

Unobservable outcomes and multiattribute preferences in the evaluation of managerial performance*

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Abstract. This paper employs a generalized principal-agent model to analyze accounting situations in which the outcome is not jointly observable and the principal's and agent's preferences are multiattribute in nature. This requires the consideration of accounting signals for risk-sharing (or insurance) information in addition to performance evaluation (or incentive) information. It is shown that precisely two factors determine whether a signal will be valuable in the agency relationship: Observability of the agent's effort and the principal's multivariate risk neutrality. Sufficient conditions for various accounting signals to have value are also developed. Furthermore, when multiple accounting signals are available, it is shown that under certain conditions, the insurance components of the multiple signals can be aggregated into a single aggregate insurance measure and the incentive components of the signals can be aggregated (via a different aggregation procedure) into another aggregate incentive measure.

Résumé. Les auteurs utilisent un modèle généralisé mandant-mandataire pour analyser les situations comptables dans lesquelles le résultat n'est pas observable conjointement et les préférences du mandant et du mandataire comportent, par nature, de multiples attributs. Cela exige la prise en considération d'indicateurs comptables relatifs à l'information sur le partage des risques (ou de l'assurance) en plus de l'information relative à l'évaluation du rendement (ou aux stimulants). Les auteurs démontrent que deux facteurs permettent de déterminer avec précision si un indicateur sera valable dans la relation de mandataire : le caractère observable de l'effort du mandataire et la neutralité multivariée du mandant à l'égard du risque. Des conditions suffisantes pour que les divers indicateurs comptables soient valables sont également établies. En outre, lorsque des indicateurs comptables multiples sont disponibles, les auteurs démontrent que dans certaines conditions, les éléments des indicateurs multiples liés à l'assurance peuvent faire l'objet d'une regroupement en une seule mesure globale d'assurance, et que les éléments des indicateurs liés aux stimulants peuvent faire l'objet d'une regroupement (au moyen d'une méthode différente) en une autre mesure globale des stimulants.

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Introduction

Applications of agency theory in accounting have typically assumed outcomes to be observable with very specific preference representations for the principal and agent. However, in many common situations in management accounting and accountability, the outcome is not jointly observable and the principal's and agent's preferences are multiattribute in nature. In these situations the role of accounting signals is considerably broader. For instance, when the outcome of interest to the principal is not jointly observable, the agent's compensation contract cannot be based on outcome and imperfect measures of outcome need to be developed.

Situations in which outcomes are not jointly observable are quite pervasive. The shareholders of a firm, typically, do not know the precise increase in the future cash flows from the firm (which is the outcome of interest to shareholders) that results from a manager's action. This occurs because the effect of a manager's action is often long term in nature. Consequently, the exact impact of a manager's action is obfuscated by many other intervening factors (including the actions of multiple managers that the firm may have hired over a period of time). This implies that the optimal compensation contract cannot be written as a function of the actual outcome but rather as a function of some accounting aggregates such as profits, revenues and costs which are imperfect measures of outcome.¹ This notion is particularly important, as emphasized by Mirrlees (1976) and Gjesdal (1982), because it suggests the use of accounting signals for optimal risk sharing or insurance purposes in addition to monitoring or incentive purposes. Much of the focus of agency theory research in accounting has been on the role of information in monitoring the agent's action. Since outcome is typically assumed to be observable, the accountant's role in providing surrogate measures of outcome, even if such measures provide no additional information about the agent's effort, has not been considered. In this paper, we explicitly model outcome as unobservable and examine the role of accounting information in providing surrogate measures of outcome for insurance purposes. These measures may of course also be useful, in addition, for incentive or monitoring purposes.

Most agency models in accounting have modeled the principal as a single owner or have implicitly assumed that multiple shareholders can be represented as a single principal. This surrogate single principal's utility function is generally assumed to be of the form $W(r(x) - \phi)$, where $x = (x_1, x_2, \dots)$ is the vector of future cash flows resulting from the manager's action, $r(x)$ is the present value of these cash flows, and ϕ is the current compensation paid to the manager. In an imperfect capital market with uncertain and unequal borrowing and lending rates, this representation may not be appropriate even for any one individual. When preferences of different individuals are aggregated under aggregation con-

1 The problem of unobservable outcome is further accentuated when evaluating individual division managers in a multidivision firm.

ditions such as those identified by Harsanyi (1955), or Keeney and Raiffa (1976, p. 528), differences in individual tax rates, intertemporal preferences, and borrowing or lending opportunities, would also in general rule out an aggregate utility function of the form $W(r(x) - \phi)$. We thus extend the basic agency model by retaining the general multiattribute form $W(x, \phi)$ for the principal's utility function.

We develop these generalizations of the principal-agent model in the context of management accounting. We identify characteristics of signals that are valuable in different situations. For instance, we show that when the agent's action is not jointly observable and cannot be directly regulated, but the principal is intertemporally risk neutral,² only signals about the agent's effort are worth generating. Information about the unobserved outcome is not of value in this case unless it permits some additional inference about the agent's action. If, however, the principal is not intertemporally risk neutral, signals on the outcome are valuable even if they permit no additional inference about the agent's action.

Our generalization of principal-agent models provides a different perspective on the value of accounting measurement and increases the applicability of agency models to a richer and broader range of settings that the accountant may encounter. For instance, when outcome is assumed to be jointly observable, the focus of the accounting system is on developing accounting signals that are useful in evaluating the agent's action. Consequently, the evaluation of accounting systems is oriented in favor of systems that provide better measures of the agent's effort (the stewardship role of accounting), since there is no need to measure outcome. When outcome is not jointly observable, there is a demand for accounting signals to provide information on outcome in order to facilitate risk sharing between the principal and agent. We examine this role in terms of the insurance component of accounting signals.

Several accounting signals, such as individual cost and revenue data, are generally available for evaluating managerial performance and as imperfect measures of unobservable outcomes. The accountant's role is to measure these signals and aggregate them for performance evaluation and outcome measurement purposes. A primary result of this paper is that when the principal's utility function belongs to a specified general class, multiple accounting signals can be reduced to just two aggregates which are economically sufficient for the multiple signals. One aggregate reflects the insurance use of the different accounting signals while the second aggregate reflects the incentive use of the signals. These dimensions are precisely the dimensions that make accounting signals valuable in a compensation contract. Our result implies that instead of reporting each signal individually for use in the optimal compensation contract of the agent, the

2 In the decision theory literature, an individual is defined to be "multivariate risk neutral" when his utility function is separable in its attributes, see Richard (1975). Since we define the principal's utility function over future and current cash flows x and ϕ , we use the term "intertemporally risk neutral" to characterize separability of the utility function in these attributes.

accountant can summarize the signals into an aggregate insurance dimension and an aggregate incentive dimension without any economic loss to the agency.

This paper is organized as follows. In the next section we describe a principal-agent model of the relationship between shareholders and a manager. In the third section we derive and discuss some results on the role and value of information in this agency relationship for evaluating managerial performance and sharing risk. The fourth section provides an analysis of how accounting signals can be combined into an aggregate insurance dimension and an aggregate incentive dimension. Concluding remarks are presented in the last section of the paper.

The basic model

We model a principal-agent relationship between multiple shareholders and a manager. The outcome of interest (x) is the change in the vector of future cash flows from the firm. The current period is the period of analysis, namely the period the manager is with the firm, takes action(s) and is compensated. The compensation is represented by ϕ .

We model shareholders as a single principal. This is akin to the standard principal-agent literature in which, implicitly at least, a single owner is assumed to aggregate shareholder preferences. Arrow's impossibility theorem suggests that such an aggregation of preferences is impossible if trade-offs across individual preferences are not permitted. However, if interpersonal comparisons are permitted, Harsanyi (1955) and Keeney and Raiffa (1976, p. 528) show that a group utility function that aggregates the preferences of various individuals can be constructed. Wilson (1968) also analyzes conditions under which a group of individuals who must make a common decision under uncertainty can be modeled as a single decision maker (that is, as a single principal) with a surrogate "group utility function" and a surrogate "group probability assessment." Our model differs from the standard principal-agent literature in that the surrogate group utility function for the multiple shareholders is assumed to be multidimensional and of the generalized form $W(x, \phi)$.

In their usual form, principal-agent models commonly assume that the outcome x is a current period monetary payoff to be shared by the two parties in accordance with the contract, with $\phi(x)$ paid to the agent and $x - \phi(x)$ retained by the principal. The principal's utility function is then correctly written as a single attribute function $W(x - \phi)$. However, when x is a vector of future cash flows and ϕ is the current period compensation paid to the manager, then the utility function can be written as a single attribute function $W(r(x) - \phi)$ only under a strong assumption. This is the assumption that the principal's preferences are such that any certain stream of future cash flows x can be converted into a current period monetary equivalent $r(x)$ independent of the level of current cash flow $-\phi$. This is equivalent to assuming that the trade-off between cash flows in any two periods, including the current one, is independent of current cash flow. (See, for instance, Keeney and Raiffa (1976, p. 86 and Ch. 9) and Banker and Maindiratta (1984).) In an imperfect capital market where borrowing and lending

opportunities depend upon current endowments, and borrowing and lending rates are both uncertain and unequal, this assumption about cash flow trade-offs will most often be invalid even for any given individual. Even if it is valid, so that the individual utility function can be written as $W_i(r_i(x) - \phi)$, the W_i s and r_i 's will in general be different for individuals because of different borrowing or lending opportunities and rates, and different risk attitudes and intertemporal preferences. When the W_i s are then aggregated into a surrogate group utility function, under aggregation conditions such as those identified by Harsanyi (1955), or Keeney and Raiffa (1976, p. 528), it is unlikely that this function will be of the form $W(r(x) - \phi)$. For instance, if Harsanyi's (1955) condition holds, it is possible to construct a linear additive group utility function of the form

$$\sum_{i=1}^N \lambda_i W_i(r_i(x) - \phi).$$

This function cannot, in general, be written in the form $W(r(x) - \phi)$. We impose no specific assumptions for the purpose of constructing an aggregate or group utility function; instead we model the group utility function as some general function $W(x, \phi)$.

A second difference between our model and those in the traditional principal-agent literature is that we allow a more general specification of the utility function of the agent as well. Many of the traditional models assume the agent's utility function to be separable in the agent's monetary payment ϕ and effort a . We model the agent's utility function U as some general function of ϕ and a .

A third departure of our model from those in the standard principal-agent literature is our assumption that the outcome of interest, the actual change in the future cash flows from the firm is not jointly observable by the shareholders and manager. Joint observability requires that the principal and agent are able to *observe, objectively measure and agree upon* the outcome. In many accounting situations, the precise outcome of a manager's action is often not jointly observable by the principal and agent, and hence the agent's compensation contract cannot be directly written in terms of the actual outcome. (See, for instance, Antle (1982), Banker and Datar (1985) and Datar (1987).) Accounting systems cannot isolate the outcome of a particular manager's action because interdependencies across various managerial functions necessitate certain arbitrary allocations to be made in determining outcome. Interperiod allocations such as depreciation are also arbitrary (in the absence of perfect and complete markets) and do not reflect the exact economic resources consumed by the manager. Indeed, in most common accounting situations, the accounting system measures the actual outcome with error, the actual outcome not being jointly observed. What is jointly observed is only the measured outcome.

In the context of the model described above, the actual impact of a manager's action(s) on the future cash flows from the firm may not be observable for several reasons. The outcome of a manager's action is often long term in nature and may never be capable of precise isolation and measurement due to the influence of

several other intervening factors (including the actions of subsequently hired managers). Second, over the period of the manager's tenure with the firm, shareholders only obtain an imperfect measure of the outcome of a particular manager's action based perhaps on the accounting system or the change in the market value of the firm. This is because market values are based only on the market's expectations of the value of the firm, given the market's imperfect information about the firm's prospects that are attributable to the combined efforts of various managers, past and present. Market values therefore provide only imperfect measures of outcome under an incomplete markets regime. Our arguments suggest that not only are actual outcomes not observable at the end of the manager's tenure with the firm, but the actual outcomes are unobservable and incapable of objective measurement at some later time as well. Thus, writing a contingent contract (such as stock options) based on future market values of the firm will not resolve the problem of the outcome of a particular manager's action being not jointly observable.

It is sometimes argued that outcome is observable at the time the firm is liquidated. This argument is valid if a single agent manages the firm from its inception to liquidation. A manager's tenure with a firm, however, rarely if ever, equals the life of the firm. The observed outcome at liquidation typically reflects the cumulative effect of the efforts of several managers and the precise outcome of any individual manager's effort cannot be separately determined.³

Although the model we describe focuses on the agency relationship between shareholders and a manager, its basic features apply to many other settings. The context of pollution can be similarly modeled with society as the principal and the polluting entrepreneur as the agent. In order to control pollution, society devises a scheme of transfers from the polluting entrepreneur to society in the form of penalties, subsidies and taxes (denoted by ϕ). The outcome of the entrepreneur's action x is the health effect of pollution on society. The health effect (due in part to its long-term nature) is rarely, if ever, jointly observable in an objective fashion by the parties involved. Moreover, society's preferences are multiattribute in nature so that W is a two-dimensional function of x and ϕ .

The model also applies in studying the relationship between an entrepreneur and a local taxing authority which desires to promote employment and economic development in the area. Another application of the model is in understanding negotiations between an entrepreneur and labor unions. The entrepreneur takes actions that affect the long term job security, health and safety of his employees. Unionized employees can negotiate transfer arrangements with the entrepreneur to influence his actions.

Our model provides some insight into performance evaluation mechanisms employed by various not-for-profit organizations such as hospitals, development agencies and county health care services. The objective of these organizations is to provide various services rather than maximize profits or cash flow. It is

³ The ultimate observed outcome may, of course, serve as an informative but noisy signal about the outcomes and actions of individual managers.

unlikely that a utility function employed to reflect the preferences of these organizations can be written in terms of a single attribute, such as money. Our model describes features of the accounting systems to monitor, evaluate and compensate managers in such organizations.

We consider a two-person, single period principal-agent model with shareholders modeled as the principal and the manager as the agent. The agent takes some action $a \in [a_1, a_2] \subseteq R$, which together with a random unobserved state of nature θ , determines an outcome x . In the rest of this paper, for expositional convenience, we model x as a scalar belonging to $[x_1, x_2] \subseteq R$. The outcome x is not jointly observable. Multiple signals y and z are however available and observable by the principal and agent. For ease of exposition and to emphasize the two sources of value for imperfect information signals, we start by assuming that the signal $z = z(a, \theta) \in Z \subseteq R$ is an imperfect signal about the unobserved outcome x and the signal $y = y(a, \theta) \in Y \subseteq R$ is an objective (though imperfect) signal on the agent's action a . In other words, we assume that the signal y given signal z provides no additional information about x , the actual outcome. In an analogous manner the signal z given signal y provides no additional information about the agent's effort a . In Proposition 2, we shall relax these assumptions about the signals and allow each signal to provide information about both x and a , given the other signal.

In the context of a retail firm operating in a competitive product market environment, the signal y may be thought of as the market share of the firm whereas z may be regarded as the change in the *market* value (including dividends) of the firm's shares as quoted on a Stock Exchange. The signal z is an imperfect signal on the outcome x , the actual change in the firm's future cash flows resulting from the manager's action. The signal y is a signal on the manager's effort. The intuition is that the market share signal provides relatively direct evidence about a when compared to z , the signal on the resultant impact of such effort on the outcome. The latter impact will depend on other random factors such as the state of the economy, inflation and interest rates. Consequently given y , the signal z provides no additional information about a .

In the context of the polluting firm discussed earlier, the signal z may be regarded as an imperfect signal about potential health effects observable to both parties. The signal y is a measure of the physical level of pollution in the environment which is a signal on the entrepreneur's effort a . Clearly, the signal y given signal z provides no additional information about x , the actual (though not jointly observed) health impact of pollution on society. Similarly, the signal z given the signal y provides no additional information about the entrepreneur's effort a to control pollution. The intuition is that physical measures of pollution provide more direct evidence about a than does the resultant impact of such pollution on society.

In order to provide the agent with incentives to take the desired action, the principal writes a contract with the agent to base the agent's compensation on realized values of the jointly observed signals y and z . The fee schedule ϕ is determined before the agent acts and is a mapping from $Y \times Z \rightarrow R$. We consider

both the case when the action a is observable by the principal and the case when the action a is not observable. If the action a is observable and directly regulated,⁴ the transfer schedule ϕ is beneficial to the extent that it exploits risk sharing opportunities between the two parties. On the other hand, if the action a is not observable and the agent is free to choose the action most beneficial to himself under a given transfer schedule, ϕ can, in addition to risk sharing, serve to influence the entrepreneur's action choice.

The notion of risk sharing used in this paper is the sharing of risk based on the realization of the signal. If the signal can take one of only two values: favorable and unfavorable, risk sharing between a risk averse principal and agent (as distinct from incentive effects) implies that a smaller compensation will be paid to the agent in case an unfavorable signal is observed and a larger compensation will be paid in the event of a favorable signal. Of course, risk still remains after the realization of the signal in the sense that even after a favorable signal a bad outcome may result.

In the single attribute case, the principal will bear no risk if the principal gets a fixed ($x - \phi$) for all realizations of the signal. In our model, however, x is unobservable and the principal's utility function is multiattribute in nature so that $(x - \phi)$ cannot be measured and is irrelevant. In this case the principal will bear no risk relative to the information signal if the principal's expected utility, $EW(x, \phi)$, is a constant for all possible realizations of the signal, where the expectation is taken over all possible values of x conditional on the observed imperfect signals.

We illustrate the notion of risk sharing in the multiattribute case using the following simple example. Suppose that the realization of the signal z (which is a signal on the outcome x) is unfavorable, that is $E(x|z)$ is low. If the agent's compensation ϕ is independent of z , lower expected realizations of the outcome $E(x|z)$ are not compensated. The entire risk of an unfavorable signal on the outcome is borne by the principal in this case. Basing the agent's compensation ϕ on the signal z (which is a signal on the other attribute x) allows the principal to "insure" himself against an unfavorable signal by paying a lower ϕ . That is, even though x and ϕ are different attributes, the correlation between x and z enables the principal to shift part of his risk by writing the compensation contract ϕ as a function of z . Note that given the signal y on the agent's effort a , z has no incentive effect, that is basing the contract ϕ on z does not by itself motivate the agent to take a higher effort. The benefit of writing ϕ as a function of z is to facilitate risk sharing as described above.

We assume that the preference orderings of the principal and agent satisfy the von Neuman–Morgenstern axioms, so that there exist multiattribute utility functions $W(x, \phi)$ and $U(a, \phi)$ that express these preferences in conformity with the expected utility rule. W and U are assumed to be thrice continuously differentiable in all their arguments. The principal and agent know the structure of the choice problem, their utility functions and the set of available options. We define

4 As in Harris and Raviv (1979), even an imperfect signal whose support is a nontrivial function of the agent's action is sufficient to enable the principal to directly regulate the agent's action choice.

ϕ as the compensation paid by the principal to the agent, and therefore assume that W_ϕ is negative and U_ϕ is positive everywhere. We further assume U_a is negative and W_x is positive everywhere. The assumption that U_a is negative implies that the agent has a disutility for taking higher actions. The assumption that W_x is positive reflects the fact that the principal prefers higher outcomes. The second order derivatives $W_{\phi\phi}$ and $U_{\phi\phi}$ are assumed to be nonpositive with at least one of $W_{\phi\phi}$ and $U_{\phi\phi}$ negative. That is, both principal and agent are conditionally weakly risk averse with respect to the transfer payments and one of them is strictly conditionally risk averse.

The partial second derivatives $W_{\phi\phi}$ and W_{xx} reflect the risk attitudes of the principal over current and future cash flows respectively. The sign of the cross partial $W_{\phi x}$, on the other hand, reflects the principal's intertemporal risk preferences. The cross partial $W_{\phi x} = 0$ is equivalent to intertemporal risk neutrality over current and future cash flows. This also implies that the principal's utility function is additively separable in x and ϕ (that is, $W(x, \phi) = G(x) - H(\phi)$). Similarly, $W_{\phi x} > 0$ and $W_{\phi x} < 0$ are equivalent to intertemporal risk aversion and intertemporal risk proneness respectively.⁵ We impose no restrictions on the intertemporal risk preferences of the principal, that is $W_{\phi x}$ is unrestricted in sign.

Our assumptions are weaker than those employed in the standard principal-agent models. For instance, unlike the traditional principal-agent model, assuming the agent to be risk neutral with respect to current monetary transfers does not trivialize the problem. A nontrivial risk-sharing problem remains even when the agent is assumed to be risk neutral. One further assumption about the agent's utility function is made. The agent's risk preference over current cash is assumed independent of the agent's action choice. That is

$$\frac{\partial}{\partial a} \left(\frac{U_{\phi\phi}}{U_\phi} \right) = 0$$

everywhere. This assumption implies that the agent's risk aversion for current cash does not depend on the agent's level of effort. It also implies that

$$\frac{\partial}{\partial \phi} \left(\frac{U_{a\phi}}{U_\phi} \right) = 0$$

everywhere.

The principal and agent share common beliefs about the joint distribution of

5 The economic interpretation of intertemporal risk preferences is best illustrated in the context of lotteries involving level streams of cash flows. Consider a one period lottery with a 50 percent chance of receiving cash flow c' and a 50 percent chance of receiving cash flow c'' . Let the certainty equivalent of this lottery be \bar{c} , where \bar{c} is obtained in the same period. Now consider a two period lottery with a 50 percent chance of receiving the cash flow vector (c', c') ; and a 50 percent chance of receiving the cash flow vector (c'', c'') ; the first element of the vector is current cash flow and the second element is future cash flow. Let the certain level cash flow vector that is considered equivalent to this lottery be (\bar{c}, \bar{c}) . If $\bar{c} = \bar{c}$ the individual is considered intertemporally risk neutral. Similarly, $\bar{c} > \bar{c}$ and $\bar{c} < \bar{c}$ imply intertemporal risk aversion and intertemporal risk proneness respectively. Thus an individual who is more risk averse for level cash flow streams than for single period cash flows is intertemporally risk averse. See Keeney and Raiffa (1976, Ch 9) and Richard (1975) for a more detailed discussion.

$x \in X, y \in Y$ and $z \in Z$ as encoded in the joint probability density function $f(x, y, z|a)$, $a \in [a_1, a_2]$. We assume $f(x, y, z|a)$ is twice continuously differentiable in y, z and a and that the likelihood ratio expression $f_a(x, y, z|a)/f(x, y, z|a)$ is continuously differentiable in y and z . We shall write $f(x, y, z|a) = g(x|y, z, a)h(y, z|a)$. The function g is the conditional density function of x (which is not jointly observed) given the signals y and z (which are jointly observed and on which the optimal compensation contract can be based). The function h is the joint density function of the signals y and z given a .

The agent's action choice is assumed to make higher levels of outcomes more probable. We invoke the monotone likelihood ratio condition [MLRC] that

$$\frac{\partial h_a(y, z|a)}{\partial y h(y, z|a)} > 0.$$

Milgrom (1981) has shown that the MLRC condition is equivalent to the statistical inference from the observation of a higher value of the signal y that the agent has taken a higher level of effort.⁶ Our assumption that z provides no information about a given y can be expressed as requiring

$$\frac{\partial h_a(y, z|a)}{\partial z h(y, z|a)} = 0 \quad \text{at all } a.$$

Our earlier assumption that z provides information about the unobserved x given y can be expressed as requiring $g_x(x|y, z, a)$ to be nonzero for some x, y, z and all a . Similarly, our assumption that y provides no information about x given z can be written as $g_y(x|y, z, a) = 0$ for all x, y and z and for all values of the parameter a .

The principal is modeled as a Stackelberg leader. He moves first by specifying the performance evaluation measure and the transfer scheme for observed values of y and z . The agent next selects his action.

We first consider the case in which the agent's action is observable and can be directly regulated. The agency problem can then be characterized as follows.

$$\text{Maximize } EW(x, \phi(y, z))$$

$$a, \phi(y, z)$$

subject to

$$EU(a, \phi(y, z)) \geq \bar{U}$$

$$\phi \in [\phi_1, \phi_2]$$

$$a \in [a_1, a_2],$$

(P1)

where the expectations are taken over the joint probability density function $f(x, y, z|a)$, and \bar{U} denotes the minimum utility level demanded by the agent to work for the principal.

As discussed in the first section, our model differs from most models in the

6 We note that Whitt (1980) has shown that MLRC implies first-order stochastic dominance.

agency literature since we assume generalized preference structures for the principal and agent and further permit outcomes to be not jointly observable. Most models in the agency literature have not modeled outcome as not jointly observable. Mirrlees (1976) was the first to model it as such and subsequently, Gjesdal (1982) also considered unobservable outcomes.

In Problem (P2) we consider the case where the agent's action is not observable and cannot be directly regulated. The Pareto-optimal arrangement $(a^*, \phi^*(y, z))$ is obtained by solving the following program.

$$\text{Maximize } EW(x, \phi(y, z))$$

$$a, \phi(y, z)$$

subject to

$$EU(a, \phi(y, z)) \geq \bar{U}$$

$$a \in \text{argmax } EU(a, \phi(y, z))$$

$$\phi \in [\phi_1, \phi_2]$$

$$a \in [a_1, a_2].$$

(P2)

The first constraint is as before and denotes that the agent is guaranteed a certain minimum utility level. The second constraint is the incentive compatibility constraint which specifies that the agent maximizes expected utility given the transfer schedule $\phi(y, z)$.

We assume the optimal solution⁷ $(a^*, \phi^*(y, z))$ for Problem (P2) is such that a^* is in the interior of $[a_1, a_2]$ and uniquely maximizes $E[U(a, \phi^*(y, z))]$ with $\partial^2\{E[U(a^*, \phi^*(y, z))]\}/\partial a^2 < 0$. The last constraint in Problem (P2) may then be replaced by the first-order condition⁸ $\partial\{E[U(a, \phi(y, z))]\}/\partial a = 0$.

A major concern in the study of agency relationships and also in the context of this section is the identification of conditions under which observation of an additional signal (say y) is valuable (given signal z). With respect to the agent's effort, this question is addressed in Holmström (1979, 1982), Amershi (1984), Amershi, Banker and Datar (1987) and Banker and Datar (1989). A signal y is said to be *valuable* if at least one of the principal and the agent can be made strictly better off, and the other kept at least as well off, with a contract of the form $\phi(y, z)$ compared to a contract of the form $\phi(z)$.

Following Banker and Maindiratta (1984) and Amershi, Banker and Datar (1987), we consider the gradients of the optimal transfer schedule ϕ^* with respect to y and z at the optimal a^* . Values of the gradients ϕ_y^* and ϕ_z^* will be nonzero for at least some values of y and z if and only if the principal and agent are better off by basing transfers on the realizations of both y and z . In other words, ϕ_y^* (or ϕ_z^*) equal to zero at a^* for all y (or z) is equivalent to the state-

7 If *ex ante* randomization is optimal, it will consist of a convex combination of optimal ϕ^* considered here. Our focus is only on the value of an information signal in a specific ϕ^* , and not as a randomizing device.

8 See Mirrlees (1975), Rogerson (1985), and Banker and Datar (1989) for a detailed discussion.

ment that the imperfect signal y (or z) is not valuable in the optimal compensation contract. Therefore, examining the gradients enables us to fully characterize the value of signals in the optimal contract. In the next section, we use the gradients to derive and discuss some results on the role and value of information in the agency relationship between shareholders and the manager.

The value of imperfect signals

Most agency theoretic models assume the outcome x to be observable. Therefore the focus of these models (as in Holmström (1979)) is to examine when the observation of a signal in addition to the outcome x is valuable. In the model described here, the outcome x is not jointly observable and the question of interest is which, if any, of the several available signals are valuable for the contract. We show that in addition to signals about the agent's action being valuable when the agent's action is unobservable (the incentive motive), other imperfect signals may also be valuable only because they provide information about the not jointly observable outcome x (the insurance motive).

We first consider the case when the agent's action is observable and can be directly regulated. If the principal is intertemporally risk neutral, that is $W_{\phi x} = 0$, we show that neither the (imperfect) signal y nor the (imperfect) signal z is valuable in structuring the incentive contract between the principal and agent. If the principal is not intertemporally risk neutral, that is $W_{\phi x} \neq 0$, only signal z on the unobserved outcome is valuable.

We next consider the case when the agent's action a is not observable and cannot be directly regulated. If the principal is intertemporally risk neutral, the optimal transfer arrangement will depend only on signal y which is a signal on the agent's action choice. The signal z on the outcome of the agent's action will not be valuable. If, however, the principal is not intertemporally risk neutral, both the signals y and z will be valuable in determining the optimal transfer arrangement between the principal and agent.

We summarize the results described above via the following 2×2 matrix.

TABLE 1
Optimal transfer arrangements

		Principal	
		Intertemporally risk neutral	Not intertemporally risk neutral
Entrepreneur's effort	Observable	Proposition 1A Neither signal y nor signal z valuable	Proposition 1B Signal y <i>not</i> valuable Signal z generally valuable
	Unobservable	Proposition 1C Signal y generally valuable Signal z <i>not</i> valuable	Propositions 1B and 1C Signals y and z generally valuable

The matrix reflects the results that precisely two factors determine which signals will be valuable in the agency relationship between the principal and agent: observability of the agent's effort and the principal's intertemporal risk neutrality. These results are useful for management accounting since they indicate which signals will be valuable in furthering performance evaluation and optimal risk sharing.

We first examine the case in which the agent's effort is observable. Pointwise optimization of the Lagrangian for Problem (P1) then yields the following necessary conditions for any optimal solution $(a^*, \phi^*(y, z))$.

$$\frac{\int W_\phi g(x|y, z, a^*) dx}{U_\phi} + \lambda = 0 \quad \text{for all } y, z. \tag{1.1}$$

$$\begin{aligned} & \iiint W(x, \phi^*(y, z)) f_a(x, y, z|a^*) dx dy dz \\ & + \lambda \delta \left\{ \iiint U(\phi^*(y, z), a^*) f(x, y, z|a^*) dx dy dz \right\} / \delta a = 0 \end{aligned} \tag{1.2}$$

The first-order conditions we obtain are a generalization of those obtained when the outcome x is jointly observable. If x is jointly observable, condition (1.1) will become

$$\frac{W_\phi}{U_\phi} + \lambda = 0 \quad \text{for all } x, y, z.$$

Since x is not jointly observable, we take the expected value of W_ϕ conditional on the observed y and z .

As in Banker and Maindiratta (1984), it follows that under the assumptions of our model, ϕ^* is differentiable. In Lemma 1 we derive expressions for the gradients ϕ_y^* , ϕ_z^* of the optimal compensation contract. As argued in the previous section, the gradients fully characterize the value of signals in the optimal contract since ϕ is a function of only y and z . Thus, the necessary and sufficient condition for the signal y (or z) to be valuable is that the gradient ϕ_y^* (or ϕ_z^*) is not equal to zero for some y (or z).

Lemma 1: The gradients of the optimal compensation contract $\phi^*(y, z)$ are given by

$$\phi_y^* = \frac{-U_\phi \int W_\phi g_y(x|y, z, a^*) dx}{U_\phi \int W_{\phi\phi} g(x|y, z, a^*) dx - U_{\phi\phi} \int W_\phi g(x|y, z, a^*) dx} \tag{2.1}$$

$$\phi_z^* = \frac{-U_\phi \int W_\phi g_z(x|y, z, a^*) dx}{U_\phi \int W_{\phi\phi} g(x|y, z, a^*) dx - U_{\phi\phi} \int W_\phi g(x|y, z, a^*) dx} \quad (2.2)$$

Proof: See Appendix.

We next state and prove Proposition 1A. We show that if the principal (shareholders) can directly regulate the action of the agent (manager), and if the principal is intertemporally risk neutral there is no value to observing the signals y and z .

Proposition 1A: When the action of the agent is jointly observable and can be directly regulated, a sufficient condition for there to be no gain to observing the signal y (on effort) and z (on outcome) is that the principal is intertemporally risk neutral over x and ϕ (that is, $W_{\phi x} = 0$).

Proof: See Appendix.

It is worth noting that the condition in Proposition 1A is not necessary for z to have no marginal value. That is, $W_{\phi x}$ may not be equal to zero yet ϕ_z^* and

$$\int W_\phi g_z(x|y, z, a^*) dx$$

may coincidentally be equal to zero.

The intuition underlying the proposition is as follows. Since the agent's action is observable, no incentive problem exists. Any possible benefit from observing the signals y and z can only arise from better risk-sharing opportunities. Intertemporal risk neutrality over x and ϕ implies that the principal's utility function is separable into these two attributes. In other words, the principal cannot be made better off in expected utility terms by contracting with the agent to make larger payments, ϕ , if the signal on x is greater, and smaller transfers if the signal on x is lower. Furthermore, since at least one of the principal or agent is assumed to be risk averse with respect to wealth, it is worth eliminating all *other* uncertainty as regards the transfer payment. Consequently, the transfer payment is made independent of the signal on future cash flows from the firm and there is no value to measuring such a signal. However, if the principal is not intertemporally risk neutral and the agent's action is observable, there is some benefit to be derived in terms of optimal risk sharing by basing the transfers on the signal z (except when ϕ_z^* is coincidentally equal to zero).

The application of this general result to the usual principal-agent models when x is a monetary outcome that is *not* jointly observed is instructive. Thus, if the

principal is risk neutral, with utility defined over $(x - \phi)$, there is no value to observing signals on x if the agent's action choice is observable. Notice, however, that in general what is important is the separability of the principal's utility function in x and ϕ rather than the risk neutrality of the principal over wealth. Indeed, in the general case, the principal's risk preference over wealth is *irrelevant*. It is the cross-partial of his utility function over x and ϕ that is relevant. Consequently, even if the principal is risk neutral over wealth, that is, $W_{\phi\phi} = 0$, but $W_{\phi x} \neq 0$, there is value to observing z . On the other hand, if the principal is risk averse over wealth, that is, $W_{\phi\phi} < 0$, but $W_{\phi x} = 0$, there is no value to observing z .

We next consider the case in which the agent's action is jointly observable but the principal is not intertemporally risk neutral. In Proposition 1B below we show that there is value to observing only the signal z about the future cash flows from the firm but no value to observing the signal y about the agent's action.

Proposition 1B: If the principal is not intertemporally risk neutral and the agent's action is jointly observable, then the signal y will have no marginal value given the signal z .

Proof: See Appendix.

It is worth noting that the condition in Proposition 1B is not necessary for y to have no marginal value. The signal y may be such that even if $g_y(x|y, z, a^*)$ is nonzero somewhere,

$$\int W_{\phi} g_y(x|y, z, a^*) dx$$

is coincidentally equal to zero. In such a case y will have no marginal value.

The significance of Proposition 1B is that when the action a is observable and $W_{\phi x} \neq 0$, only information about the future cash flows x , is useful. The signal z provides information about x and is therefore generally valuable. Since signal y provides no additional information about x given signal z (that is $g_y(x|y, z, a^*) = 0$ everywhere), it has no marginal value.

In the standard agency literature in accounting, the outcome x is assumed to be jointly observable. Consequently, as discussed earlier, the first order condition (1.1) becomes

$$\frac{W_{\phi}}{U_{\phi}} + \lambda = 0,$$

and the question of which signal provides "better" information about x does not arise. If the outcome x is not jointly observable as is the case in our model, Proposition 1B suggests that if the signal y (on action) given the signal z (on outcome), does not alter the conditional probability distribution of x , there will be no value to observing y .

It is instructive to contrast this result with results on the value of signals that provide information about the agent's action choice a when the agent's action is unobservable. The notion there is one of sufficiency of signals with respect to the parameter a . Here we are concerned with information about the probability distribution of the unobservable future cash flows x . Proposition 1B states that a sufficient condition for the signal y to have no value is that the conditional density function $g(x|y, z, a^*)$ is independent of y . This implies that if the action of the agent can be directly regulated and the principal is not intertemporally risk neutral, the principal need only measure signals such as z that provide information about the future cash flows rather than signals such as y that are signals on the agent's action.

Of course, the signal y may provide important information in monitoring the agent's action choice when such action cannot be directly observed and regulated. We next focus attention on such a case. We continue to assume that the future cash flows, x , are not jointly observable. Two interesting questions arise. First, when is additional information not worth observing? Second, if additional information is worth observing, what are the characteristics of such information? To address these questions, we first write and solve the Lagrangian for Problem (P2):

$$\begin{aligned} \mathcal{L} = & \int \int \int W(x, \phi(y, z)) f(x, y, z|a) dx dy dz \\ & + \lambda \left[\int \int \int U(a, \phi(y, z)) f(x, y, z|a) dx dy dz - \bar{U} \right] \\ & + \mu \left[\int \int \int U(a, \phi(y, z)) f_a(x, y, z|a) dx dy dz \right. \\ & \left. + \int \int \int U_a(a, \phi(y, z)) f(x, y, z|a) dx dy dz \right], \end{aligned}$$

where λ and μ are the Lagrange multipliers of the first and second constraints respectively. Note that since $f(x, y, z|a) = g(x|y, z, a)h(y, z|a)$ for all (y, z) , we have

$$f_a(x, y, z|a) = g_a(x|y, z, a)h(y, z|a) + g(x|y, z, a)h_a(y, z|a).$$

Pointwise optimization of the Lagrangian for problem (P2) yields the following necessary conditions for any optimal solution $(a^*, \phi^*(y, z))$.

$$\begin{aligned} h(y, z|a^*) \int W_\phi g(x|y, z, a^*) dx + \lambda \left[h(y, z|a^*) U_\phi \int g(x|y, z, a^*) dx \right] \\ + \mu \left\{ U_\phi h(y, z|a^*) \int g_a(x|y, z, a^*) dx + U_\phi h_a(y, z|a^*) \int g(x|y, z, a^*) dx \right. \\ \left. + U_{a\phi} h(y, z|a^*) \int g(x|y, z, a^*) dx = 0 \quad \text{for all } y, z \quad (3.1) \right. \end{aligned}$$

$$\begin{aligned} & \iiint W(x, \phi^*(y, z)) f_a(x, y, z|a^*) dx dy dz \\ & + \mu \frac{\partial^2}{\partial a^2} \left[\iiint U(a^*, \phi^*(y, z)) f(x, y, z|a^*) dx dy dz \right] = 0. \end{aligned} \quad (3.2)$$

Noting that

$$\int g(x|y, z, a^*) dx = 1 \quad \text{and} \quad \int g_a(x|y, z, a^*) dx = 0$$

we can rewrite (3.1) as

$$\frac{\int W_\phi g(x|y, z, a^*) dx}{U_\phi} = -\lambda - \mu \left[\frac{U_{a\phi}}{U_\phi} + \frac{h_a(y, z|a^*)}{h(y, z|a^*)} \right] \quad \text{for all } y, z. \quad (3.1')$$

In contrast to much of the agency literature in accounting, the term

$$\int W_\phi g(x|y, z, a^*) dx$$

arises because x is not observable by both parties. The term $U_{a\phi}/U_\phi$ enters because the agent's utility function is not separable in ϕ and a . As in Holmström (1979), and Banker and Maindiratta (1984), it follows that under the assumptions of our model $\mu > 0$ and ϕ^* is continuous and differentiable.

We consider the value of the signals y and z when the agent's action is not observable and the principal is intertemporally risk neutral. In Proposition 1C, we next show that there is value to observing the signal y on the agent's action but no value to observing the signal z on future cash flows.

Proposition 1C: If the agent's action a is not jointly observable and if the principal is intertemporally risk neutral, then the signal y will be valuable. Further, the signal z will have no marginal value given that signal y is already being observed.

Proof: See Appendix.

The intuition behind Proposition 1C is as follows. When the principal is intertemporally risk neutral, as in Proposition 1A, signals (such as z) about x , have no value. However, since the agent's effort is not jointly observable, a signal such as y that provides information at the margin about the agent's action a^* is worth observing. Proposition 1C states that if signal z does not provide any additional information about a^* given the signal y

$$\left(\text{because } \frac{\partial}{\partial z} \frac{h_a(y, z|a^*)}{h(y, z|a^*)} = 0 \text{ everywhere} \right),$$

there is no value to measuring z .

As discussed in Amershi (1984), Amershi, Banker, and Datar (1987) and Banker and Datar (1989), we are only concerned with the marginal information of the signal z given y with respect to a , at a^* , the agent's optimal action choice. If at a^* , the optimal contract, given y , does not depend on z , z has no marginal value. For y to be a sufficient statistic for (y, z) with respect to a , z must have no marginal value *at all* a .

It is instructive to distinguish Proposition 1C from Proposition 1B. In Proposition 1B, the agent's action choice a is directly regulated. Consequently, signals about a are of no value. However, since the principal is *not* intertemporally risk neutral, signals about the outcome x are generally valuable. Consequently, if the signal y (on the agent's action) given the signal z (on outcome) does not alter the probability distribution of x , there is no value to measuring y . In Proposition 1C, the principal is assumed to be intertemporally risk neutral and as in Proposition 1A, signals about x are of no value. However, since the agent's action is not jointly observable, signals about a are valuable. Consequently, if at the optimal action a^* , the signal z given the signal y does not provide any additional information about a^* , signal z will have no value. In other words, if the principal is not intertemporally risk neutral, information about x is generally valuable. If the agent's action is not observable, information about a is generally valuable.

Finally, in the case where the principal is not intertemporally risk neutral *and* the agent's action is not jointly observable, our discussion above suggests that both the signals y and z will be valuable. Signal z will be valuable since it provides additional information about the conditional probability distribution of x (as in Proposition 1B), and signal y will be valuable since it provides information about a at the optimal a^* (as in Proposition 1C).

Aggregation of multiple signals

In this section we first generalize the results of the earlier section by considering two general signals β and η . Unlike the specific signals y (on a) and z (on x) considered earlier, each of the signals β and η provide information about both the agent's action a and the unobserved future cash flows x . We show that the value of a signal β (or η) can be attributed to two components: an *insurance* component and an *incentive* component.⁹ In Proposition 2 we provide a sufficient (and almost necessary) condition for the signal β (or η) to be valueless given the other signal. We further demonstrate that under certain conditions, when multiple signals exist, the insurance component of the individual signals can be aggregated into a single aggregate insurance measure and the incentive component of the signals can be aggregated (via a different aggregation procedure) into another single aggregate incentive measure. Thus, the accounting consolidation process can reduce the multiple accounting signals to just two aggregate

⁹ The insurance and incentive values of a signal also arise when the outcome x is unobservable and the principal is risk averse. The results we derive can be immediately applied to this special case. Our use of the terms "insurance" and "incentive" components in the general case parallels the usage in Gjesdal (1982).

gate performance measures, one used for insurance purposes and the other used for incentive purposes in the optimal compensation contract.

We first generalize Problem (P2) to address the case of the general signals β and η . Pointwise optimization of the Lagrangian with β substituted for y and η substituted for z in Problem (P2) yields a condition analogous to condition (3.1') as follows.

$$\frac{\int W_\phi g(x|\beta, \eta, a^*) dx}{U_\phi} = -\lambda - \mu \left[\frac{U_{a\phi}}{U_\phi} + \frac{h_a(\beta, \eta|a^*)}{h(\beta, \eta|a^*)} \right] \text{ for all } \beta, \eta. \quad (3.1'')$$

In Lemma 2 we derive expressions for the gradients ϕ_β^* and ϕ_η^* of the optimal compensation contract. These gradients fully characterize the optimal contract based on the signals β and η .

Lemma 2: The gradients of the optimal compensation contract $\phi^*(\beta, \eta)$ are given by

$$\phi_\beta^* = - \frac{\frac{\int W_\phi g_\beta(x|\beta, \eta, a^*) dx}{U_\phi} + \mu \frac{\partial}{\partial \beta} \left[\frac{h_a(\beta, \eta, |a^*)}{h(\beta, \eta|a^*)} \right]}{\frac{\partial}{\partial \phi} \left[\frac{\int W_\phi g(x|\beta, \eta, a^*) dx}{U_\phi} \right]}. \quad (4.1)$$

$$\phi_\eta^* = - \frac{\frac{\int W_\phi g_\eta(x|\beta, \eta, a^*) dx}{U_\phi} + \mu \frac{\partial}{\partial \eta} \left[\frac{h_a(\beta, \eta|a^*)}{h(\beta, \eta|a^*)} \right]}{\frac{\partial}{\partial \phi} \left[\frac{\int W_\phi g(x|\beta, \eta, a^*) dx}{U_\phi} \right]}. \quad (4.2)$$

Proof: See Appendix.

Proposition 2: A sufficient condition for the signal β (or η), given the other signal, to be valueless (that is, $\phi_\beta^* = 0$ for all β, η) is that the principal is intertemporally risk neutral (i.e., $W_{\phi x} = 0$ as in Richard (1975)) and the signal β (or η) is not informative about the agent's action a at a^* given the other signal

$$\left(\text{i.e., } \frac{\partial}{\partial \beta} \frac{h_a(\beta, \eta; a^*)}{h(\beta, \eta; a^*)} = 0 \text{ as in Holmström (1979)} \right).$$

Proof: See Appendix.

It is evident from Lemma 2 that each of ϕ_β^* and ϕ_η^* comprises of two additive components. We shall label the first component of each of these gradients as the insurance component and the second component of each of the gradients as the incentive component. Our motivation for so labeling the two components of ϕ_β^* and ϕ_η^* is that the insurance component of the gradient of the optimal compensation contract becomes zero when, as in Proposition 2, the principal is intertemporally risk neutral and no potential benefits from improved risk sharing exist, or when the signal does not influence the conditional distribution of the future cash flows x . Similarly, the incentive component of the gradient of the optimal compensation contract becomes zero when at the margin the signal does not provide additional information about the agent's effort as in Proposition 2. Such is also the case, if the agent's action is observable or can be directly regulated (as in the case of moving support).

If there is the potential for use of information for insurance purposes, the signal β (or η) will be required for insurance purposes if and only if the insurance component of the gradient of the compensation function with respect to the signal is not zero. A sufficient (and almost necessary) condition for the signal β (or η) to be useless for insurance purposes is that the conditional distribution of $g(x|\beta, \eta, a^*)$ be independent of β (or η).

Similarly, when there is the potential for use of accounting information signals for incentive purposes, the signal β (or η) will be useful for incentive purposes if and only if the incentive component of the gradient of the compensation contract with respect to the signal β (or η) is not zero. A necessary and sufficient condition for the signal β (or η) to be useless for incentive purposes may be represented in terms of whether the signal β (or η) is uninformative about the agent's effort at the optimal a^* ; that is whether the likelihood ratio is independent of β (or η) at a^* .

The conditions in Proposition 2 are sufficient but not necessary. For instance, in the case of the signal β , the insurance component, the incentive component and μ may be such that the numerator is coincidentally equal to zero without any of the individual terms being equal to zero. Second, as discussed in the immediately preceding section, the insurance component may coincidentally be zero even when the principal is not intertemporally risk neutral. Third, the incentive component $\partial/\partial\beta(h_a/h)$ may equal zero only at a^* but not at all a (See Amershi, Banker and Datar (1987) for a detailed discussion.).

Multiple accounting signals such as β and η are often available in the design of the optimal compensation contract. An interesting question for accounting research is how multiple accounting signals can be aggregated into a small number of performance evaluation and risk sharing measures. In the next proposition we identify conditions under which the various accounting signals can be aggregated into just two aggregates, one for insurance purposes and another, via a separate aggregation procedure, for incentive purposes without any economic loss to the agency. The accounting consolidation process can thereby reduce the dimensionality of the available accounting signals by reporting just two aggregate dimensions, an insurance aggregate and an incentive aggregate.

A sufficient condition for the aggregation procedure described above to be optimal is that the two attributes x and ϕ in the principal's utility function $W(\cdot)$ be mutually utility independent. This is equivalent to assuming that the conditional preferences for lotteries on x given ϕ do not depend on the particular level of ϕ , and lotteries on ϕ given x do not depend on the particular level of x ; see Keeney and Raiffa (1976, p. 226). Keeney and Raiffa (1976, p. 224) also note that "the utility independence assumptions are appropriate in many realistic problems and are operationally verifiable in practice. Utility functions exploiting utility independence have been used in a number of important problems. ... In group decision problems, the decision makers might have different utility functions but they might jointly agree on the appropriateness of various utility independence assumptions." From Theorem 5.2 of Keeney and Raiffa (1976, p. 234) it follows that if x and ϕ are mutually utility independent (as we have assumed), then the utility function is multilinear, that is $W(x, \phi)$ can be written as

$$W(x, \phi) = B_1(\phi) + B_2(x) + B_3(\phi) \cdot B_4(x) \quad (5)$$

Thus, in many realistic situations, including group decision problems, the assumption of a multilinear specification of the utility function may be quite reasonable.

We next state and prove Proposition 3 on the optimality of aggregating individual accounting signals into an insurance aggregate and an incentive aggregate assuming the principal's utility function is multilinear.

Proposition 3: If $W(x, \phi) = B_1(\phi) + B_2(x) + B_3(\phi) \cdot B_4(x)$, then the optimal incentive contract ϕ^* can be written simply as a function of the two aggregates $\nu = E[B_4(x)|\beta, \eta, a^*]$ and $\tau = h_a(\beta, \eta|a^*)/h(\beta, \eta|a^*)$; the actual realized values of β and η are not required in addition to ν and τ for computing ϕ^* .

Proof: See Appendix.

It follows from Proposition 3 that the incentive aggregate τ is equal to $h_a(\beta, \eta, a^*)/h(\beta, \eta, a^*)$ and the insurance aggregate ν is equal to $E[B_4(x)|\beta, \eta, a^*]$. We label ν the insurance aggregate because the signal β (or η) is useful for insurance purposes if and only if the aggregate ν is not independent of β (or η), that is $\partial\nu/\partial\beta \neq 0$ for a given ϕ . Similarly, we refer to τ as the incentive aggregate because the signal β (or η) will be useful for incentive purposes if and only if the aggregate τ is not independent of β (or η), that is $\partial\tau/\partial\beta \neq 0$ at a^* . The optimal compensation contract ϕ^* is some function of these aggregates τ and ν .

The design of the optimal compensation contract can be regarded as comprising of two stages. The first stage consists of two aggregation procedures. One aggregate combines individual accounting signals into a managerial performance evaluation measure for incentive purposes. A second (and different) aggregate combines the individual accounting signals into a risk sharing measure (given by the expected value of a function of the outcome) for insurance purposes. The second stage in the design of the optimal compensation contract involves choosing the

compensation contract based on the two aggregate dimensions discussed above. This distinction into two stages is interesting because it is the first stage which is an important element of the management accountant's function.

The results of Proposition 3 can be directly extended to the case of multiple accounting signals say $\beta_1, \beta_2 \dots \beta_N$. Under the condition of Proposition 3, the management accountant's function is to construct two aggregates, the insurance aggregate $\nu = E[B_4(x)|\beta_1, \dots \beta_N, a^*]$ and the incentive aggregate $\tau = h_a(\beta_1 \dots \beta_N, a^*)/h(\beta_1 \dots \beta_N, a^*)$. Having constructed these two measures, the compensation contract of the agent can next be computed as a function $\psi(\nu, \tau, a^*)$. The aggregates ν and τ may each be (different) linear aggregates of the basic accounting signals $\beta_1 \dots \beta_N$ even though the optimal compensation function $\psi(\cdot)$ may be nonlinear.

The aggregates in Proposition 3 are not in general linear aggregates of the individual accounting signals β and η . In many commonly encountered accounting settings, we observe that accounting signals are linearly aggregated. For instance, profits are computed as revenues minus costs; the total cost of a cost center is calculated as the sum of the individual cost categories; residual income linearly combines operating profits with the level of investment.

In order to examine conditions under which linear aggregation of accounting signals in constructing the insurance aggregate and the incentive aggregate will be optimal, we introduce some more structure on the problem. We further assume as in Banker and Datar (1987) that the joint density function h of the signals β and η is of the form

$$h(\beta, \eta; a) = \exp \{p(a)\beta + q(a)\eta - r(a) + s(\beta) + t(\eta - \gamma\beta)\}, \tag{6}$$

where a is the agent's action choice, $p(\cdot), q(\cdot), r(\cdot), s(\cdot)$ and $t(\cdot)$ are arbitrary functions of a, β or η as indicated and γ is a scalar parameter. For this class of joint probability density functions, the conditional distribution of the signal $\beta|\eta$ (and also $\eta|\beta$) includes many of the common distributions such as (truncated) normal, exponential, gamma, *chi*-square and inverse Gaussian. Indeed, these conditional distributions include most of the common continuous parametric functional forms for probability distributions that have been considered in the agency literature in accounting.

For the class of joint density functions defined in (6), Banker and Datar (1989, Proposition 1) show that linear aggregation of the accounting signals β and η is optimal for incentive purposes. They further show (see Banker and Datar (1989, Proposition 4)), that the ratio of the gradients with respect to the signals β and η of the incentive component of the transfer payment ϕ is given by

$$\frac{\rho_1^2 \xi_{1a}}{\rho_2^2 \xi_{2a}} \text{ when } \phi^*(\beta, \eta) \in (\underline{\phi}, \bar{\phi}),$$

where

$$\rho_1^2 = \frac{1}{\text{Var}(\beta)}$$

is the precision of the signal β ,

$$\rho_2^2 = \frac{1}{\text{Var}(\eta)}$$

is the precision of the signal η , ξ_{1a} is the (adjusted) sensitivity of the signal β (see Banker and Datar (1989)) and ξ_{2a} is the (adjusted) sensitivity of the signal η . The (adjusted) sensitivity of a signal β is the relative change in the expected value of β in response to a change in the level of the agent's effort, adjusted for the correlation with the other signal η which may also change with the agent's effort. The value $\xi_{1a} = \mu_{1a} - k\rho_2^2\mu_{2a}$, where

$$\mu_{1a} = \frac{\partial}{\partial a} E(\beta), \quad \mu_{2a} = \frac{\partial}{\partial a} E(\eta) \quad \text{and} \quad k = \text{cov}(\beta, \eta).$$

Similarly $\xi_{2a} = \mu_{2a} - k\rho_1^2\mu_{1a}$. Note that ρ_i^2 and ξ_{ia} for $i = 1, 2$ are evaluated at $a = a^*$. Banker and Datar (1989) define the intensity τ_i of a signal to be equal to its sensitivity multiplied by its precision, that is, $\tau_i = \rho_i^2\xi_{ia}$ for $i = 1, 2$. The weight on each signal in the optimal linear incentive aggregate is proportional to the intensity of each signal.

If we further assume that $E(B_4(x)|\beta, \eta, a^*)$ is linear in β and η , then each of the insurance aggregate and the incentive aggregate will be a (different) linear aggregate of the accounting signals β and η . We formalize this result in the following corollary which follows directly from Proposition 3.

Corollary 1: If the principal's utility function is multilinear, $E(B_4(x)|\beta, \eta, a^*)$ is linear in β and η and the density function h belongs to the class specified in (6), then two separate linear aggregates of the accounting signals, one an insurance aggregate and the other an incentive aggregate will be economically sufficient for the optimal compensation contract.

Proposition 3 and Corollary 1 show that the optimal compensation contract can be written as the function of two accounting aggregates, an aggregate insurance measure ν and an aggregate incentive measure τ . The analysis of Proposition 3 can be directly extended to the case of multiple signals. The optimal compensation contract based on multiple signals can be summarized into just two aggregate measures, one aggregate corresponding to the insurance dimension and the second aggregate corresponding to the incentive dimension, without any economic loss to the agency. Thus, instead of reporting the values of each of the individual signals with which to share risk and motivate the agent, the accountant need only report the aggregate insurance measure and the aggregate incentive measure.

The aggregation procedure substantially reduces the amount of information that the accountant must report without any economic loss accruing to the agency from either an insurance or incentive perspective. In the statistics literature there is a considerable volume of work on minimal sufficient statistics that considers

how signals may be combined without any loss of information in a statistical sense. Our results here are similar in spirit, to the extent that we are looking for the maximum reduction in information by means of aggregation of various signals. In our context, the aggregates do result in a loss of information in a statistical sense. However, our concern is with the aggregation of information without any economic loss rather than statistical loss. Our results suggest that the volume of information on which the agent's optimal compensation contract is based can be substantially reduced.

Concluding remarks

Many common situations in management accounting are characterized by the outcome being not jointly observable and the principal's and agent's preferences being multiattribute in nature. Our generalization of principal-agent models to these settings provides a different perspective on the value of accounting measurement and extends the applicability of agency models in accounting to a richer and broader context.

When outcome is assumed to be jointly observable, agency models recommend only accounting systems that develop accounting signals to provide information about the agent's performance, since no demand for measuring outcome exists. However, when outcome is not jointly observable, the accounting system has an additional role: to provide measures of outcome that facilitate risk sharing between the principal and agent. The value of accounting signals can therefore arise either on account of the insurance dimension or on account of the incentive dimension.

Our analysis indicates that signals on the outcome x (that is not jointly observed) are of no value if the principal is multivariate (intertemporally) risk neutral, that is, the principal's utility function is separable in x and ϕ . In other words, there is no risk-sharing (or insurance) demand for information if the principal is multivariate risk neutral. In contrast with the traditional principal agent literature, risk neutrality of the principal over wealth is irrelevant when multiattribute preferences are considered.

Our results provide insights into the nature of information that is valuable in management accounting for performance evaluation and risk sharing. We show that precisely two factors determine whether a signal will be valuable in the agency relationship between the principal and agent: observability of the agent's effort and the principal's multivariate risk neutrality.

The condition obtained for one signal (say y) to be informative about the *unobserved outcome* given a second signal (say z) is different from the condition for one signal to be informative about the *agent's action choice* given another signal. With respect to the agent's action, the notion employed is that of sufficiency of one signal given the other. When the outcome is not jointly observed, the relevant notion is that of the additional information provided by the signal about the probability distribution of the outcome x .

Multiple accounting signals are often available for both risk sharing (insur-

ance) and performance evaluation (incentive) purposes. We show that under certain conditions, the insurance components of the multiple signals can be aggregated into a single aggregate insurance measure and the incentive components of the signals can be aggregated (via a different aggregation procedure) into another aggregate incentive measure. Thus, the accounting consolidation process can reduce multiple accounting signals to just two aggregate performance measures, an aggregate insurance dimension and an aggregate incentive dimension without any economic loss to the agency.

Appendix

Proof of Lemma 1

We first differentiate condition (1.1) with respect to y to obtain

$$U_\phi \left[\int W_{\phi\phi} g(x|y, z, a^*) dx \right] \phi_y^* + U_\phi \int W_{\phi} g_y(x|y, z, a^*) dx - \left[\int W_{\phi} g(x|y, z, a^*) dx \right] U_{\phi\phi} \phi_y^* = 0.$$

Therefore,

$$\phi_y^* = \frac{-U_\phi \int W_{\phi} g_y(x|y, z, a^*) dx}{U_\phi \int W_{\phi\phi} g(x|y, z, a^*) dx - U_{\phi\phi} \int W_{\phi} g(x|y, z, a^*) dx}.$$

Similarly, differentiating with respect to z we obtain

$$\phi_z^* = \frac{-U_\phi \int W_{\phi} g_z(x|y, z, a^*) dx}{U_\phi \int W_{\phi\phi} g(x|y, z, a^*) dx - U_{\phi\phi} \int W_{\phi} g(x|y, z, a^*) dx}.$$

Proof of Proposition 1A

The assumption that the principal is intertemporally risk neutral, i.e. $W_{\phi x} = 0$, implies that the principal's utility function can be written as $W(\phi, x) = G(\phi) + H(x)$. Condition (1.1) then becomes

$$\frac{\int G_\phi(\phi)g(x|y, z, a^*) dx}{U_\phi} + \lambda = 0 \quad \text{for all } y, z.$$

Therefore, $G_\phi/U_\phi = -\lambda$ (a constant) for all y, z . Differentiating with respect to each of y and z , we note that $\phi_y^* = \phi_z^* = 0$. That is, the optimal transfer schedule is independent of the realizations of the signals y and z . Hence there is no value to observing the signals y and z .

Proof of Proposition 1B

Since we have assumed that y provides no additional information about the probability distribution of x , $g(x|y, z, a^*) = g(x|z, a^*)$ which is independent of y . Therefore, $g_y(x|y, z, a^*) = 0$ everywhere, and condition (2.1) yields $\phi_y^* = 0$ everywhere. Therefore, the Pareto optimal transfer schedule $\phi^*(y, z)$ is independent of y . In other words, y has no marginal value.

Proof of Proposition 1C

The assumption that the principal is intertemporally risk neutral, i.e., $W_{\phi x} = 0$, implies that the principal's utility function can be written as $W(\phi, x) = G(\phi) + H(x)$. Condition (3.1) then becomes

$$\frac{G_\phi(\phi) \int g(x|y, z, a^*) dx}{U_\phi} = -\lambda - \mu \left[\frac{U_{a\phi}}{U_\phi} + \frac{h_a(y, z|a^*)}{h(y, z|a^*)} \right];$$

that is,

$$\frac{G_\phi(\phi)}{U_\phi(\phi)} = -\lambda - \mu \left[\frac{U_{a\phi}}{U_\phi} + \frac{h_a(y, z|a^*)}{h(y, z|a^*)} \right].$$

Differentiating with respect to y and noting that

$$\frac{\partial}{\partial \phi} \frac{U_{a\phi}}{U_\phi} = \frac{\partial}{\partial a} \frac{U_{\phi\phi}}{U_\phi} = 0$$

by assumption, condition (4.2) becomes

$$\phi_z^* = - \frac{\mu \frac{\partial}{\partial z} \left(\frac{h_a(y, z|a^*)}{h(y, z|a^*)} \right)}{\frac{\partial}{\partial \phi} \left(\frac{G_\phi(\phi)}{U_\phi(\phi)} \right)}. \tag{A.1}$$

Since z is uninformative about a given y ,

$$\frac{\partial}{\partial z} \left(\frac{h_a(y, z|a^*)}{h(y, z|a^*)} \right) = 0$$

everywhere. Therefore, $\phi_z^* = 0$. That is, the optimal contract is independent of the signal z .

Next, differentiating with respect to y condition (4.1) becomes

$$\phi_y^* = - \frac{\mu \frac{\partial}{\partial y} \left(\frac{h_a(y, z|a^*)}{h(y, z|a^*)} \right)}{\frac{\partial}{\partial \phi} \left(\frac{G_\phi(\phi)}{U_\phi(\phi)} \right)}. \tag{A.2}$$

Therefore, ϕ_y^* is nonzero for some y, z . That is the signal y will be employed in the optimal contract.

Proof of Lemma 2

Differentiating condition (3.1'') with respect to β we have

$$\begin{aligned} \frac{\partial}{\partial \phi} \left[\frac{\int W_\phi g(x|\beta, \eta, a^*) dx}{U_\phi} \right] \phi_\beta^* + \frac{\int W_\phi g_\beta(x|\beta, \eta, a^*) dx}{U_\phi} \\ = -\mu \frac{\partial}{\partial \beta} \left(\frac{U_{a\phi}}{U_\phi} \right) - \mu \frac{\partial}{\partial \beta} \left(\frac{h_a(\beta, \eta|a^*)}{h(\beta, \eta|a^*)} \right). \end{aligned}$$

Now

$$\frac{\partial}{\partial \beta} \left(\frac{U_{a\phi}}{U_\phi} \right) = \left[\frac{\partial}{\partial \phi} \left(\frac{U_{a\phi}}{U_\phi} \right) \right] \phi_\beta = \left[\frac{\partial}{\partial a} \left(\frac{U_{\phi\phi}}{U_\phi} \right) \right] \phi_\beta$$

which is equal to zero, by assumption. Therefore,

$$\phi_\beta^* = - \frac{\frac{\int W_\phi g_\beta(x|\beta, \eta, a^*) dx}{U_\phi} + \mu \frac{\partial}{\partial \beta} \left(\frac{h_a(\beta, \eta|a^*)}{h(\beta, \eta|a^*)} \right)}{\frac{\partial}{\partial \phi} \left[\frac{\int W_\phi g(x|\beta, \eta, a^*) dx}{U_\phi} \right]}.$$

Similarly, ϕ_η^* is obtained by differentiating (3.1'') with respect to η .

Proof of Proposition 2

The proof follows immediately from Lemma 2. If $W_{\phi x} = 0$ then W_ϕ is independent of x . Therefore, we can write

$$\begin{aligned} \int W_\phi g_\beta(x|\beta, \eta, a^*) dx &= W_\phi \int g_\beta(x|\beta, \eta, a^*) dx \\ &= W_\phi \frac{\partial}{\partial \beta} \int g(x|\beta, \eta, a^*) dx \\ &= 0, \end{aligned}$$

since

$$\int g(x|\beta, \eta, a^*) dx = 1.$$

Furthermore, since β is not informative given η ,

$$\frac{\partial}{\partial \beta} \frac{h_a}{h} = 0$$

and the second term in the numerator of the expression (4.1) for ϕ_β^* also vanishes. Hence, $\phi_\beta^* = 0$ for all (β, η) .

Proof of Proposition 3

Since $W = B_1(\phi) + B_2(x) + B_3(\phi) \cdot B_4(x)$ we have

$$W_\phi = B_{1\phi}(\phi) + B_{3\phi}(\phi) \cdot B_4(x),$$

and

$$\begin{aligned} \int W_\phi g(x|\beta, \eta, a) dx &= B_{1\phi}(\phi) \int g(x|\beta, \eta, a) dx + B_{3\phi}(\phi) \int B_4(x)g(x|\beta, \eta, a) dx \\ &= B_{1\phi}(\phi) + B_{3\phi}(\phi) \cdot E[B_4(x)|\beta, \eta, a]. \end{aligned}$$

Therefore, constraint (3.1'') becomes

$$\frac{B_{1\phi}(\phi)}{U_\phi} + \frac{B_3(\phi) \cdot E[B_4(x)|\beta, \eta, a^*]}{U_\phi} = -\lambda - \mu \frac{U_{a\phi}}{U_\phi} - \mu \frac{h_a(\beta, \eta|a^*)}{h(\beta, \eta|a^*)}. \quad (\text{A.3})$$

Writing

$$\nu = \nu(\beta, \eta, a^*) = E[B_4(x)|\beta, \eta, a^*]$$

and

$$\tau = \tau(\beta, \eta, a^*) = h_a(\beta, \eta, a^*)/h(\beta, \eta, a^*)$$

we note that the above equation in (A.3) is in three variables ϕ , ν , and τ only.

Therefore, the optimal compensation function ϕ^* is determined by ν and τ only, the signals β and η are not required. That is, we must have

$$\phi^* = \phi^*(\nu, \tau, a^*).$$

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