in parentheses indicate expected frequencies.

$$\text{Capacity Utilization} = \frac{\text{Total Patient Days}}{365 \times \text{Number of Beds}}.$$

the two techniques are in broad agreement. Exhibit 5C presents a comparison between the purely technical efficiency ratings and the combined technical and scale efficiency ratings from DEA. The χ^2 statistic is 105.76, which is significant at the 0.1 percent level.

Next we turn to a comparison of these efficiency estimates with the degree of observed capacity utilization, computed as the ratio of total number of patient days to 365 times the number of beds for each hospital. Exhibit 6A compares the technical efficiency ratings from DEA with capacity utilization, categorized into three equal sized groups. The χ^2 statistic of 25.27 is significant at the 0.1 percent level. This indicates that the degree of capacity utilization is closely related to the DEA measure of technical efficiency. The comparison between the combined technical and scale efficiency ratings and capacity utilization is reported in Exhibit 6B. The χ^2 statistic of 54.38 suggests a similar relationship. However, the comparison between the translog estimates and capacity utilization, reported in Exhibit 6C, does not reveal a close relationship. The χ^2 statistic of 6.46 is not significant at the 10 percent level.

While there is broad agreement between the translog and DEA efficiency estimates that impose linear homogeneity, the other differences in the two sets of estimates illustrate the divergence between the direct and indirect estimation of production correspondences. In particular, the absence of a statistical relationship between the translog estimates and capacity utilization, in contrast to the close correspondence of capacity utilization with DEA estimates, is of interest. This may be due to the assumption of no allocative inefficiency invoked in translog, which may not be valid. Furthermore, the estimates obtained from these two deterministic frontier estimation methods are sensitive to outliers, and possible specification, measurement or data errors can confound our inferences. These results suggest the need for further comparative studies of DEA and translog methods, and in particular the application of such models to synthetically generated data which would be the subject of an extensive Monte Carlo study.

7. Conclusion

In this paper, we have compared the characterization of cost and production correspondences through the use of the translog and DEA models applied to the same empirical data. Application of these models to a sample of North Carolina hospitals reveals several interesting differences and similarities in the results. In particular, we may infer from the translog estimates that constant returns to scale are present in the industry, while the DEA estimates identify a richer, and more diverse set of behavior. Both increasing returns and decreasing returns to scale in different segments of the production correspondence may be inferred from the DEA estimates.

Both models find the production of hospital care for children to be more resource intensive than the production of care for adults or the elderly. However, the technical efficiency estimates from the translog model do not appear to be as closely related to the degree of capacity utilization of individual hospitals as the corresponding DEA estimates. This could be because the assumption of no allocative inefficiency in the translog model may not be valid.

While there are numerous empirical studies employing the translog and DEA models, the literature is lacking in comparative studies of several well-known estimation models applied to the same empirical data. Our findings suggest the need for further studies using both empirical and simulated data which compare DEA with translog and other econometric methods, including stochastic frontier estimation models. A more detailed examination of the reasons for inconsistency in the estimates of different types of models is indicated, for the finding of constant rather than increasing/decreasing returns to scale can have significant public policy implications.[9]

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