

Sensitivity, Precision, and Linear Aggregation of Signals for Performance Evaluation

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1. Introduction

Several accounting and other signals are generally available for the construction of a managerial performance evaluation measure on which an optimal compensation contract is based. The demand for aggregation in evaluating managerial performance arises because reporting all the basic transactions and other nonfinancial information about performance is costly and impracticable (see Ashton [1982], Casey [1978], and Holmstrom and Milgrom [1987]). We identify necessary and sufficient conditions on the joint density function of the signals under which linear aggregation, a simple and commonly employed way to construct a performance evaluation measure, is optimal. This characterization suggests that the linear form of aggregation is optimal for a large class of situations. Focusing on performance measures that are linear aggregates enables us to determine the relative weights on the individual signals in the optimal linear aggregate, since these weights are invariant for all realizations of the signals. We interpret these weights in terms of statistical characteristics (sensitivity and precision) of the joint distribution of the signals.

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We break the design of the optimal compensation contract into two stages: (1) constructing a managerial performance evaluation measure, and (2) choosing a compensation contract based on that measure. We focus on the first stage, which is more closely identified with the management accountants' role of structuring and implementing a measurement system that first generates imperfect signals (denoted in our model by y and z)¹ about the performance of the manager and then aggregates these signals into a single evaluation measure, $\pi = \pi(y, z)$. The principal then chooses a compensation contract $\psi(\pi) = \psi(\pi(y, z))$ as a function of the performance evaluation measure, π , rather than the individual signals y and z . Emphasizing the first stage allows us to examine when individual signals may be linearly aggregated even though compensation $\psi = \psi(\pi(\cdot))$ under the optimal incentive contract may be nonlinear in π for optimal risk-sharing purposes. The usual agency models suppress the evaluation measure π and write the compensation function ϕ as a direct function of y and z .

The agency literature in accounting and economics, or the related literature in statistics, provides little guidance about how signals should be combined in constructing an evaluation measure.² Holmstrom's [1979] informativeness condition provides little insight about the optimal relative weights ($l:m$) on each signal y and z in the optimal linear aggregate $ly + mz$. For a subclass of distributions of the stochastic signals y and z for which some linear aggregation is optimal, we interpret the relative weights in terms of the sensitivity and precision of the signals. The sensitivity of a signal measures the extent to which the expected value of a signal changes with the agent's action, adjusted for the correlation with the other signal which may also change with the agent's effort. Precision indicates the lack of noise in a signal. The relative weight on each signal in the optimal performance evaluation measure for incentive purposes is directly proportional to the product of the sensitivity and precision of the signal.³

Our paper relates to earlier work in agency theory, in particular, Holmstrom [1979]. His informativeness condition suggests that the signal z will be used in the agent's compensation contract no matter how noisy the signal, provided y is not a sufficient statistic for the pair (y, z) with respect to a . We show, for the subclass considered by us, an information

¹ In much of the agency literature in accounting, one signal has typically been assumed to be the output x from which the principal derives utility. More generally, we can model this as a situation where the output x is simply a deterministic, or even a stochastic, function of y and z .

² An exception is Holmstrom's [1982, theorem 8] result on relative performance evaluation.

³ The optimal relative weights should not be interpreted as measures of the economic value of signals. Instead, the optimal weights characterize the manner in which individual signals must be aggregated to obtain the performance evaluation measure when linear aggregation is optimal.

signal z will be valuable, as in Holmstrom [1979], if it is sensitive, but the weight on it in the linear aggregate for incentive purposes will be small if it is noisy, that is, if it lacks precision. Holmstrom's [1979] result states that aggregation will be optimal provided the aggregate is a sufficient statistic for the individual signals y and z . In this paper, the optimal linear aggregates are *not* sufficient statistics for the individual signals. In our subclass, no scalar aggregate $\pi = \pi(y, z)$ is a sufficient statistic for the individual signals (y, z) except under certain specific conditions discussed in Amershi, Banker, and Datar [1988]. The key difference between Holmstrom's [1979] result and ours is that we examine conditions for *some* linear aggregation to be optimal in all agencies while, as we elaborate in section 3, Holmstrom's result can be interpreted as identifying conditions for the *same* linear aggregate to be optimal for all agencies.

In section 2, we describe the basic model and derive expressions for the gradients of the agent's compensation function with respect to the signals. In section 3, we introduce the idea of consolidating multiple signals to construct a performance evaluation measure to be used in the agent's compensation contract. We identify necessary and sufficient conditions on the joint probability distribution of the signals for which the performance evaluation measure can be optimally constructed by combining the signals linearly for all agencies. For a subclass of these joint probability distributions, we interpret, in section 4, the relative weights with which the signals are linearly aggregated as being directly proportional to the product of the sensitivity and the precision of the respective signals. Section 5 discusses our results and their accounting implications. Concluding remarks are presented in section 6.

2. The Basic Model

We consider a two-person, single-period principal-agent model where the agent takes some action⁴ $a \in [a, \bar{a}] \subseteq R$, not observed by the principal, which together with a random unobserved state of nature θ generates two signals y and z , observed by the principal. Thus $y = y(a, \theta) \in Y \subseteq R$ and $z = z(a, \theta) \in Z \subseteq R$. The actual monetary outcome of interest to the principal, denoted by x , is measured in terms of the signals y and z by a known function⁵ $x = x(y, z) \in X \subseteq R$, such that $x_y(y, z), x_z(y, z) \geq 0$. The agent's compensation ϕ is a function of the jointly observed signals y and z , that is $\phi: Y \times Z \rightarrow R$ with the residual $x(y, z) - \phi(y, z)$ accruing

⁴ For ease of exposition and consistency with much of the agency literature in accounting, we consider a single-dimensional action a . However, as we demonstrate in section 4, the extension of our model to the case of multidimensional actions is direct.

⁵ More generally, x can be modeled as a stochastic function of y and z as in Mirrlees [1976], Gjesdal [1982], and Banker, Datar, and Maindiratta [1988]. We illustrate this in section 5. Note that in particular one of the two signals, say y , may be considered to be the outcome x itself, so that $x_y(y, z) = 1$ and $x_z(y, z) = 0$.

to the principal. We write the principal's utility function as $W(x - \phi)$, with $W' > 0$ and $W'' \leq 0$. The agent is assumed to be strictly risk and work averse with a separable, twice continuously differentiable utility function $U(\phi) - V(a)$ with $U'(\cdot) > 0$, $U''(\cdot) < 0$, $V'(\cdot) > 0$, and $V''(\cdot) \geq 0$. The agent's compensation is restricted⁶ to the real interval $[\underline{\phi}, \bar{\phi}]$.

The function $f_a(y, z; a)/f(y, z; a)$ is continuously differentiable in y and z and has a support independent of $a \in A$. We invoke the monotone likelihood ratio condition (*MLRC*) for the signals y and z ⁷ to show that the optimal compensation contract is monotonic in y and z . The principal and agent know the structure of the choice problem, their utility functions, and the set of available options. They jointly observe only the accounting signals $y \in Y$ and $z \in Z$ and of course the outcome $x = x(y, z)$ assumed here to be a deterministic function of y and z . They also share identical state beliefs encoded in the density function $f(\cdot)$.

The principal's problem is characterized by the following program:

$$\max_{\phi, a} \int \int W[x(y, z) - \phi(y, z)]f(y, z; a)dydz \quad (1)$$

subject to:

$$\int \int U[\phi(y, z)]f(y, z; a)dydz - V(a) \geq \underline{U} \quad (1a)$$

$$a \in \operatorname{argmax}_a \int \int U[\phi(y, z)]f(y, z; a)dydz - V(a) \quad (1b)$$

$$a \in [a, \bar{a}], \phi \in [\underline{\phi}, \bar{\phi}]. \quad (1c)$$

To characterize the optimal contract, we employ the first-order approach, and replace (1b) with the following first-order condition assuming the usual regularity conditions.⁸

$$\int \int U[\phi(y, z)]f_a(y, z; a)dydz - V'(a) = 0. \quad (1b')$$

Let λ and μ denote the Lagrange multipliers for the constraints (1a) and (1b'). Point-matched optimization of the Lagrangian yields the following characterization of the optimal compensation contracts for a^* and ϕ^* in

⁶ If the compensation function is not restricted to a finite interval, the existence of an optimal solution to the principal-agent problem cannot be guaranteed. See Mirrlees [1974]. For a more precise discussion of the existence of a solution to the agency problem, see Holmstrom [1979, p. 77].

⁷ Whitt [1980] has shown that *MLRC* implies first-order stochastic dominance. Milgrom [1981] has shown that this is equivalent to the statistical inference from the observation of a higher value of the signal that the agent has taken a higher level of effort.

⁸ For a discussion of the validity of the mathematically more tractable first-order approach, see Mirrlees [1979], Rogerson [1985], and Banker and Datar [1987].

the interior:

$$\frac{W'}{U'} = \lambda + \mu \frac{f_a(\cdot)}{f(\cdot)}. \quad (2)$$

Furthermore, we have the following adjoint condition:

$$\begin{aligned} \frac{\partial}{\partial a} \iint W[x(y, z) - \phi(y, z)]f(y, z; a)dydz \\ + \mu \frac{\partial^2}{\partial a^2} \left\{ \iint U[\phi(y, z)]f(y, z; a)dydz - V(a) \right\} = 0. \end{aligned} \quad (3)$$

Note, however, that if f_a/f is linear in y and z and if, for instance, y is unbounded below, the right-hand side of equation (2) will be negative for some y , while the left-hand side is always positive. In this case, an interior solution is clearly not possible and the optimal ϕ^* will be at its lower bound value ϕ of y . The Kuhn-Tucker conditions imply that such a boundary solution with $\phi^*(y, z) = \phi$ will occur for values of y and z for which $\lambda + \mu f_a/f \leq W'(x - \phi)/U'(\phi)$. For further discussion, see Mirrlees [1976, p. 125]. However, note that for other values of y and z for which an interior solution obtains, equation (2) can be employed to characterize the optimal ϕ^* .

To ensure that the agency problem is nontrivial we assume that the second-best optimal action a^* is greater than the minimal action a . Then, as in Holmstrom [1979], it follows that $\mu > 0$. Banker and Maindiratta [1986] also show that ϕ^* is differentiable on $(\underline{\phi}, \bar{\phi})$. Differentiating (2) with respect to y and z , we obtain:

$$\phi_y^* = \frac{\rho^P}{\rho^P + \rho^A} x_y(y, z) + \mu \frac{U'/W'}{\rho^P + \rho^A} \frac{\partial f_a(y, z; a)}{\partial y f(y, z; a)} \quad \text{and} \quad (4)$$

$$\phi_z^* = \frac{\rho^P}{\rho^P + \rho^A} x_z(y, z) + \mu \frac{U'/W'}{\rho^P + \rho^A} \frac{\partial f_a(y, z; a)}{\partial z f(y, z; a)} \quad (5)$$

where $\rho^P = -W''/W'$ and $\rho^A = -U''/U'$ are the Arrow-Pratt risk-aversion functions for the principal and the agent respectively.

We use these characterizations of the gradients to explore the reliance of the compensation contract on each of the signals. For expositional clarity, we develop the results for the case when the principal is risk neutral ($\rho^P = 0$). We also indicate the complexities introduced and the modifications required when the principal is risk averse ($\rho^P > 0$).

3. Linear Aggregation of Accounting Signals

In this section, we examine conditions for the optimality of the simple, commonly observed linear aggregation of signals in constructing a performance evaluation measure. The optimal linear aggregate for a specific agency (which is the only problem of interest to the principal) is a

function of the distribution of signals at the optimal action a^* , induced by the principal, which in turn depends on the specific utility function of the agent. Consequently, for different actions a^* that the principal may induce, the optimal solution to the principal's problem causes the relative weights in the linear aggregate to vary. In Proposition 1, we provide sufficient conditions under which linear aggregation is robust with respect to any action a^* induced by the principal, though the relative weights in the linear aggregate may vary with a^* .

Let β parameterize different agencies with $U^\beta(\cdot) - V^\beta(\cdot)$ representing the utility function of the agent in the agency β . Let a^β represent the optimal (second-best) action for agency β . If linear aggregation is to be optimal for all agencies, that is for all $a^\beta \in A$, then there must exist some arbitrary functions $l(a^\beta)$ and $m(a^\beta)$ such that the optimal contract ϕ^β corresponding to any given agency β can *always* be written in the form $\phi^\beta = \psi^\beta(\pi^\beta)$, where $\pi^\beta = l(a^\beta)y + m(a^\beta)z + n(a^\beta)$ is *some* linear aggregate of y and z .

PROPOSITION 1. When the principal is risk neutral, a sufficient condition for the optimal compensation contract to be written as $\phi^\beta = \psi^\beta(\pi^\beta)$, $\pi^\beta = l(a^\beta)y + m(a^\beta)z$ for all $a^\beta \in A$ is that the joint density function is of the form:

$$f(y, z; a) = \exp \left\{ \int g[l(a)y + m(a)z, a] da + t(y, z) \right\} \quad (6)$$

where $g(\cdot)$, $l(a)$, $m(a)$, $t(y, z)$ are arbitrary functions. Further, in this case $\phi_y^\beta / \phi_z^\beta = l(a^\beta) / m(a^\beta)$.

Proof. See Appendix A.

A broad subclass of the class in (6) is given by:

$$f(y, z; a) = \exp\{p(a)y + q(a)z - r(a) + s(y) + t(z - \gamma y)\} \quad (7)$$

where a is the agent's action choice, $p(\cdot)$, $q(\cdot)$, $r(\cdot)$, $s(\cdot)$, and $t(\cdot)$ are arbitrary functions of a , y , or z as indicated, and γ is a scalar parameter. For this class of joint probability density functions, the conditional distribution⁹ of the signal $y|z$ (and also $z|y$) includes many common distributions such as (truncated) normal, exponential, gamma, chi-square, and inverse Gaussian. These conditional distributions include many of the common parametric functional forms for continuous probability distributions that have been considered (to our knowledge) in the agency literature¹⁰ in accounting.

COROLLARY 1. If $f(y, z; a)$ belongs to the class in (7), then ϕ^* can be written as $\psi^*(\pi)$ where π is a linear combination of y and z .

Proof. See Appendix A.

⁹ These univariate conditional density functions were first considered by Darmais [1935] and Koopman [1936].

¹⁰ See, for instance, Mirrlees [1976], Holmstrom [1979; 1982], Baiman and Demski [1980], Basu et al. [1985], and Amershi [1985].

REMARK 1

We see from Proposition 1 that linear consolidation of signals is optimal for a large class of joint density functions, which includes most of the common continuous probability distributions considered in agency models in accounting. But the compensation contract ψ itself need not be linear in y and z for this class of distributions. That is, even though $\pi = ly + mz$ is a linear function of y and z , the compensation function $\psi(\pi)$ need not be linear in π (and hence in y and z). However, as observed in Banker and Datar [1986], if the principal is risk neutral and the agent's utility function is logarithmic, $U' = 1/\phi$ and for the class in (7) the optimal $\phi^* = \lambda + \mu[p_a(a^*)y + q_a(a^*)z - r_a(a^*)]$ and the compensation contract itself is linear in y and z .¹¹

REMARK 2

The condition in (6) for the joint density function is sufficient to ensure that *some* but not necessarily the *same* linear aggregate of y and z is optimal for all agencies. However, it is *not* a necessary condition for a linear aggregate of y and z to be optimal for a *specific* agency with given utility functions for the principal and for the agent. Consider the joint density function f of the form given by:

$$\ln f(y, z; a) = \int g_1[l(a)y + m(a)z]da + t(y, z) + g_2(a - a^*)g_3(y, z),$$

where $g_1(\cdot)$, $g_2(\cdot)$, and $g_3(\cdot)$ are arbitrary functions with $g'_2(0) = 0$, and a^* is the optimal action for the specific agency under consideration. A linear aggregate $l(a^*)y + m(a^*)z$ is indeed optimal because:

$$f_a(y, z; a^*)/f(y, z, a^*) = g_1[l(a^*)y + m(a^*)z],$$

since $g'_2(a - a^*)g_3(y, z)$ vanishes at $a = a^*$. Thus, we can construct any number of joint density functions, outside the class specified in (6) for which f_a/f is coincidentally a function of a linear aggregate of y and z at $a = a^*$.

Instead of evaluating the optimal linear aggregate for a specific agency (and action), our focus in Proposition 2 is to identify necessary conditions on the joint density functions of y and z with the more robust property which ensures that *some* linear aggregation will always be optimal, *whatever* the utility function of the agent and the action induced by the risk-neutral principal. Note that the joint density function considered in Remark 2 does not satisfy this more robust property. For another agency, in which the optimal action induced is other than the a^* considered in the example, a linear aggregate of y and z will not be optimal, since the

¹¹ In fact, since $\phi^* = U'^{-1}[1/(\lambda + \mu f_a/f)]$, a logarithmic utility function for the agent and joint density function $f(y, z; a)$ of the class described in (7) with $t(y, z)$ in place of $t(z - \gamma y)$, constitutes "almost necessary" and, of course, sufficient conditions for the optimal compensation contract ϕ^* to be linear in the signals y and z .

expression $g_2(a - a^*)g_3(y, z)$ will not vanish. The class of joint density functions identified in Proposition 2 is precisely the class described in Proposition 1. We are thus able to characterize the class of joint density functions for which some linear aggregation is optimal for *all* agencies, although the relative weights in the optimal aggregation will vary with each agency.

PROPOSITION 2. A necessary (and sufficient) condition for the optimal compensation contract to be written as $\phi^\beta = \psi^\beta(\pi^\beta), \pi^\beta = l(a^\beta)y + m(a^\beta)z$ for all possible actions $a^\beta \in A$ is that the joint density function is of the form in (6), that is, $f(y, z; a) = \exp\{\int g[l(a)y + m(a)z, a]da + t(y, z)\}$, where $g(\cdot), l(a), m(a), t(y, z)$ are arbitrary functions.

Proof. See Appendix A.

REMARK 3

While Proposition 2 provides necessary and sufficient conditions for *some* linear aggregation to be optimal for *all* agencies β , the weights $l(a^\beta)$ and $m(a^\beta)$ vary with the optimal action a^β for each agency β . If, on the other hand, we are interested in necessary and sufficient conditions for the optimality of a specific $ly + mz$, with l and m independent of a^β , we can proceed as in Proposition 2 and obtain:

$$f(y, z; a) = \exp \left\{ \int g(ly + mz, a)da + t(y, z) \right\}$$

as the necessary and sufficient condition on the joint density function. We can then rewrite this condition as:

$$f(y, z; a) = f_1(ly + mz, a)f_2(y, z) \quad \text{for all } a, y, z, \quad (8)$$

where $f_1(\cdot) = \exp\{\int g(ly + mz, a)da\}$ and $f_2(\cdot) = \exp\{t(y, z)\}$, which is precisely the condition for $ly + mz$ to be a sufficient statistic for y and z with respect to a (see DeGroot [1970] and Holmstrom [1979]). The notion of a sufficient statistic is not adequate for examining when some linear aggregation is optimal across all agencies and actions induced by the principal since the joint density function $f(y, z; a)$ in (6) that characterizes this class cannot always be written in the form in (8).

REMARK 4

If the principal is risk averse, then the signals y and z will be required for determining the value of the outcome $x = x(y, z)$ for optimal risk-sharing arrangements, in addition to the information required for motivating the agent as considered earlier in the case of a risk-neutral principal. That is, even when the density function f is of the form in (6), the optimal compensation function ϕ^β will depend on $x = x(y, z)$ in addition to $\pi^\beta = l(a^\beta)y + m(a^\beta)z$. Therefore, the single linear aggregate π^β will be adequate only if we can write $x(y, z) = \hat{x}(\pi^\beta)$. However, in many situations x will be some (other) linear function of y and z . In such cases, *two* linear aggregates π^β and x , will together be optimal.

4. Sensitivity and Precision of Signals

We interpret the relative weights $l(a^j)$ and $m(a^j)$ in the optimal linear aggregate in terms of statistical measures summarizing the joint probability distribution of the accounting signals. For reasons of analytical tractability we focus on a subclass of the distributions in (6) identified as the necessary and sufficient conditions in Proposition 1 for some linear aggregation to be optimal in all agencies. Johnson and Kotz [1970, p. 32] note that this subclass defined by (7) is itself a fairly broad class that includes many common distributions. The class imposes few severe restrictions in that the distribution may be convex or concave and skewed on either side of the mean. It permits consideration of a large number of parameters since $p(\cdot)$, $q(\cdot)$, $r(\cdot)$, $s(\cdot)$, and $t(\cdot)$ are arbitrary functions. Furthermore, the class allows means, variances, and covariance to be functions of the agent's action a .¹²

It follows immediately from (4) and (5) that when the principal is risk neutral and linear aggregation is optimal for the class in (7):

$$\frac{\phi_y^*}{\phi_z^*} = \frac{l(a)}{m(a)} = \frac{\partial f_a(y, z; a)}{\partial y f(y, z; a)} \bigg/ \frac{\partial f_a(y, z; a)}{\partial z f(y, z; a)}$$

where the relative weights $l(a):m(a)$ are independent of the actual realized values of the signals y and z . We interpret the relative weights in terms of the means, variances, and covariance of the joint probability distribution of the signals. Note, however, that our interpretations apply only to distributions belonging to the class in (7) and not to all distributions for which linear aggregation is optimal as identified in (6).

For our exponential-type bivariate distributions of signals we denote the precision of signal y by $\rho_1^2 = 1/\text{Var}(y)$, and the precision of signal z by $\rho_2^2 = 1/\text{Var}(z)$. Sensitivity measures the change in the expected value of the signal with changes in the level of effort of the agent. For uncorrelated signals y and z , we define the (unadjusted) sensitivity of signal y by $\mu_{1a} = \partial E(y)/\partial a$, and the (unadjusted) sensitivity of signal z by $\mu_{2a} = \partial E(z)/\partial a$. For correlated signals y and z , we modify the notion of sensitivity to measure the sensitivity of one signal relative to the other. We define the (adjusted) sensitivity of signal y as $\zeta_{1a} = \mu_{1a} - k\rho_2^2\mu_{2a}$, where k is $\text{Cov}(y, z)$ and ρ_2^2 is the precision of z as defined above. We similarly define $\zeta_{2a} = \mu_{2a} - k\rho_1^2\mu_{1a}$ as the (adjusted) sensitivity of signal z .

We show, for the class of density functions defined in (7), that the weight on a particular signal in the optimal linear aggregate used for incentive purposes is directly proportional to its sensitivity and precision.

¹² Although the variances and the correlation coefficient must be independent of a in the case of a normal distribution, this restriction is relaxed for some other members of the class in (7). For instance, in the case of a gamma, exponential, chi-square, or truncated normal distribution, the variances and the correlation coefficient may depend on a .

We first prove the result for the case of uncorrelated signals, that is, with $\gamma = 0$ in (7).¹³

PROPOSITION 3. If the joint density function of y and z is of the form $f(y, z; a) = \exp\{p(a)y + q(a)z - r(a) + s(y) + t(z)\}$ defined over $y \in Y \subseteq R$ and $z \in Z \subseteq R$:

$$\frac{l}{m} = \frac{\rho_1^2 \mu_{1a}}{\rho_2^2 \mu_{2a}}$$

when ϕ^* is in the interior of $[\underline{\phi}, \bar{\phi}]$. Note that l , m , and ρ_i^2 , μ_{ia} for $i = 1, 2$, are evaluated at $a = a^*$.¹⁴

Proof. See Appendix A.

We next consider the general case of correlated accounting signals. We show that the optimal weight on a particular signal in the linear aggregate used for incentive purposes is directly proportional to its precision and (adjusted) sensitivity measured relative to the other signal, as defined earlier.

PROPOSITION 4. If the joint density function $f(y, z; a)$ is of the form $f(y, z; a) = \exp\{p(a)y + q(a)z - r(a) + s(y) + t(z - \gamma y)\}$, $\gamma \neq 0$, defined over $y \in Y \subseteq R$ and $z \in R$, $l/m = \rho_1^2 \zeta_{1a} / \rho_2^2 \zeta_{2a}$ when $\phi^*(y, z) \in (\underline{\phi}, \bar{\phi})$. Note that ρ_i^2 and ζ_{ia} for $i = 1, 2$ are evaluated at $a = a^*$.

Proof. See Appendix A.

COROLLARY 2. If the joint density function f belongs to the class defined in (7) over $y \in Y \subseteq R$ and $z \in R$, then $\gamma = \text{Cov}(y, z) / \text{Var}(y)$.

It is evident that linear transformations of the original signals belong to the class in (7). Moreover, the relative weights on the signals y and z in the optimal linear aggregate are scale-invariant, that is, the relative weights do *not* vary when the *same* linear transformation is applied to both the signals y and z .

We have developed our results on sensitivity, precision, and linear aggregation for the case of multiple signals about the agent's action, modeled as single dimensional. Our results, however, extend directly to the case where the agent's action is multidimensional (see also Matsu-mura [1985]). Suppose the agent selects actions a_1 and a_2 that affect the distribution of both the signals y and z . The joint density function can then be written in the form:

$$f(y, z; a_1, a_2) = \exp\{p(a_1, a_2)y + q(a_1, a_2)z - r(a_1, a_2) + s(y) + t(z - \gamma y)\}.$$

¹³ Bildikar and Patil [1968, p. 1317] show that for this class y and z are independent if and only if they are uncorrelated.

¹⁴ The above results generalize directly to the case when the signals y and z are independent and $f(y, z; a) = p(a)h(y) + q(a)k(z) - r(a) + s(y) + t(z)$. In this case:

$$\frac{l}{m} = \frac{\frac{\partial E[h(y)]}{\partial a} \text{Var}[k(z)]}{\frac{\partial E[k(z)]}{\partial a} \text{Var}[h(y)]} \frac{h_y(y)}{k_z(z)}.$$

If the principal is risk neutral, the first-order condition for optimization is:

$$\frac{1}{U'(\phi(y, z))} = \lambda + \mu_1 \frac{f_{a_1}(y, z; a_1, a_2)}{f(y, z; a_1, a_2)} + \mu_2 \frac{f_{a_2}(y, z; a_1, a_2)}{f(y, z; a_1, a_2)}$$

where μ_1 and μ_2 are the Lagrange multipliers on the incentive compatibility constraints for a_1 and a_2 respectively. The optimal linear aggregate is $(\mu_1 p_{a_1} + \mu_2 p_{a_2})y + (\mu_1 q_{a_1} + \mu_2 q_{a_2})z$. The modified sensitivity measure for each signal is now the weighted sum of the sensitivity of the signal to each action a_i where the weights are the Lagrange multipliers μ_i . Due to the Envelope Theorem these multipliers are interpreted as $\partial EW/\partial a_i$, the marginal importance of each action to the principal. The relative weight on each signal is then equal to the weighted sensitivity measure times the precision of the signal.

5. Discussion and Accounting Implications

In many accounting contexts, the outcome x of the agent's effort is not jointly observed,¹⁵ but two distinct jointly observable signals y and z are available which measure the outcome x with error. The incentive contract can then be based on these signals. The question of interest is the relative weights on the signals y and z in an optimally designed incentive contract. If the joint density function of y and z belongs to the class in (7), with $y = x + \eta$ and $z = x + \tau$ where x , η , and τ are independent stochastic variables,¹⁶ $\text{Var}(y) = \text{Var}(x) + \text{Var}(\eta)$ and $\text{Var}(z) = \text{Var}(x) + \text{Var}(\tau)$. Furthermore, since $y - z = \eta - \tau$ we have $\text{Var}(y) + \text{Var}(z) - 2\text{Cov}(y, z) = \text{Var}(\eta) + \text{Var}(\tau) = \text{Var}(y) - \text{Var}(x) + \text{Var}(z) - \text{Var}(x)$. Therefore, $\text{Cov}(y, z) = \text{Var}(x)$. Next we compute the (adjusted) sensitivity of each of the signals y and z as follows:

$$\zeta_{ya} = [\partial E(x)/\partial a] \text{Var}(\tau) / [\text{Var}(x) + \text{Var}(\tau)] \quad \text{and}$$

$$\zeta_{za} = [\partial E(x)/\partial a] \text{Var}(\eta) / [\text{Var}(x) + \text{Var}(\eta)].$$

Hence, $l/m = \text{Var}(\tau)/\text{Var}(\eta)$. In other words, the relative weights for the two signals y and z will be inversely proportional to the variances of their corresponding (measurement) errors η and τ respectively.

Our model provides some insight into the practice of adjusting divisional profits by corporate overheads in evaluating divisional performance. Consider a divisional manager whose action a directly affects divisional profits, y , such that $\partial E(y)/\partial a > 0$. The corporate overhead, z , is not affected by the agent's action, so that $\partial E(z)/\partial a = 0$. Divisional profits and corporate overheads are correlated with a correlation coeffi-

¹⁵ These situations have been considered by Mirrlees [1976, p. 122], Gjesdal [1982], and Banker, Datar, and Maindiratta [1988].

¹⁶ The variables η and τ may be interpreted as "measurement errors."

cient, r , $0 \leq r < 1$. In the following proposition we show that z will be used in the optimal performance evaluation measure if and only if z and y are correlated, that is, $r \neq 0$. Further, the relative weight $m:l$ on the signal z relative to signal y is directly proportional to the correlation coefficient.

PROPOSITION 5. If the joint density function of y and z belongs to the class in (7), and $\mu_{1a} > 0$, $\mu_{2a} = 0$, $\rho_1^2 > 0$ and $\rho_2^2 > 0$, then the relative weight:

$$R = -\frac{m}{l} = +k\rho_2^2 = +\frac{r\sigma_y}{\sigma_z}$$

and $\partial R/\partial E(y) = 0$, $\partial R/\partial E(z) = 0$, $\partial R/\partial \sigma_z = -r\sigma_y/\sigma_z^2 < 0$, $\partial R/\partial \sigma_y = r/\sigma_z > 0$, $\partial R/\partial r = +\sigma_y/\sigma_z > 0$.

Proof. See Appendix A.

Proposition 5 suggests that noncontrollable corporate overhead expense will be used in the construction of the optimal performance evaluation measure if and only if this accounting signal is correlated with controllable divisional profits. This is in contrast to Baiman and Demski [1980] who develop a demand for overhead cost allocation from the additional information conveyed by the cost allocation basis which serves as an additional signal. In our model, the correlation of overhead costs with divisional profits is the key. Overhead costs, z , provide information about the agent's action choice a even though a does not directly influence z .

The absolute value of the relative weight ($-m/l$) on the overhead cost signal in the optimal performance evaluation measure is independent of $E(y)$ and $E(z)$. It increases if the signal y is noisier (i.e., σ_y increases) or if y and z are more positively correlated (i.e., r increases) and decreases if the signal z is noisier (i.e., σ_z increases).

Our results on the use of corporate overhead costs in the divisional manager's optimal performance evaluation measure are similar in spirit to the results on relative performance evaluation, as in Baiman and Demski [1980] and Holmstrom [1982], where the output of one agent is used in the performance evaluation of a second agent when the outputs of the two agents are correlated. Thus, the notions of sensitivity and precision, which we have employed in determining the relative weights in optimal linear aggregation, extend directly to the case of relative performance evaluation.

We next apply the results of our model to evaluate the performance of a profit center (divisional) manager whose action a affects revenue measured by ($\$y$) and costs measured by ($\$ - z$). The joint distribution of y and z belongs to the class in (7). It follows from Proposition 4 that the relative weight on each signal $l:m$ in the optimal performance evaluation measure is directly proportional to the sensitivity times precision of each signal. The noisier a signal, the smaller the weight on that

signal in the agent's performance evaluation measure.¹⁷ The more sensitive a signal to changes in the manager's level of effort, the greater the weight on that signal in the manager's performance evaluation measure.¹⁸

We refer to the product of the sensitivity and precision as the intensity of the signal. The performance evaluation measure $ly + mz$ can be written as a function of the profit number $(y + z)$ alone only when the intensity of the revenue signal equals the intensity of the cost signal. If the weight on the cost signal $(-z)$ is greater than the weight on the revenue signal (y) , the optimal performance evaluation measure may be written as $l(y + z) + (m - l)z$. The optimal performance evaluation measure then is based on the profit number $(y + z)$ and on the more heavily weighted cost signal.

Following Holmstrom [1979], a necessary condition for a manager to be evaluated as a profit center rather than as a cost center is that revenues be marginally informative about the manager's effort given costs. Our discussion above illustrates that this condition is not sufficient. When the sensitivity times precision of revenue (y) and costs $(-z)$ are not equal, it is not optimal to evaluate the profit center manager on the basis of the *pure* profit number alone.

Individual elements of costs within a cost center are often added together to form an aggregate cost measure. In the cost accounting literature, a commonly suggested criterion for aggregation is the *homogeneity* of the individual cost components. Homogeneity means that the costs assigned to a particular cost pool should exhibit the same pattern of response to the various determinants of cost behavior (Shillinglaw [1972, p. 77]).¹⁹ Feltham [1977] develops this notion of homogeneity with respect to aggregation of costs into cost pools within an information economics framework to evaluate the sufficiency of the cost aggregate for a single decision maker. In a two-person setting, our results indicate that a simple equally weighted aggregation of individual cost components into a single cost pool for performance evaluation purposes is optimal if and only if the intensity (sensitivity times precision) of all individual components is the same. If the intensity of any individual cost signal is greater, the optimal performance evaluation measure will include *both* the aggregate cost and the cost signal whose intensity is greater.²⁰

¹⁷ Holmstrom [1982] obtains a similar result in the case of a multivariate normal distribution.

¹⁸ Our results provide a formal economic basis for extending laboratory experiments involving accounting aggregation for single-person decision making (Barefield [1972]) to multiperson economic settings involving evaluation of performance.

¹⁹ Shillinglaw [1977, p. 21] suggests that accounting data should be collected and aggregated in ways that make it easier to identify the relationships between costs and their determinants. Horngren and Foster [1987, p. 457] also note that Cost Accounting Standards Board regulations require homogeneous cost pools for product costing purposes.

²⁰ Another example of linear aggregation of signals occurs in the case of an investment center. In this case, the manager's effort influences both the operating income (y) and the level of investment $(-z)$. If the joint density function of y and z belongs to the class in (7),

6. Concluding Remarks

Multiple signals are commonly available for evaluating managerial performance. We distinguish between the performance evaluation measure based on basic signals and the compensation schedule based on the evaluation measure. We focus on the management accountants' role in combining detailed records (signals) into an aggregate performance evaluation measure. In Propositions 1 and 2 we identify necessary and sufficient conditions under which some linear aggregation is optimal for all agencies, and for a subclass we obtain an interpretation (in terms of the sensitivity, precision, and intensity of the individual signals) of the optimal weights in such linear aggregates.²¹

An implication of our analysis is that evaluating the performance of a profit center (divisional) manager on the basis of a pure profit number will be optimal only if the sensitivity times precision (intensity) of revenue (y) and costs ($-z$) is equal. If the intensity of either signal is greater, the optimal performance evaluation measure will depend on the profit number and the more intense signal.

Our analysis also demonstrates that it is optimal to include noncontrollable corporate overhead expenses in the construction of the optimal divisional performance evaluation measure if corporate overheads are correlated with controllable divisional profits, a result in the spirit of relative performance evaluation, as in Holmstrom [1982]. The notions of sensitivity and precision extend directly to the case of relative performance evaluation. The absolute value of the relative weight on the signal not controllable by the agent in the optimal performance evaluation measure is directly proportional to the precision of that signal as well as the correlation coefficient between it and the other signal controllable by the agent.

the optimal performance evaluation measure will look like residual income. However, the relative linear weight (m/l) for the investment signal in the optimal linear aggregate for managerial performance evaluation will depend on the intensity of the two signals and will, in general, differ from the interest rate. Moreover, for joint density functions of y and z that belong to the class in (7), the ratio measure return on investment will not be an optimal performance evaluation measure for incentive purposes. In fact, proceeding as in Proposition 2, the necessary and sufficient condition for return on investment to be the optimal performance evaluation measure for all agencies β is that $f_{\alpha}(y, z; \alpha^{\beta})/f(y, z; \alpha^{\beta}) = g(y/z, \alpha^{\beta})h(y, z)$ for all $\alpha^{\beta} \in A$, where $g(\cdot)$ and $h(\cdot)$ are any arbitrary functions. In other words y/z is a sufficient statistic for (y, z) with respect to the agent's action a .

²¹ We do not investigate the amount or extent of loss (a) if equal weights are used in the linear aggregate instead of the optimal weights or (b) if accounting signals outside the class for which linear aggregation is optimal are aggregated linearly. Determining the extent of loss analytically is difficult because it entails a comparison of integrals. In a single-person decision-making context, Feltham [1977] estimates losses resulting from aggregation using simulation techniques.

APPENDIX A

Proof of Proposition 1

We show that if $f(y, z; a)$ is indeed of the form in (6), then $\phi^\beta(y, z)$ can be written as $\psi^\beta[l(a^\beta)y + m(a^\beta)z]$ for all agencies β . If f belongs to this class of joint density functions, then $f_a/f = g[l(a)y + m(a)z]$. Furthermore, since the optimal compensation function $\phi^\beta(y, z)$ is characterized by the equation $\frac{1}{G(\phi)} = \lambda + \mu \frac{f_a(y, z, a^\beta)}{f(y, z, a^\beta)}$, where $G = (U^\beta)'$, we can write $\phi^\beta = G^{-1}[1/(\lambda + \mu g(\pi^\beta))] = \psi^\beta(\pi^\beta)$, where $\pi^\beta = l(a^\beta)y + m(a^\beta)z$.

Proof of Corollary 1

We simply show that if f is in class (7), then it is in the class in (6). This is seen immediately by writing $l(a) = p_a(a)$, $m(a) = q_a(a)$, $g(\cdot) = p_a(a)y + q_a(a)z - r_a(a)$, and the constant of integration with respect to a as $s(y) + t(z - \gamma y)$, in particular.

Proof of Proposition 2

Suppose $\phi^\beta(y, z) = \psi^\beta[l(a^\beta)y + m(a^\beta)z]$. Differentiating with respect to each of y and z , we have $\phi_{y^\beta} = (\psi^\beta)' l(a^\beta)$ and $\phi_{z^\beta} = (\psi^\beta)' m(a^\beta)$. Therefore, $\phi_{y^\beta}/\phi_{z^\beta} = l(a^\beta)/m(a^\beta)$. From equations (4) and (5), we also have:

$$\frac{l(a^\beta)}{m(a^\beta)} = \frac{\phi_{y^\beta}}{\phi_{z^\beta}} = \frac{\partial[f_a(y, z; a^\beta)/f(y, z; a^\beta)]/\partial y}{\partial[f_a(y, z; a^\beta)/f(y, z; a^\beta)]/\partial z}.$$

Next, writing $h^\beta = h(y, z; a^\beta) = f_a(y, z; a^\beta)/f(y, z; a^\beta)$, we have $m(a^\beta)h_{y^\beta} - l(a^\beta)h_{z^\beta} = 0$. This is a linear homogeneous partial differential equation whose *subsidiary equation* can be written as:

$$\frac{l(a^\beta)}{dy} = \frac{m(a^\beta)}{dz} = \frac{0}{dh}.$$

Therefore, any solution to it must be of the form $h(y, z, a^\beta) = g[l(a^\beta)y + m(a^\beta)z, a^\beta]$, where g is any arbitrary function. See, for instance, Scarborough [1965, p. 95], and Sneddon [1957, p. 50]. Note further that $\partial \ln f(y, z; a^\beta)/\partial a = h(y, z; a^\beta) = g[l(a^\beta)y + m(a^\beta)z, a^\beta]$ holds for all $a^\beta \in A$. Therefore, integrating both sides of the equation with respect to a^β , we obtain:

$$f(y, z; a) = \exp \left\{ \int g[l(a)y + m(a)z, a] da + t(y, z) \right\}.$$

Proof of Proposition 3

We begin by proving that for this class of joint density functions:

$$\frac{\partial f_a(y, z; a^*)}{\partial y f(y, z; a^*)} = p_a(a^*) = \rho_1^2 \mu_{1a}$$

and:

$$\frac{\partial f_a(y, z; a^*)}{\partial z f(y, z; a^*)} = q_a(a^*) = \rho_2^2 \mu_{2a}.$$

The proposition will then follow immediately since:

$$l:m = \frac{\partial f_a}{\partial y f} \bigg/ \frac{\partial f_a}{\partial z f}.$$

Note that for the class of joint density functions considered in this proposition, y and z are independent stochastic variables, with density functions:

$$f_1(y) = \exp\{p(a)y - r_1(a) + s(y)\} \quad \text{and}$$

$$f_2(z) = \exp\{q(a)z - r_2(a) + t(z)\},$$

where $r_1(a) + r_2(a) = r(a)$. Note that for simplicity in exposition we suppress the parameter a in $f_1(\cdot)$, $f_2(\cdot)$, and also in $E(\cdot)$, $\text{Var}(\cdot)$.

In order to next derive expressions for $E(y)$ and $\text{Var}(y)$, we shall adopt the method employed in two-sided Laplace transform theory. See, for instance, Holbrook [1966], Jaeger and Newstead [1969], and Widder [1941].

Since $\int f_1(y) dy = 1$, we have $\int \exp\{p(a)y + s(y)\} dy = \exp\{r_1(a)\}$. Therefore, on differentiating with respect to $p(a)$, we have:

$$\int y \exp\{p(a)y + s(y)\} dy = r_1'(a) \exp\{r_1(a)\}/p'(a)$$

and hence, $E(y) = r_1'(a)/p'(a)$. To obtain an expression for $E(y^2)$, we once again differentiate the above integral with respect to $p(a)$. Thus:

$$\begin{aligned} \int y^2 \exp\{p(a)y + s(y)\} dy \\ = [r_1'/p']^2 \exp r_1 + \frac{1}{p'} [(p'r_1'' - p''r_1')/p'^2] \exp r_1 \end{aligned}$$

and hence:

$$E(y^2) = [r_1'/p']^2 + [(p'r_1'' - p''r_1')/p'^3].$$

Therefore, $\text{Var}(y) = E(y^2) - [E(y)]^2 = (p'r_1'' - p''r_1')/p'^3$.

In addition, differentiating $E(y)$ with respect to a , we obtain:

$$\frac{\partial}{\partial a} E(y) = \left[\frac{p' r_1'' - p'' r_1'}{p'^2} \right].$$

Thus, $\frac{\partial}{\partial a} E(y) = p' \text{Var}(y)$, or equivalently:

$$\frac{\partial}{\partial y} \frac{f_a}{f} = p_a(a) = p' = \frac{\partial}{\partial a} E(y)/\text{Var}(y) = \rho_{1\mu_{1a}}^2.$$

Similarly, it can be shown that:

$$\frac{\partial}{\partial z} \frac{f_a}{f} = q_a(a) = \rho_{2\mu_{2a}}^2.$$

Proof of Proposition 4

We begin by making the linear transformation $\omega = z - \gamma y$, so that $E(\omega) = E(z) - \gamma E(y)$, and $\text{Var}(\omega) = \text{Var}(z) + \gamma^2 \text{Var}(y) - 2\gamma \text{Cov}(y, z)$.

Furthermore, note that the joint density function $g(y, \omega; a)$ can be expressed as $g(y, \omega; a) = \exp\{[p(a) + \gamma q(a)]y + q(a)\omega - r(a) + s(y) + t(\omega)\}$ and, therefore, it is evident that y and ω are independent stochastic variables and their joint density function belongs to the class considered in Proposition 3.

Since $z = \omega + \gamma y$, we also have $\text{Var}(z) = \text{Var}(\omega) + \gamma^2 \text{Var}(y)$. Comparing this expression with the earlier expression relating variances of y , z , and ω , it follows that $2\gamma^2 \text{Var}(y) = 2\gamma \text{Cov}(y, z)$, and hence, $\gamma = \text{Cov}(y, z)/\text{Var}(y) = k\rho_{1z}^2$, where $k = \text{Cov}(y, z)$.

Next consider the optimal linear aggregate of the two signals y and ω for incentive purposes. Let l' and m' be the weights for y and ω in the optimal linear aggregate.

Since y and ω are independent, from Proposition 3 we have:

$$\frac{l'}{m'} = \frac{\partial E(y)/\partial a}{\text{Var}(y)} \frac{\text{Var}(\omega)}{\partial E(\omega)/\partial a}.$$

Furthermore, we also have:

$$\begin{aligned} l'/m' &= \frac{\partial}{\partial y} \frac{g_a}{g} \bigg/ \frac{\partial}{\partial \omega} \frac{g_a}{g} \\ &= [p_a(a) + \gamma q_a(a)]/q_a(a) \\ &= [p_a(a)/q_a(a)] + \gamma \\ &= [l/m] + \gamma. \end{aligned}$$

Therefore:

$$\begin{aligned}
 (l/m) &= (l'/m') - \gamma \\
 &= \frac{\text{Var}(\omega)[\partial E(y)/\partial a] - \gamma \text{Var}(y)[\partial E(\omega)/\partial a]}{\text{Var}(y)[\partial E(\omega)/\partial a]} \\
 &= \frac{\rho_1^2 \mu_{1a} - \gamma \rho_2^2 \mu_{2a}}{\rho_2^2 (\mu_{2a} - \gamma \mu_{1a})} \\
 &= \frac{\rho_1^2 (\mu_{1a} - k \rho_2^2 \mu_{2a})}{\rho_2^2 (\mu_{2a} - k \rho_1^2 \mu_{1a})}.
 \end{aligned}$$

Proof of Proposition 5

From Proposition 4, the adjusted sensitivity of $z = -k\rho_1^2\mu_{1a}$. Therefore:

$$\begin{aligned}
 R &= -m/l = +k\rho_1^2\mu_{1a}\rho_2^2/\rho_1^2\mu_{1a} \\
 &= +k\rho_2^2 \\
 &= +r\sigma_y/\sigma_z.
 \end{aligned}$$

Comparative statics results follow immediately by differentiating R with respect to $E(y)$, $E(z)$, σ_y , σ_z , and r .

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