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Analysis of trends in technical and allocative efficiency: An application to Texas public school districts

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Abstract

We present a new DEA based method to analyze efficiency trends over time and differences across subgroups in a panel data setting. We employ a result that the aggregate technical and allocative inefficiency score equals the technical inefficiency when input quantities are aggregated into a single total input cost variable, and develop test procedures to evaluate the presence of allocative inefficiency. We apply these methods to test for the presence of allocative inefficiency in Texas school districts over 1993–99, and analyze shifts and trends in both technical and allocative inefficiencies over time for different regions. Our empirical results indicate the existence of statistically significant allocative inefficiencies. While technical inefficiency increased over the six year sample period, allocative inefficiency remained relatively stable during this period. These results for the full sample obtain also when we repeat the analysis for different regions.

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1. Introduction

In twenty years since its inception, data envelopment analysis (DEA) has become a widespread analytical tool for evaluating the relative efficiency of comparable organizations (Seiford, 1996). While the original DEA models specify the production set relating inputs to outputs only in terms of properties such as convexity and monotonicity and do not impose any parametric structure on the distribution of the inefficiency of individual observations, statistical properties can be derived for the DEA estimator and a variety of statistical tests

can be devised if additional structure is specified (Banker, 1993, 1996).

When data are available in panel form with several observations for each decision making unit (DMU) in the sample over several time periods, the researcher may be interested in evaluating efficiency trends over time as well as across DMUs. Techniques such as window analysis (Charnes et al., 1985) and Malmquist index (Fare et al., 1994) have been used for this purpose. In this paper, we present an alternative way of analyzing efficiency trends over time and efficiency differences across DMUs in a panel data setting.

To illustrate our method, we analyze technical and allocative efficiency trends and differences over a six year period using data on 585 Texas Public School districts. Real expenditures for public K-12 education in the United States increased from \$2

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billion in 1890 to nearly \$190 billion in 1990 (Hanushek et al., 1994). During this period, the growth in education expenditures was more than three times the growth in the Gross National Product. As a result, considerable attention has centered on whether the public school system efficiently utilizes resources.

DEA had been used in the past to analyze the education production function (Bessent et al., 1982; Ahn et al., 1988; Ray, 1991) and more recently in Grosskopf et al. (1999). Many of these studies focus on the cross-sectional relationship between expenditures and student enrollment or expenditure per student and outcomes such as test scores. We contribute to this literature by providing a statistical analysis of efficiency scores and their trends for a large number of school districts over an extended period of time. We focus on two different dimensions of efficiency in a multiple output–multiple input production environment and examine the technical and allocative efficiencies for school districts located in three different geographical regions of Texas over a six year period.

We use the statistical tests developed in Banker (1993, 1996) to examine the trends in technical and allocative inefficiencies. We adapt the statistical tests developed in Banker et al. (1999) to examine the existence of allocative inefficiencies associated with input utilization. We employ the result that the DEA technical inefficiency measure using a single aggregate cost variable, constructed from multiple inputs weighted by their unit costs, reflects the aggregate technical and allocative inefficiency. Thus, our analysis provides an intuitive interpretation for the technical inefficiency estimated by studies that use aggregate cost expenditure information. Based on this analysis, we construct statistical tests of the null hypothesis of no allocative inefficiency analogous to those of the null hypothesis of no scale inefficiency described in Banker (1996).

Our empirical results indicate statistically significant allocative inefficiencies. While technical inefficiency increased over the six year sample period, allocative inefficiency remained relatively stable during this period. These results for the whole sample obtain also when we repeat the analysis for different regions. Pairwise comparison of the per-

formance of the three regions indicates that Texas school districts in the North region operate at the lowest levels of technical inefficiency followed by the South-East and finally the West region.

The rest of the paper is organized as follows. In Section 2, we describe the DEA methodology used in the estimation of technical and allocative inefficiencies using panel data. In Section 3, we describe our data and, in Section 4, we discuss the results of our empirical analysis. In Section 5, we present a summary and concluding remarks.

2. Evaluating shifts in technical and allocative inefficiencies using DEA

Let $Y_{jt} = (y_{1jt}, \dots, y_{rjt}, \dots, y_{Rjt}) \geq 0$ and $X_{jt} = (x_{1jt}, \dots, x_{ijt}, \dots, x_{Ijt}) \geq 0$, $j = 1, \dots, N$, $t = 1, \dots, T$, be the observed output and input vectors generated from an underlying production possibility set $\mathbf{S} = \{(X, Y) \mid \text{outputs } Y \text{ can be produced from inputs } X\}$ for a sample of N firms in the same industry over T time periods.¹ Each input i , $i = 1, \dots, I$, is bought by all firms in the same competitive market at a price p_i . Let $P = (p_1, \dots, p_I)$ be the vector of input prices. The cost of input i for firm j in period t is then $c_{ijt} = p_i x_{ijt}$. Denote the total cost of inputs for firm j in period t as $c_{jt} = \sum_i p_i x_{ijt}$.

The input-oriented technical inefficiency $\theta_{jt}^B \geq 1$ of an observation $(X_{jt}, Y_{jt}) \in \mathbf{S}$ for the period t , measured radially by the reciprocal of Shephard's (1970) distance function, is given by $\theta_{jt}^B \equiv \theta^B \times (X_{jt}, Y_{jt}) = \sup\{\theta \mid (X_{jt}/\theta, Y_{jt}) \in \mathbf{S}\}$. Assume that the production set \mathbf{S} is monotonically increasing [i.e. $(X_{jt}, Y_{jt}) \in \mathbf{S}, X_{kt} \geq X_{jt}, Y_{kt} \leq Y_{jt} \Rightarrow (X_{kt}, Y_{kt}) \in \mathbf{S}$] and convex [i.e. $(X_{jt}, Y_{jt}), (X_{kt}, Y_{kt}) \in \mathbf{S} \Rightarrow \lambda(X_{jt}, Y_{jt}) + (1 - \lambda)(X_{kt}, Y_{kt}) \in \mathbf{S}$ for all $0 \leq \lambda \leq 1$] and that the probability density function $f^B(\theta^B)$ is such that $f^B(\theta^B) = 0$ if $\theta < 1$ and $\int_1^{1+\delta} f^B(\theta^B) d\theta^B > 0$ for $\delta > 0$.² Then, following Banker (1993), a consis-

¹ For each observation, at least one output y_{rj} and one input x_{ij} are assumed to be strictly positive with a positive price.

² The data generating process comprises the generation of the output vector Y_j , the direction vector representing the relative mix of inputs, and the scalar θ_j^B that determines the distance traversed by the observed input vector X_j along the direction vector representing the relative input mix.

tent estimator of θ_{jt}^B is obtained as $\hat{\theta}_{jt}^B$ by solving the following Banker et al. (BCC 1984) model:

$$\hat{\theta}_{jt}^B = \text{Max } \theta \tag{1.0}$$

subject to

$$\sum_t \sum_k \lambda_{kt} y_{rkt} \geq y_{rjt} \quad \forall r = 1, \dots, R, \tag{1.1}$$

$$\sum_t \sum_k \lambda_{kt} x_{ikt} \leq x_{ijt} / \theta \quad \forall i = 1, \dots, I, \tag{1.2}$$

$$\sum_t \sum_k \lambda_{kt} = 1, \tag{1.3}$$

$$\theta, \lambda_{kt} \geq 0 \quad \forall k = 1, \dots, K, \quad t = 1, \dots, T. \tag{1.4}$$

2.1. Measuring and evaluating allocative inefficiency

Banker and Maindiratta (1988) describe the calculation of aggregate, technical and allocative inefficiency using DEA-based linear programs to ascertain whether the set of observed output, input and price data is not consistent with profit maximization (or cost minimization) for at least one firm in the sample. They define allocative inefficiency as aggregate inefficiency divided by technical inefficiency. Here, we address the situation when information about input prices is not available, except for the knowledge that the firms procure the inputs in the same competitive market place. We show how an aggregate technical and allocative inefficiency measure, equivalent to the one described by Banker and Maindiratta (1988) can be calculated when the available data consists of only quantities of output and total costs of various inputs. We measure allocative inefficiency as aggregate inefficiency divided by technical inefficiency as in Banker and Maindiratta (1988). More importantly, we develop statistical tests, along the lines of those developed for the revenue-maximizing case in Banker et al. (1999), to determine the presence of allocative inefficiency in the observed sample of DMUs.

We first define a new production set $\bar{\mathbf{S}}$ derived from the original production set \mathbf{S} , for a given input price vector P . This derived set is based on the aggregate cost, $c_t = \sum_i p_i x_{it} > 0$, as a single input

and the same multiple outputs as in \mathbf{S} . Formally, we define $\bar{\mathbf{S}} = \{(c_t, Y_t) \mid \text{there exists some } (X_t, Y_t) \in \mathbf{S} \text{ such that } c_t = P'X_t = \sum_i p_i x_{it}\}$. It immediately follows that if \mathbf{S} is a monotone increasing and convex set, then so is $\bar{\mathbf{S}}$.

We next consider the two measures defined by $\theta_{jt}^A = \text{Max}\{\sum_i p_i x_{ijt} / \sum p_i x_i \mid (X_{jt}, Y_{jt}) \in \mathbf{S}\}$ and $\theta_{jt}^Z = \text{Max}\{\theta \mid (c_{jt}/\theta, Y_{jt}) \in \bar{\mathbf{S}}\}$. Here, θ_{jt}^A is the aggregate technical and allocative inefficiency, for the observation $(X_{jt}, Y_{jt}) \in \mathbf{S}$, as defined by Banker and Maindiratta (1988), relative to the production set \mathbf{S} , while θ_{jt}^Z is the technical inefficiency for the observation $(c_{jt}, Y_{jt}) \in \bar{\mathbf{S}}$, as defined by Banker et al. (1984), relative to the production set $\bar{\mathbf{S}}$. Also, since $(X_{jt}/\theta_{jt}^B, Y_{jt}) \in \mathbf{S}$, $\theta_{jt}^B = \sum_i p_i x_{ijt} / \sum_i p_i (x_{ijt}/\theta_{jt}^B) \leq \theta_{jt}^A$. It is straightforward to adapt the proof by Banker et al. (1999) to show that $\theta_{jt}^Z = \theta_{jt}^A \geq \theta_{jt}^B$ and the probability density function $f^Z(\theta^Z)$ is such that $f^Z(\theta^Z) = 0$ for all $\theta^Z < 1$ and $\int_1^{1+\delta} f^Z(\theta^Z) d\theta^Z > 0$ for all $\delta > 0$.

The allocative inefficiency of an observation (X_j, Y_j) relative to the set \mathbf{S} is then given by $\theta_{jt}^V = \theta_{jt}^A / \theta_{jt}^B = \theta_{jt}^Z / \theta_{jt}^B \geq 1$. The result $\theta_{jt}^Z = \theta_{jt}^A \geq \theta_{jt}^B$ holds true for any monotone increasing and convex set \mathbf{S} and the corresponding derived set $\bar{\mathbf{S}}$. In particular, it holds for the DEA free disposable convex hull of sample observations and the corresponding derived set based on aggregate cost observations. Specifically, consider the following linear program, used by Banker and Maindiratta (1988), to estimate aggregate technical and allocative efficiency:

$$\text{Max } \hat{\theta}_{jt}^A = \sum_i p_i x_{ijt} / \sum_i p_i x_i \tag{2.0}$$

subject to

$$\sum_t \sum_k \lambda_{kt} y_{rkt} \geq y_{rjt} \quad \forall r = 1, \dots, R, \tag{2.1}$$

$$\sum_t \sum_k \lambda_{kt} x_{ikt} \leq x_{ijt} \quad \forall i = 1, \dots, I, \tag{2.2}$$

$$\sum_t \sum_k \lambda_{kt} = 1, \tag{2.3}$$

$$\theta, \lambda_{kt} \geq 0 \quad \forall k = 1, \dots, K, \quad t = 1, \dots, T. \tag{2.4}$$

Consider also the linear program in (1), after replacing the I constraints in (1.2) by a single constraint, $\sum_t \sum_k \lambda_{kt} c_{kt} \leq c_{jt}/\theta$ for the aggregate cost. That is, consider the following modified version of (1):

$$\hat{\theta}_{jt}^Z = \text{Max } \theta \tag{3.0}$$

subject to

$$\sum_t \sum_k \lambda_{kt} y_{rkt} \geq y_{rjt} \quad \forall r = 1, \dots, R, \tag{3.1}$$

$$\sum_t \sum_k \lambda_{kt} c_{kt} \leq c_{jt}/\theta, \tag{3.2}$$

$$\sum_t \sum_k \lambda_{kt} = 1, \tag{3.3}$$

$$\theta, \lambda_{kt} \geq 0 \quad \forall k = 1, \dots, K, \quad t = 1, \dots, T. \tag{3.4}$$

Applying the result $\theta_{jt}^Z = \theta_{jt}^A$ to the DEA estimators $\hat{\theta}_{jt}^A$ and $\hat{\theta}_{jt}^Z$ we observe that the estimated aggregate technical and allocative inefficiency $\hat{\theta}_{jt}^A$, calculated using (2), is identical to $\hat{\theta}_{jt}^Z$, the estimated technical inefficiency relative to the derived production set with a single aggregate output, calculated using (3). Further, using a proof analogous to that in Banker et al. (1999), it can be shown that $\hat{\theta}_{jt}^Z$ is a consistent estimator of θ_{jt}^Z , the technical inefficiency of observation j in period t relative to the derived production set \bar{S} with the same multiple outputs in the original production set S and the aggregate cost as a single input.

Combining the earlier result ($\theta_{jt}^A = \theta_{jt}^Z$) and the above result, it can be seen that $\hat{\theta}_{jt}^Z$ is a statistically consistent estimator of θ_{jt}^A , the aggregate technical and allocative inefficiency measure. Since the constraint (3.2) can be obtained by summing the I constraints in (1.2) weighted by the corresponding prices, p_i , and the remaining structure of the program in (3) is identical to that in (1), it is evident that $\hat{\theta}_{jt}^B$ is a feasible solution to (3), and therefore $\hat{\theta}_{jt}^B \leq \hat{\theta}_{jt}^Z$. The estimator for the allocative inefficiency, $\hat{\theta}_{jt}^V$, is then calculated as $\hat{\theta}_{jt}^Z/\hat{\theta}_{jt}^B$.

Based on Banker (1993) and analogous to the tests of no scale inefficiency described in Banker (1996), we develop a variety of procedures to test the null hypothesis of no allocative inefficiency against the alternative of the presence of such inefficiency in a particular period t based on assumed

structure for the distribution of $\tau(\theta)$, where $\tau(\cdot)$ is an appropriate transformation function. The following illustrate the test procedures for the case when $\tau(\cdot) = \ln(\cdot)$ and $\tau(\cdot) = (\ln(\cdot))^2$:

- (i) If $\ln(\theta_{jt}^B)$ is distributed as exponential over $[0, \infty)$, then under the null hypothesis of no allocative inefficiency, the test statistic is calculated as $\sum_{j=1}^N \ln(\hat{\theta}_{jt}^Z) / \sum_{j=1}^N \ln(\hat{\theta}_{jt}^B)$ and evaluated relative to the critical value of the half- F distribution with $(2N, 2N)$ degrees of freedom.
- (ii) If $\ln(\theta_{jt}^B)$ is distributed as half-normal over the range $[0, \infty)$, then under the null hypothesis of no allocative inefficiency, the test statistic is calculated as $\sum_{j=1}^N (\ln(\hat{\theta}_{jt}^Z))^2 / \sum_{j=1}^N (\ln(\hat{\theta}_{jt}^B))^2$ and evaluated relative to the critical value of the half- F distribution with (N, N) degrees of freedom.
- (iii) If no such assumptions are maintained about the probability distribution of inefficiency, a non-parametric Smirnov's test statistic given by the maximum vertical distance between $F(\ln(\hat{\theta}_{jt}^Z))$ and $F(\ln(\hat{\theta}_{jt}^B))$, the empirical distributions of $\ln(\hat{\theta}_{jt}^Z)$ and $\ln(\hat{\theta}_{jt}^B)$, respectively, is used. This statistic, by construction, takes values between 0 and 1 and a high value is indicative of the existence of allocative inefficiency.

However, as Banker (1993, 1996) points out, while $\ln(\hat{\theta}_{jt}^Z)$ and $\ln(\hat{\theta}_{jt}^B)$ are asymptotically independent of each other, they need not be independently distributed for finite samples. For finite samples, they need not also follow the true distribution of $\ln(\theta_{jt}^Z)$ and $\ln(\theta_{jt}^B)$. An important caveat, therefore, is that the above tests are designed for large samples, their small sample performance needs to be evaluated with Monte Carlo experimentation.

2.2. Evaluating shifts in inefficiencies over time and across groups

To examine shifts in inefficiencies over time or across groups in the sample we use test statistics similar to those described in Banker (1993) and Banker and Chang (1995) to evaluate the null

hypothesis of no difference in the inefficiency distributions of two sub-samples. For N_1 and N_2 , DMUs in subgroups G_1 and G_2 , respectively, the null hypothesis of no difference in technical inefficiency between the two subgroups can be tested using the following procedures:

- (i) If $\ln(\theta_{jt}^B)$ is distributed as exponential over $[0, \infty)$, then under the null hypothesis that there is no difference between the two groups, the test statistic is calculated as

$$\sum_{j \in G_1} (\ln(\hat{\theta}_{jt}^B)/N_1) / \sum_{j \in G_2} (\ln(\hat{\theta}_{jt}^B)/N_2)$$

and evaluated relative to the critical value of the F distribution with $(2N_1, 2N_2)$ degrees of freedom.

- (ii) If $\ln(\theta_{jt}^B)$ is distributed as half-normal over the range $[0, \infty)$, then under the null hypothesis that there is no difference between the two groups, the test statistic is calculated as

$$\sum_{j \in G_1} ((\ln(\hat{\theta}_{jt}^B))^2/N_1) / \sum_{j \in G_2} ((\ln(\hat{\theta}_{jt}^B))^2/N_2)$$

and evaluated relative to the critical value of the F distribution with (N_1, N_2) degrees of freedom.

- (iii) If no such assumptions are maintained about the probability distribution of inefficiency, a non-parametric Smirnov's test statistic given by the maximum vertical distance between $F^{G_1}(\ln(\hat{\theta}_{jt}^B))$ and $F^{G_2}(\ln(\hat{\theta}_{jt}^B))$, the empirical distributions of $\ln(\hat{\theta}_{jt}^B)$ for the groups G_1 and G_2 , respectively, is used. This statistic, by construction, takes values between 0 and 1 and a high value for this statistic is indicative of significant differences in technical inefficiency between the two groups.³

The above tests can be suitably modified to examine shifts in inefficiencies between two differ-

ent time periods k and m for DMUs in a particular subgroup, say G_1 . Finally, while the above tests were described for estimated technical inefficiencies, analogous tests can be constructed for estimated allocative efficiencies. We use these various tests in our empirical analysis that follows to examine trends and changes in the efficiency of Texas school districts.

Additional tests can be constructed based on the multivariate distribution of inefficiency scores retrieved empirically from DEA estimation. We can specify a suitable structure to represent the evolution of inefficiency scores over time to reflect potential serial correlation and covariation, and estimate its parameters in a second stage procedure based on the empirical distribution of the estimated inefficiency scores. While differences over time and across groups can be evaluated in this manner, in this paper we shall restrict our attention to relatively simple test procedures employed in prior DEA literature.

3. Education production functions and Texas school districts data

Many studies have been undertaken over the last three decades to estimate cost and production functions in education. Hanushek (1979, 1986) provides comprehensive reviews of the literature on the educational production function. Until the early 1980s single output educational production functions were estimated using ordinary least squares (Polachek et al., 1978). More recent work recognizes the need to specify a production function with multiple outputs (Hanushek, 1979, 1986; Bessent et al., 1982; Chizmar and Zak, 1983; Callan and Santerre, 1990; Ray, 1991). To estimate multiple output production technologies, studies in education have employed the translog cost function (Jimenez, 1986; Gyimah-Brempong and Gyapong, 1992) or DEA (Bessent et al., 1982; Ahn et al., 1988; Fare et al., 1989; Ray, 1991) methodologies.

Research on educational production functions can be divided into two broad groups. The first group includes several studies, such as Butler and Monk (1985), Ahn et al. (1988), and

³ In our empirical tests we report the value for the asymptotic statistic KS_a . This statistic is given by $0.5D/\sqrt{N_1 + N_2}$ where D is the maximum vertical distance and N_1 and N_2 are DMUs in subgroups G_1 and G_2 , respectively.

Cohn et al. (1989), which involve the estimation of the input–output production correspondence to evaluate the relative efficiency of educational institutions. The second group includes Gyimah-Brempong and Gyapong (1992), Ray (1991) and others that focus on the input and outcome (rather than output) relation to evaluate effectiveness. Input measures used typically include instructional expenditures, full time equivalents of teachers, and other physical measures such as number of square feet of space. Output measures generally comprise full time student equivalents. Outcome measures include scores from standard achievement tests, or other proficiency tests.

From an application point of view, our study belongs to the first group of studies such as Butler and Monk (1985), Ahn et al. (1988), and Cohn et al. (1989). In the spirit of these studies, we concentrate on the production correspondence between outputs (student enrollment) and inputs (operating expenditures). More importantly, we use the estimated production correspondence to analyze shifts and trends in inefficiencies across regions as well as over time.

The data for this study is collected from the AEIS (Academic Excellence Indicator System) reports provided to the public by the Texas Education Agency (TEA). These reports are an integral part of the school accountability system in Texas. Texas was one of the five states to earn a spot on the Thomas B. Fordham Foundation's "honor roll" in 1999 because of its "solid academic standards" and "strong school accountability system". TEA has been making available to the public detailed data on students, staff, performance, revenue and expense data since 1990–1991. Six years of data, for the period 1993–94 to 1998–99, both academic years inclusive, are currently available from TEA in electronic form. We use these six years of data in our analysis.

In 1998–99, Texas had 1042 independent school districts. The districts are serviced by 20 regional education service centers. Total student enrollment in Texas schools was 3.95 million and the school districts spent a total of \$20.5 billion in operating expenditures. We collect and analyze data at the school district level because resource allocation decisions are generally made at this level. TEA

mentions in its "Snapshot'98" report that while the SBOE (State Board of Education) and the commissioner of education provide leadership for education, much of the control of public schools resides with the local districts and charters.

We measure three outputs: total student enrollment in elementary schools (K–5), middle schools (6–8) and high schools (9–12) in every district. These output measures are denoted as ELEMENTARY, MIDDLE and HIGH. We use three types of operating expenditures as our input variables. All expenditures are converted into 1993–94 dollars using the CPI (U) data for the Southern Region for the sample period available from the Bureau of Labor Statistics (BLS). TEA provides district-level operating expenditures for various groups of functions. We condense these various categories into three broad groups of expenditures comparable over the six years of sample data. The three inputs are (a) INSTRUCTION: measured as the expenditures associated with all activities dealing directly with the interaction between teachers and students, including instruction aided with computers (b) ADMINISTRATION: measured as the sum of expenditures related to instructional, campus and central administration and data processing services and (c) SUPPORT: measured as the sum of expenditures corresponding to instruction related services, student support services, student transportation, food services, school-sponsored cocurricular or extracurricular activities, plant maintenance and operations and security and monitoring services. Our choice of outputs and inputs is in line with measures used in prior research (Butler and Monk, 1985; Jimenez, 1986; Ahn et al., 1988; Callan and Santerre, 1990; Ray, 1991).

We impose the following restrictions to arrive at our final sample: (i) All output and input data for the district should be available for all six years of the sample period. (ii) There should be at least one student enrolled in each of the grades K–12. (iii) There should be at least 50 students in both the 6th and 10th grades during the academic year 1998–99. This restriction is identical to that used by Grosskopf et al. (1999) to avoid sampling problems.

Our final sample consists of a balanced panel of data for 585 Texas school districts over a six year period from 1993 to 1999. We assign the school



Fig. 1. The three regions of Texas school districts.

districts in our sample to one of three regions, SOUTH-EAST, NORTH or WEST, described in the map of Texas in Fig. 1, by consolidating the geographical regions served by the 20 regional education service centers (ESCs). For this purpose, ESCs 1–6, 7–12 and 13–20, respectively were assigned to the regions SOUTH-EAST, NORTH and WEST, respectively. Based on 1998–99 student enrollment, the three regions are of similar

size accounting for 38%, 34% and 28% of total students in the state. With a total enrollment of 3.64 million students and total operating expenditures of \$17.2 billion, the 585 school districts account for 92% of the total students in the state and 84% of the total expenditures.

Table 1 provides descriptive statistics on the output and input measures for the year 1993–94. The distributions were similar for other years except for growth in student enrollment as well as real expenditures which we will discuss shortly. The representative median school in the sample had a total student enrollment of 1880 and total operating expenditures of \$8.11M. Elementary, Middle and High school enrollments were 47%, 25% and 28% of the total student body, respectively. Instruction accounted for 59% of the operating expenditures. Support expenses with 30% and administration with 11% accounted for the remainder. In 1993–94, the Texas school districts spent \$4419 per student.

Table 2 provides six years of data for statewide averages for the various measures. The total student enrollment grew by a total of 9.0% during the sample period. Total expenditures, though, grew faster, by 16.5%. As a result, the cost per student in real dollars grew by 9.2% and was \$4901 in 1998–99. High school student enrollment grew at a much faster pace compared to the total student

Table 1
Descriptive statistics on student enrollment and expenditures (based on 1993–94 data, $N = 585$)

	Mean	Std. Dev.	25%	Median	75%
Total students	5664	13,101	1066	1880	4560
Elementary	2790	6794	498	892	2173
Middle	1371	3055	271	477	1102
High	1502	3274	296	527	1214
Elementary (%)	47.50	2.60	45.92	47.43	49.08
Middle (%)	24.88	1.37	23.96	24.79	25.76
High (%)	27.62	2.32	26.21	27.53	28.93
Total expenses ^a	\$25.28 M	\$60.34 M	\$4.82 M	\$8.11 M	\$19.49 M
Instruction ^a	\$14.83 M	\$35.61 M	\$2.84 M	\$4.75 M	\$11.33 M
Administration ^a	\$3.24 M	\$7.74 M	\$0.65 M	\$1.10 M	\$2.40 M
Support ^a	\$7.21 M	\$17.09 M	\$1.39 M	\$2.40 M	\$5.68 M
Instruction (%)	58.42	2.85	56.66	58.63	60.34
Administration (%)	13.04	1.59	11.96	12.91	14.00
Support (%)	28.54	2.48	26.86	28.41	30.08
Expense per student	\$4490	\$601	\$4104	\$4419	\$4814

^aThe symbol M indicates that the amounts are presented in millions of dollars.

Table 2
Trends in student enrollment and expenditures

	1993–94	1994–95	1995–96	1996–97	1997–98	1998–99
Total students	5664	5762	5870	6003	6101	6176
Elementary	2790	2819	2863	2902	2938	2974
Middle	1371	1395	1403	1436	1450	1464
High	1502	1549	1604	1665	1713	1739
Elementary (%)	47.50	46.93	46.67	46.22	46.02	45.93
Middle (%)	24.88	24.87	24.63	24.48	24.24	24.22
High (%)	27.62	28.19	28.70	29.30	29.74	29.85
Total expenses ^a	\$25.28 M	\$25.85 M	\$27.01 M	\$26.58 M	\$28.30 M	\$29.46 M
Instruction ^a	\$14.83 M	\$15.15 M	\$15.84 M	\$15.42 M	\$16.36 M	\$16.95 M
Administration ^a	\$3.24 M	\$3.31 M	\$3.50 M	\$3.17 M	\$3.44 M	\$3.62 M
Support ^a	\$7.21 M	\$7.39 M	\$7.67 M	\$8.00 M	\$8.50 M	\$8.89 M
Instruction (%)	58.42	58.47	58.72	57.93	58.02	57.56
Administration (%)	13.04	12.84	12.81	12.10	12.10	12.30
Support (%)	28.54	28.68	28.47	29.97	29.88	30.14
Expense per student	\$4490	\$4490	\$4647	\$4527	\$4764	\$4901

^a The symbol M indicates that the amounts are presented in millions of dollars.

enrollment. This may have been a contributing factor to the higher cost per student in 1998–99.

4. Efficiency analysis

We apply DEA to the full panel of 3510 observations for 585 districts over six years. We calculated two inefficiency measures for each school district for every year. The first is the technical inefficiency estimator, $\hat{\theta}_{jt}^B$, calculated using the program in (1). The program is based on the three student outputs and the three inputs. We

also calculate the aggregate inefficiency estimator, $\hat{\theta}_{jt}^Z$, using the program in (3). We replace the three individual input constraints by a single constraint using the total expenditure as a single input variable. As described in Section 2, the technical inefficiency estimator for the three output and the single aggregate input case, $\hat{\theta}_{jt}^Z$, is identical to the aggregate technical and allocative inefficiency for the three output and three input case. We calculate the allocative inefficiency $\hat{\theta}_{jt}^V$ for each school district j in year t as $\hat{\theta}_{jt}^Z / \hat{\theta}_{jt}^B$.

Table 3 provides the distribution of estimated technical and allocative inefficiencies on a year by

Table 3
Distribution of technical and allocative inefficiencies

	Mean	Std. Dev.	25%	Median	75%
<i>Panel A: Technical</i>					
1993–94	1.305	0.188	1.178	1.285	1.400
1994–95	1.304	0.179	1.188	1.286	1.402
1995–96	1.346	0.176	1.221	1.326	1.454
1996–97	1.294	0.166	1.181	1.281	1.390
1997–98	1.355	0.179	1.234	1.338	1.458
1998–99	1.391	0.188	1.274	1.372	1.493
<i>Panel B: Allocative</i>					
1993–94	1.031	0.032	1.009	1.024	1.041
1994–95	1.031	0.031	1.010	1.022	1.040
1995–96	1.030	0.030	1.009	1.022	1.040
1996–97	1.037	0.039	1.010	1.027	1.051
1997–98	1.038	0.038	1.010	1.029	1.054
1998–99	1.037	0.038	1.010	1.028	1.056

Table 4
Tests of existence of allocative inefficiency in Texas school districts—based on full sample

Year	Banker's sum ratio test	Banker's sum of squares ratio test	Kolmogorov–Smirnov test
1993–94	1.12*	1.19**	1.93**
1994–95	1.12*	1.19**	2.10**
1995–96	1.10*	1.17**	1.87**
1996–97	1.14**	1.25**	2.34**
1997–98	1.12*	1.22**	2.40**
1998–99	1.11*	1.20**	2.40**

$N = 585$. Test statistics significant at 5% (10%) level are marked with ** (*).

year basis. The distributions are quite revealing. The school districts in Texas appear to be operating under significant technical inefficiency. The median school district was operating at an inefficiency level of 1.285 (this corresponds to 78% efficiency) in 1993–94. There also seems to have been a steady increase in technical inefficiency over time. In 1998–99, the median technical inefficiency had increased to 1.372 (73% efficiency). Lower quartile values of technical inefficiency of the order of 1.2 for all the six years suggest that the technical inefficiency is quite widespread. The allocative inefficiencies are much lower. The median school district was operating at an inefficiency level of 1.024 (98% allocative efficiency) in 1993–94. There appears to be no significant trend in the allocative inefficiency levels over time. Finally, there also appears to be a significant departure from the steady increase in technical inefficiency in 1996–97. While the distributions are

quite informative, additional assumptions on the inefficiency variables are necessary to make inferences about shifts, trends and changes in efficiencies.

Table 4 describes the statistical analysis testing for the existence of allocative inefficiency in school districts. Recall from Section 2 that these tests are based on a comparison of the observed distribution of aggregate and technical inefficiencies because the true distributions for the two are identical under the null hypothesis that there is no allocative inefficiency. We provide details on three different tests. These tests, namely, Banker's (1993, 1996) sum ratio and sum of squares ratio tests and the Kolmogorov–Smirnov test were described earlier in Section 2. The tests conclusively reject the null hypothesis of no allocative inefficiency for all six years. Together, they indicate that although the allocative inefficiencies are small in magnitude they are statistically significant.

Table 5
Tests of shifts over time in technical and allocative inefficiency in Texas school districts

Year	Banker's sum ratio test	Banker's sum of squares ratio test	Kolmogorov–Smirnov test
<i>Panel A: Technical inefficiency</i>			
1993–94 vs. 1994–95	1.00	1.02	0.52
1994–95 vs. 1995–96	0.89**	0.83**	2.46**
1995–96 vs. 1996–97	1.16**	1.28**	2.51**
1996–97 vs. 1997–98	0.85**	0.75**	2.81**
1997–98 vs. 1998–99	0.92	0.86*	1.96**
1993–94 vs. 1998–99	0.80**	0.70**	4.15**
<i>Panel B: Allocative inefficiency</i>			
1993–94 vs. 1994–95	1.02	1.06	0.64
1994–95 vs. 1995–96	1.02	1.04	0.50
1995–96 vs. 1996–97	0.81**	0.63**	1.93**
1996–97 vs. 1997–98	0.99	1.01	0.56
1997–98 vs. 1998–99	1.01	1.02	0.50
1993–94 vs. 1998–99	0.84**	0.72**	2.13**

$N = 585$. Test statistics significant at 5% (10%) level are marked with ** (*).

Table 6
Trends in regional averages of student enrollment and expenditures

	1993–94	1994–95	1995–96	1996–97	1997–98	1998–99
<i>Panel A: South-East (N = 182)</i>						
Total students	7158	7267	7370	7538	7657	7747
Elementary	3487	3515	3574	3633	3693	3756
Middle	1749	1772	1752	1793	1800	1812
High	1922	1980	2044	2112	2164	2180
Total expenses ^a	\$32.90 M	\$33.65 M	\$35.30 M	\$34.02 M	\$36.25 M	\$37.58 M
Instruction ^a	\$19.06 M	\$19.51 M	\$20.49 M	\$19.54 M	\$20.71 M	\$21.41 M
Administration ^a	\$4.16 M	\$4.25 M	\$4.53 M	\$4.04 M	\$4.35 M	\$4.54 M
Support ^a	\$9.69 M	\$9.89 M	\$10.27 M	\$10.44 M	\$11.18 M	\$11.63 M
Expense per student	\$4668	\$4653	\$4808	\$4614	\$4844	\$5000
<i>Panel B: North (N = 233)</i>						
Total students	4688	4795	4917	5053	5163	5251
Elementary	2347	2387	2440	2486	2521	2550
Middle	1124	1148	1166	1199	1227	1256
High	1217	1259	1311	1367	1415	1445
Total expenses ^a	\$19.85 M	\$20.42 M	\$21.41 M	\$21.74 M	\$23.21 M	\$24.38 M
Instruction ^a	\$11.79 M	\$12.08 M	\$12.68 M	\$12.73 M	\$13.52 M	\$14.13 M
Administration ^a	\$2.57 M	\$2.64 M	\$2.78 M	\$2.58 M	\$2.89 M	\$3.09 M
Support ^a	\$5.49 M	\$5.70 M	\$5.95 M	\$6.42 M	\$6.80 M	\$7.16 M
Expense per student	\$4248	\$4247	\$4419	\$4357	\$4583	\$4683
<i>Panel C: West (N = 170)</i>						
Total students	5400	5478	5572	5661	5720	5764
Elementary	2652	2665	2683	2691	2701	2717
Middle	1306	1329	1354	177	182	1379
High	143	1483	1535	1593	1637	1668
Total expenses ^a	\$24.56 M	\$24.93 M	\$25.80 M	\$25.26 M	\$26.76 M	\$27.73 M
Instruction ^a	\$14.46 M	\$14.67 M	\$15.20 M	\$14.67 M	\$15.59 M	\$16.04 M
Administration ^a	\$3.18 M	\$3.23 M	\$3.37 M	\$3.04 M	\$3.20 M	\$3.37 M
Support ^a	\$6.92 M	\$7.03 M	\$7.23 M	\$7.55 M	\$7.97 M	\$8.32 M
Expense per student	\$4633	\$4649	\$4787	\$4666	\$4928	\$5094

^a The symbol M indicates that the amounts are presented in millions of dollars.

We next turn our attention to testing for shifts and trends in inefficiencies over time. Table 5 provides details on the tests used to examine trends in technical and allocative inefficiencies over time. The increases in technical inefficiency between consecutive years except the change from 1995–96 to 1996–97, are all statistically significant. The decline in technical inefficiency from 1995–96 to 1996–97 is also significant.⁴ Overall,

however, the technical inefficiency of Texas school districts, over a six-year period, has increased considerably. Allocative inefficiency has remained relatively stable from year to year, except for a significant increase from 1995–96 to 1996–97. This appears to be the main reason for the overall increase in allocative inefficiency from 1993–94 to 1998–99.

Next we focus our attention on analysis at the regional level. Our objective is to examine whether region-specific patterns exist in school district performance. Table 6 provides details on trends in regional averages of student enrollment and expenditures. The school districts in the South-East region are considerably bigger than in the other two regions and North has the lowest number of

⁴ TEA changed its categories of operating expenditures in 1996–97. It is possible that, as a result, some items of expenditures were classified as non-operating expenditures in 1996–97. It is worthwhile noting that operating expenditures per student declined from \$4647 in 1995–96 to \$4527 in 1996–97.

students per school district. North also appears to have a much lower cost per student. While cost per student in real dollars has increased over time for all three regions, all of them show a decline in cost per student from 1995–96 to 1996–97. This pattern is consistent with that observed for the full sample.

Table 7 provides details on trends in mean inefficiencies by region. The changes over time ob-

served for the whole sample also obtain at the regional level. Additionally, it is seen that North has the lowest technical inefficiency of the three regions followed by South-East and then West. No specific pattern could be observed for changes in allocative inefficiency across regions. Table 8 provides details on tests of existence of allocative inefficiency by region over time. The test results confirm the widespread presence of allocative

Table 7
Trends in mean technical and allocative inefficiencies

	1993–94	1994–95	1995–96	1996–97	1997–98	1998–99
<i>Panel A: Technical</i>						
Full sample	1.305	1.304	1.346	1.294	1.355	1.391
South-East	1.346	1.342	1.384	1.309	1.363	1.399
North	1.236	1.237	1.285	1.251	1.310	1.344
West	1.355	1.357	1.390	1.337	1.406	1.448
<i>Panel B: Allocative</i>						
Full sample	1.031	1.031	1.030	1.037	1.038	1.037
South-East	1.032	1.031	1.029	1.038	1.039	1.041
North	1.033	1.031	1.030	1.039	1.040	1.037
West	1.028	1.029	1.031	1.034	1.032	1.032

Table 8
Tests of existence of allocative inefficiency—regional analysis

Year	Banker's sum ratio test	Banker's sum of squares ratio test	Kolmogorov–Smirnov test
<i>Panel A: South-East (N = 182)</i>			
1993–94	1.11	1.19	1.26*
1994–95	1.11	1.18	1.15
1995–96	1.09	1.16	1.42**
1996–97	1.14	1.25	1.83**
1997–98	1.12	1.23	1.62**
1998–99	1.12	1.23	1.52**
<i>Panel B: North (N = 233)</i>			
1993–94	1.16	1.25*	1.34**
1994–95	1.15	1.23	1.90**
1995–96	1.12	1.20	1.53**
1996–97	1.17*	1.30**	1.53**
1997–98	1.15	1.28*	1.71**
1998–99	1.13	1.23	1.67**
<i>PANEL C: West (N = 170)</i>			
1993–94	1.09	1.15	1.14
1994–95	1.10	1.16	1.30*
1995–96	1.09	1.16	1.36**
1996–97	1.12	1.20	1.14
1997–98	1.10	1.16	1.19
1998–99	1.09	1.15	1.08

Test statistics significant at 5% (10%) level are marked with ** (*).

Table 9
Tests of shift in technical efficiency over time—regional analysis

Year	Banker's sum ratio test	Banker's sum of squares ratio test	Kolmogorov–Smirnov test
<i>Panel A: South-East (N = 182)</i>			
1993–94 vs. 1994–95	1.01	1.02	0.31
1994–95 vs. 1995–96	0.90	0.85	1.42**
1995–96 vs. 1996–97	1.21**	1.40**	2.57**
1996–97 vs. 1997–98	0.87	0.77*	1.83**
1997–98 vs. 1998–99	0.92	0.86	1.47**
1993–94 vs. 1998–99	0.88	0.80	1.68**
<i>Panel B: North (N = 233)</i>			
1993–94 vs. 1994–95	0.99	1.01	0.51
1994–95 vs. 1995–96	0.84*	0.77**	1.95**
1995–96 vs. 1996–97	1.13	1.20	1.20
1996–97 vs. 1997–98	0.82**	0.74**	1.95**
1997–98 vs. 1998–99	0.91	0.85	1.39**
1993–94 vs. 1998–99	0.71**	0.58**	3.94**
<i>Panel C: West (N = 170)</i>			
1993–94 vs. 1994–95	0.99	1.01	0.54
1994–95 vs. 1995–96	0.92	0.88	1.36**
1995–96 vs. 1996–97	1.14	1.25	1.25**
1996–97 vs. 1997–98	0.85	0.75*	1.79**
1997–98 vs. 1998–99	0.92	0.86	1.08
1993–94 vs. 1998–99	0.81*	0.71**	2.12**

Test statistics significant at 5% (10%) level are marked with ** (*).

Table 10
Tests of shift in allocative efficiency over time—regional analysis

Year	Banker's sum ratio test	Banker's sum of squares ratio test	Kolmogorov–Smirnov test
<i>Panel A: South-East (N = 182)</i>			
1993–94 vs. 1994–95	1.03	1.01	0.79
1994–95 vs. 1995–96	1.08	1.21	0.63
1995–96 vs. 1996–97	0.76**	0.59**	1.57**
1996–97 vs. 1997–98	0.97	1.00	0.58
1997–98 vs. 1998–99	0.96	0.78**	0.63
1993–94 vs. 1998–99	0.78**	0.56**	1.52**
<i>Panel B: North (N = 233)</i>			
1993–94 vs. 1994–95	1.06	1.14	0.46
1994–95 vs. 1995–96	1.02	1.03	0.46
1995–96 vs. 1996–97	0.79**	0.59**	1.20
1996–97 vs. 1997–98	0.96	0.94	0.65
1997–98 vs. 1998–99	1.07	1.27*	0.51
1993–94 vs. 1998–99	0.88	0.83	1.02
<i>Panel C: West (N = 170)</i>			
1993–94 vs. 1994–95	0.94	0.98	0.60
1994–95 vs. 1995–96	0.96	0.90	0.49
1995–96 vs. 1996–97	0.90	0.77**	1.03
1996–97 vs. 1997–98	1.06	1.22	0.38
1997–98 vs. 1998–99	1.00	1.01	0.02
1993–94 vs. 1998–99	0.86	0.84	1.36

Test statistics significant at 5% (10%) level are marked with ** (*).

Table 11
Comparison of regions—technical inefficiency

Year	Banker's sum ratio test	Banker's sum of squares ratio test	Kolmogorov–Smirnov test
<i>Panel A: South-East (N=182) vs. North (N=233)</i>			
1993–94	1.42**	1.75**	3.29**
1994–95	1.39**	1.73**	3.30**
1995–96	1.30*	1.56**	3.32**
1996–97	1.21	1.34**	1.89**
1997–98	1.15	1.28**	1.72**
1998–99	1.14	1.26**	2.04**
<i>Panel B: North (N=233) vs. West (N=170)</i>			
1993–94	0.70**	0.52**	2.84**
1994–95	0.70**	0.52**	3.32**
1995–96	0.77*	0.60**	2.86**
1996–97	0.77*	0.62**	2.70**
1997–98	0.80	0.64**	2.96**
1998–99	0.80	0.64**	2.68**
<i>Panel C: West (N=170) vs. South-East (N=182)</i>			
1993–94	1.01	1.10	0.75
1994–95	1.03	1.11	0.91
1995–96	1.00	1.07	0.90
1996–97	1.07	1.20*	1.33*
1997–98	1.09	1.22*	1.63**
1998–99	1.09	1.23**	1.19

Test statistics significant at 5% (10%) level are marked with ** (*).

Table 12
Comparison of regions—allocative inefficiency

Year	Banker's sum ratio test	Banker's sum of squares ratio test	Kolmogorov–Smirnov test
<i>Panel A: South-East (N=182) vs. North (N=233)</i>			
1993–94	0.97	0.90	0.62
1994–95	1.00	1.02	0.78
1995–96	0.95	0.88	0.68
1996–97	0.98	0.88	0.60
1997–98	0.97	0.83*	0.73
1998–99	1.09	1.35**	0.72
<i>Panel B: North (N=233) vs. West (N=170)</i>			
1993–94	1.17	1.40**	1.07
1994–95	1.05	1.20*	0.98
1995–96	0.98	1.04	1.24*
1996–97	1.12	1.37**	0.63
1997–98	1.23	1.78**	0.85
1998–99	1.15	1.41**	0.74
<i>Panel C: West (N=170) vs. South-East (N=182)</i>			
1993–94	0.88	0.79	0.87
1994–95	0.95	0.82	0.63
1995–96	1.07	1.09	1.14
1996–97	0.91	0.83	0.81
1997–98	0.83	0.68**	1.05
1998–99	0.80	0.53**	0.93

Test statistics significant at 5% (10%) level are marked with ** (*).

inefficiencies in all the regions. One can, therefore, conclude that the allocative inefficiency is prevalent across the state with no single region contributing in a singular way.

We repeat the tests for examining shifts over time at the regional level. These tests are reported in Tables 9 and 10. The results reported in Table 9 indicate that the statistically significant increase in technical inefficiency observed at the state level is also a regional phenomenon. It also confirms the blip in 1995–96 to 1996–97. The overall increase for the six year period appears to be more in North compared to the other regions. In other words, the more efficient North region appears to be moving down to catch up with its less technically efficient counterparts! Table 10 results indicate that while allocative inefficiency has increased over time in all three regions, the increase in the South-East region is more pronounced than in the other regions. The year to year increases in allocative inefficiency are also much smaller than the increases observed in technical inefficiency.

Tables 11 and 12 provide details on whether the regions are different in the way they utilize their resources. We compare each region with each of the other two. The tests for technical inefficiency differences across regions confirm the pattern observed in Table 7. North has the lowest technical inefficiency for all six years. South-East, in a relative sense, has decreased its technical inefficiency in the last two years, 1997–98 and 1998–99. West, however, has consistently trailed the other two regions with high technical inefficiency. Table 12 indicates that there is no significant difference in allocative inefficiency between North and South-East. West has a lower allocative inefficiency when compared to both North and South-East.

5. Summary and conclusion

This paper contributes to the relatively sparse literature in DEA on the analysis of technical and allocative inefficiency for panel data settings. First, we adapted the tests presented in Banker et al. (1999) to assess the presence of allocative inefficiency in input consumption. We employed a result that the DEA technical inefficiency measure

using a single aggregate cost variable, constructed from multiple input quantities weighted by their unit costs, equals the aggregate technical and allocative inefficiency for the detailed multiple input model. We also presented new test procedures for researchers interested in examining time trends and inter-group differences in inefficiencies present in panel data.

We then demonstrated an application of the tests developed in this paper to Texas school district data for the period 1993–99, analyzing the trends in inefficiency in resource utilization by public schools in different regions. Our empirical results indicated the presence of statistically significant allocative inefficiencies. We found that median technical inefficiency increased from 1.285 to 1.372 over the six year period. Median allocative inefficiency remained relatively stable at around 1.025 over the six year period. We also discovered that Texas school districts in the North region operate at the lowest levels of technical inefficiency followed by the South-East and finally the West region.

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