

# Optimal transfer pricing under postcontract information\*

RAJIV D. BANKER *University of Minnesota*

SRIKANT M. DATAR *Stanford University*

*Abstract.* This paper analyzes a formal principal-agents model of resource allocation and coordination in which demand for transfer pricing arises endogenously within a decentralized environment characterized by asymmetric information and divergence of preferences. It is shown that a modified Groves scheme achieves full information efficiency in a setting of the type considered by Harris, Kriebel, and Raviv (1982) and Cohen and Loeb (1984) only if the information asymmetry is postcontract and collusion is precluded. Conditions for the optimality of a coordination mechanism that is immune to collusion are also examined. It is shown that demand for a collusion-free marginal cost-type transfer pricing scheme arises if the agents are risk neutral, the cost function is separable but not necessarily linear, and the information asymmetry is postcontract.

*Résumé.* Les auteurs analysent un modèle structuré d'affectation des ressources et de coordination mandant-mandataire, dans lequel la demande de prix de cession interne est issue, de façon endogène, d'un contexte décentralisé caractérisé par une information asymétrique et une divergence des préférences. Les auteurs démontrent qu'un schéma Groves modifié permet d'atteindre l'efficacité maximum de l'information dans un contexte semblable à celui qu'utilisent Harris, Kriebel et Raviv (1982) et Cohen et Loeb (1984), uniquement si l'asymétrie de l'information est postérieure au contrat et si la collusion est rendue impossible. Ils examinent également les conditions d'optimalité d'un mécanisme de coordination qui est à l'abri de la collusion. Les auteurs démontrent qu'il y a demande de prix de cession interne, à l'abri de la collusion, du type coût marginal si les mandataires sont neutres à l'égard du risque, si la fonction de coûts peut être isolée sans être nécessairement linéaire, et si l'asymétrie de l'information est postérieure au contrat.

## Introduction

In this paper, we analyze the resource allocation and transfer pricing problem under conditions of asymmetric information and divergence of preferences when the demand for resource allocation and transfer pricing arises endogenously. We identify conditions under which the principal can implement mechanisms

\* Comments and suggestions by participants in the research workshops at Carnegie Mellon University, Stanford University, the University of Minnesota, and the University of Pennsylvania are gratefully acknowledged.

to achieve collusion-immune first best allocations<sup>1</sup> when agents have private information but commit to stay with the firm after they receive their private information (postcontract information). If collusion is precluded, the optimal contracting arrangements reward managers on the basis of accounting variances of actual performance less budgeted performance. Furthermore, under certain conditions, the simple marginal cost transfer price is optimal and achieves the first best allocation.<sup>2</sup>

The postcontract private information assumption is in contrast to a precontract private information assumption, when each agent can quit the firm after receiving private information. The postcontract private information setting arises in situations in which agents gain firm-specific experience, knowledge, and expertise about their activities after they start working for the owner.<sup>3</sup> It is also applicable in contexts in which there are other high costs to leaving the firm such as search costs, loss of benefits, or penalties for breach of contracts.

Harris and Raviv (1979) show that in a model with a single risk-neutral agent acquiring postcontract private information, the first best outcome is always achievable.<sup>4</sup> With multiple risk-neutral agents making decisions based on their postcontract private information, however, Banker (1981) shows that the first best outcome is, in general, not achievable without a mechanism to communicate information and coordinate the action choices of agents. When collusion between the agents is precluded, we show that a Groves-like transfer scheme is sufficient to produce optimal first best resource allocations as a dominant strategy equilibrium so that the agents collect no rent from their private information. The modified Groves' schemes can be interpreted as budget-based reward structures that endogenously require accounting variance computations. This scheme does not yield first best allocations in the precontract information case. Furthermore, it is susceptible to collusion among various agents.

In the particular case of quasilinear technologies and preferences, we construct a transfer pricing mechanism that not only achieves full information efficiency but also is immune to collusion. Under more general assumptions about the technology and distribution of the agents' private information, we derive transfer price-type mechanisms that are immune to collusion and achieve full information efficiency. In particular, for the usual accounting model comprising fixed and variable costs, resources are transferred to agents at the marginal cost. Note that unlike earlier models of marginal cost transfer pricing (Hirschleifer, 1956) where the need for transfer pricing was assumed and the marginal transfer pricing scheme was suggested as the one that is consistent with headquarters'

1 That is, the owner can implement the same allocations that he or she would with full information.

2 It also demonstrates that when the specific conditions do not obtain, marginal cost pricing is not optimal.

3 Baiman (1982, p. 181) refers to this as *predecision state information asymmetry* and lists several papers in the agency literature in which the private information acquired by the agent(s) is modeled in this manner.

4 Banker and Maindiratta (1986) show that this result, in general, does not hold if the principal has general multiattribute preferences, such as when the outcome is nonmonetary.

preferences, in our model there is an endogenous demand for marginal cost transfer pricing. Our result should also be contrasted with the results of Harris, Kriebel, and Raviv (henceforth HKR, 1982) where the transfer price is set above the marginal cost for incentive compatibility reasons.<sup>5</sup> The HKR result flows from their assumption of precontract private information and the need to trade off optimal resource allocation decisions against informational rents that accrue to the agent.

The next section describes the basic model. The third section discusses the modified Groves-Vickrey-Clark scheme and its properties. Demonstration of the achievability of a first best allocation using a collusion-proof transfer pricing mechanism is carried out in section four. The fifth section analyzes the optimality of marginal cost transfer prices. The sixth section offers a conclusion.

### The basic model

We consider a firm consisting of  $(N + 2)$  divisions indexed  $i = 0, 1, \dots, (N + 1)$ , and a corporate headquarters. The division indexed 0 provides a resource used by  $N$  intermediate divisions indexed  $1, \dots, N$ . This resource cannot be purchased externally and is produced at a cost  $c$ , which depends on the quantities  $x_i, i = 1, \dots, N$ , of the resource demanded by the intermediate divisions, the amount of managerial effort  $a_0$  provided by the manager of division 0, and the productivity  $\theta_0$  of division 0; that is,  $c = c(\underline{x}, a_0, \theta_0)$  where  $\underline{x} = (x_1, \dots, x_N)$ . The productivity parameter  $\theta_0$  in producing the resource is known only to the divisional manager. Each of the divisions indexed  $i = 1, \dots, N$ , uses  $x_i$  units of the resource along with effort  $a_i$  to produce  $y_i$  units of output. The quantity of  $y_i$  produced depends on the productivity  $\theta_i$  of division  $i$  so that  $y_i = t_i(x_i, a_i, \theta_i)$ . As before, the productivity  $\theta_i$  of  $x_i$  in producing  $y_i$  is known only to division manager  $i$ . The units of  $y_i$  produced are transferred to the sales division, indexed by  $(N + 1)$ , which is responsible for selling  $\underline{y} = (y_1, \dots, y_N)$ . The revenue  $r(\underline{y}, a_{N+1}, \theta_{N+1})$  generated by this division depends on the sales manager's effort  $a_{N+1}$  and the division's productivity parameter  $\theta_{N+1}$ , which is known only to the sales division manager, all private information being acquired postcontract.

We assume that each divisional manager has disutility for effort, prefers more rewards to less, and is risk neutral in rewards. If allocations are made based on reported productivity, managers may have incentives to report incorrectly to maximize their utility. Our objective is to find an intrafirm resource allocation scheme to coordinate activities and maximize firm profits in a decentralized decision-making environment. The scheme will determine the outputs  $y_i, i = 1, \dots, N$ , to be produced; the resources  $x_i, i = 1, \dots, N$ , allocated to the intermediate divisions; and the payments  $\Phi_i, i = 0, 1, \dots, (N + 1)$  to each divisional manager.

The only role of headquarters is to install a coordination mechanism to

5 Furthermore, in invoking the revelation principle (see Myerson, 1979, among others) to search for the optimal allocation mechanism, HKR preclude collusion among various agents.

determine the outputs to be produced, the resources to be allocated to the intermediate divisions, and the rewards to be paid to managers. As noted earlier, the total cost to produce the resource required by divisions,  $1, \dots, N$ , is given by  $c(\underline{x}, a_0, \theta_0)$  where  $\underline{x} = (x_1, \dots, x_N)$ . The other costs of production are the rewards  $\Phi_i, i = 0, 1, \dots, (N+1)$ , paid to division managers. The total revenue to the firm from selling  $y_i, i = 1, \dots, N$ , is given by  $r(\underline{y}, a_{N+1}, \theta_{N+1})$ , where  $\underline{y} = (y_1, \dots, y_N)$ .

We also assume that the utility function of each division manager is separable in wealth and effort:

$$u_i(\Phi_i, a_i) = \Phi_i(\cdot) - v_i(a_i) \quad (1)$$

Without loss of generality, we assume that the minimum utility level of each agent is zero.

With respect to the information structure and sequence of moves in the game, we assume that headquarters cannot observe either the effort levels or the productivity parameters of the divisions. We assume also that at the time of contracting, headquarters and all division managers share common beliefs about the joint probability distribution of  $\theta_i, i = 0, 1, \dots, N, N+1$ . After agreeing to the contract offered by the principal, each division manager is assumed to observe perfectly his or her own productivity parameter but not those of other managers. Each division manager commits to remaining with the firm even after observing his or her private information  $\theta_i$ . All other aspects of the environment such as the structure of the revenue and cost functions and the utility functions of the agents are common knowledge.

The sequence of moves in the game are as follows. Headquarters moves first by specifying the resource allocation and coordination mechanism (the rules of the game) and offering contracts to the divisional managers. The divisional managers accept the contracts before they acquire private information and commit to remaining with the firm after acquiring private information. After contracting, division managers observe their private productivity parameters and send messages to headquarters about their private information. The message is chosen by each division manager to maximize his or her expected utility. Resources are allocated in accordance with the prespecified mechanism, followed by action choice by each division manager to maximize expected utility. Finally, payments to division managers are determined based on the messages received and any other jointly observable information signals, such as resources allocated, costs incurred, outputs produced, and revenues generated.

If headquarters were in a position to obtain full information about the actual realizations of  $\underline{\theta} = (\theta_0, \dots, \theta_{N+1})$  acquired after the initial contracting, then its problem could be formulated as follow:

$$\text{Maximize } E \left[ r(\underline{y}, a_{N+1}, \theta_{N+1}) - c(\underline{x}, a_0, \theta_0) - \sum_{i=0}^{N+1} \Phi_i(\cdot) \right] \quad (2a)$$

subject to

$$E[\Phi_i(\cdot) - v_i(a_i)] \geq 0 \quad \forall i = 0, \dots, (N + 1) \tag{2b}$$

where the expectations are taken over all  $\theta_i, i = 0, \dots, (N + 1)$ , and  $\Phi_i(\cdot)$  may be a function of divisional messages  $\underline{m} = (m_0 \dots m_{N+1})$ , allocations  $\underline{x}, \underline{y}$ , and observed  $c(\cdot)$  and  $r(\cdot)$ . The maximization problem reflects the fact that headquarters maximizes expected profit based on the probability distribution over all  $\theta_i, i = 0, \dots, (N + 1)$  at the start of the game. Inequality (2b) indicates that the minimum utility constraint for each divisional manager  $i$  need be satisfied only in expectational terms, since manager  $i$  commits to stay with the firm even if an unfavorable  $\theta_i$  subsequently occurs. We shall refer to the resource allocation achieved as a solution to the problem in (2a) and (2b) as a first best allocation.

Under our assumptions about the postcontract nature of information and the commitment on the part of the agent to remain with the firm, constraint (2b) is binding—see Baiman and Demski (1980). Therefore,  $E[\Phi_i(\cdot)] = E(v_i(a_i)) \forall i = 0, \dots, (N + 1)$ . Substituting for  $E[\Phi_i(\cdot)]$  in (2a), headquarters' maximization problem in the full information case can be written as the following unconstrained problem:

$$\underset{\underline{x}, \underline{y}, a}{\text{Maximize}} E \left[ r(\underline{y}, a_{N+1}, \theta_{N+1}) - c(\underline{x}, a_0, \theta_0) - \sum_{i=0}^{N+1} v_i(a_i) \right] \tag{2'}$$

where the expectation is over all  $\theta_i, i = 0, \dots, (N + 1)$ . Since, in the full information case, information about the particular realization of  $\underline{\theta}$  is obtained before the allocations  $\underline{x}, \underline{y}$  are made and efforts chosen, maximizing the above expectation requires the expression

$$\pi = r(\underline{y}, a_{N+1}, \theta_{N+1}) - c(\underline{x}, a_0, \theta_0) - \sum_{i=0}^{N+1} v_i(a_i) \tag{2''}$$

to be maximized for each realization of  $\underline{\theta}$ . Indeed, any mechanism that achieves the first best (full information) allocation must maximize (2'') for all possible realizations of  $\underline{\theta}$ .

If the division managers' information  $\underline{\theta}$  is private and not directly observed by headquarters, headquarters must establish a mechanism to elicit the division managers' information if the allocations are to reflect optimally this private information. A general mechanism to achieve this is a two-round communication mechanism with divisions sending messages  $m_i, i = 0, \dots, (N + 1)$  to headquarters about  $\theta_i$  and headquarters allocating resources based on the messages  $\underline{m}$  in accordance with a prespecified decision rule. Thus, additional constraints need to be imposed on the full information problem described by (2a) and (2b) to implement a Bayesian-Nash equilibrium as in Myerson (1984).

Headquarters first announces the decision rules to determine the allocations  $\underline{x}$  and  $\underline{y}$  based on the possible messages that it may receive from the divisions. That is, the decision rules are specified as

$$x_i^* = x_i^*(\underline{m}), \quad y_i^* = y_i^*(\underline{m}), \quad \forall i = 1, \dots, N \tag{2c}$$

The domain of each of these functions corresponds to the set of all possible values of  $\theta_i$ . After observing  $\theta_i$ , each divisional manager  $i, i = 0, 1 \dots N + 1$ , will choose a message  $m_i$  that maximizes his or her expected utility, based on his or her expectation about the productivity parameters of all other division managers and the messages to be sent by the other division managers, given his or her private information,  $\theta_i$  and the prespecified decision rules for allocations. In other words, the message  $m_i$  actually selected by each division manager  $i$  must satisfy the following self-selection constraint:

$$E[\Phi_i(\cdot) - v_i(\cdot)|\theta_i, m_i] \geq E[\Phi_i(\cdot) - v_i(\cdot)|\theta_i, \mu_i]$$

$$\text{for all possible messages } \mu_i = \mu_i(\theta_i) \quad (2d)$$

where  $\mu_i(\theta_i)$  is any other message strategy and expectations are taken over the productivities, message strategies, and consequent effort choices of all other division managers conditional on  $\theta_i$  and his or her message  $m_i$  (or  $\mu_i$ ).

On receiving the messages from individual division managers, headquarters announces the allocations  $\underline{x}, \underline{y}$ , according to the prespecified decision rules. Each division manager then chooses an action  $a_i$  to maximize his or her expected utility given his or her own  $\theta_i$ , the messages  $\underline{m}$ , and the allocations  $\underline{x}, \underline{y}$ . The self-selection constraint on each divisional manager's action choice can be expressed as

$$a_i \in \operatorname{argmax} E[\Phi_i(\cdot) - v_i(\cdot)|\theta_i, \underline{m}, \underline{x}, \underline{y}] \quad \forall_i \quad (2e)$$

where the expectation is taken over the possible action choices of the other agents.

Headquarters' problem can then be restated as maximize (2a) subject to (2b), (2c), (2d), and (2e). In the next section, we consider the optimal solution to this agency problem.

### A budget-based mechanism

In this section we observe that a modification of the well-known Groves-Vickrey-Clark scheme provides an optimal solution to the problem in (2a)–(2e) (see also Amershi and Cheng, 1990). In fact, it achieves the first best allocations and can be implemented as a dominant (Nash) strategy equilibrium with each divisional manager reporting truthfully. Here the strategy for each divisional manager  $i$  can be represented as the vector  $(m_i(\theta_i), a_i(\underline{m}))$ . A dominant strategy equilibrium implies that there exists a best message and action choice strategy for each division manager that is independent of the message (and effort level) selected by the other managers and results in each manager truthfully reporting his or her productivity parameter  $\theta_i$  and then taking the first best optimal action  $a_i$ . We next briefly describe the proposed scheme.

The headquarters selects the optimal  $x_i^*(\underline{m}), y_i^*(\underline{m})$ , such that they solve:

$$\text{Maximize}_{\underline{x}, \underline{y}, \underline{a}} \left[ r(\underline{y}, a_{N+1}, m_{N+1}) - c(\underline{x}, a_0, m_0) - \sum_{i=0}^{N+1} v_i(a_i) \right] \tag{3}$$

subject to the technology constraints:  $y_i = t_i(x_i, a_i, \theta_i), i = 1, \dots, N$ . Let  $a_i^*(\underline{m})$  denote the optimal  $a_i, i = 0, \dots, (N + 1)$ , solving (3). Note that the expression in (3) is similar to that in (2'') with  $m_i$  substituted for  $\theta_i$ .

The reward functions  $\Phi_i$  for the managers of each of the  $(N + 2)$  divisions are given by

$$\Phi_{N+1} = -c(\underline{x}^*, a_0^*, m_0) - \sum_{i=0}^N v_i(a_i^*) + r(\underline{y}^*, a_{N+1}, \theta_{N+1}) - E(\pi^*) \tag{4a}$$

$$\Phi_i = r(\underline{y}^*, a_{N+1}^*, m_{N+1}) - c(\underline{x}^*, a_0^*, m_0) - \sum_{\substack{j=1 \\ j \neq i}}^N v_j(a_j^*) - E(\pi^*), \tag{4b}$$

for  $i = 1, \dots, N$ .

$$\Phi_0 = r(\underline{y}^*, a_{N+1}^*, m_{N+1}) - \sum_{j=1}^{N+1} v_j(a_j^*) - c(\underline{x}^*, a_0, \theta_0) - E(\pi^*) \tag{4c}$$

where

$$\pi^* = r(\underline{y}^*, a_{N+1}^*, \theta_{N+1}) - c(\underline{x}^*, a_0^*, \theta_0) - \sum_{i=0}^{N+1} v_i(a_i^*)$$

and the expectation is taken over possible realizations of  $\underline{\theta} = (\theta_0, \dots, \theta_{N+1})$ . The constant  $E(\pi^*)$  in the reward function of each of the divisional managers is chosen to ensure that the division manager's expected utility at the time he or she contracts is equal to zero, his or her minimum utility level.

We can rewrite the division managers' reward functions as

$$\Phi_{N+1} = [\pi^*(\underline{m}) - E\pi^*(\underline{\theta})] + [r(\underline{y}^*, a_{N+1}, m_{N+1}) - r(\underline{y}^*, a_{N+1}^*, m_{N+1})] + v_{N+1}(a_{N+1}^*) \tag{5a}$$

$$\Phi_i = [\pi^*(\underline{m}) - E\pi^*(\underline{\theta})] + v_i(a_i^*) \text{ for } i = 1, \dots, N \tag{5b}$$

$$\Phi_0 = [\pi^*(\underline{m}) - E\pi^*(\underline{\theta})] + [c(\underline{x}^*, a_0^*, m_0) - c(\underline{x}^*, a_0, \theta_0)] + v_0(a_0^*) \tag{5c}$$

It is interesting to note that the modified Groves mechanism uses procedures commonly used in budgeting. The budgeting cycle starts with individual managers reporting on their environment much like the message strategies in the modified Groves scheme. Using the reports, headquarters determines the allocation of resources to individual divisions. After the allocations are made, division managers choose their actions.

The evaluation of managerial performance in budget-based environments is usually done by analyzing variances that measure the deviation of actual from budgeted performance. Indeed, we can interpret the reward functions of division managers to be equal to the aggregate of (in general) three components: (1) compensation for effort under the budget plus the minimum utility level (i.e.,  $v_i(a_i^*) + 0$ ), (2) a profit variance equal to  $\pi^*(\underline{m}) - E\pi^*(\underline{\theta})$ , which represents the difference in budgeted profits based on the messages and expected profits at the time of contracting before division managers acquire their private information, and (3) a revenue (or a corresponding cost) variance equal to  $r(\underline{y}^*, a_{N+1}, \theta_{N+1}) - r(\underline{y}^*, a_{N+1}^*, m_{N+1})$ , which represents the difference between actual revenues generated from the outputs transferred to the sales division and budgeted revenues. In the case of the standard principal agent model, there is no economic reason that compensation should be based on variances (see Baiman and Demski, 1980). In the model developed here, the desire to get truth-inducing reports from privately informed agents results in a compensation structure that requires accounting variance computations.

From (1) and (4a), we observe that the  $(N + 1)$ th division (sales) manager selects  $(m_{N+1}^*(\theta_{N+1}), a_{N+1}^*(\underline{m}))$  to maximize the following function:

$$D_{N+1}(m_{N+1}, a_{N+1}) = -c(\underline{x}^*, a_0^*, m_0) - \sum_{j=0}^N v_j(a_j^*) + r(\underline{y}^*, a_{N+1}, \theta_{N+1}) - E(\pi^*) - v_{N+1}(a_{N+1}). \quad (6)$$

Similar functions  $D_i, i = 0, \dots, N$ , can be constructed for the other divisions.

As in the well-known Groves-Vickrey-Clark scheme, it follows that under the proposed scheme, there exists a dominant strategy equilibrium in which the best message for the  $i$ th division manager is to report truthfully his or her productivity parameter  $\theta_i$  and choose the best effort level  $a_i$ , given the allocations, regardless of the message (and effort levels) of the other managers. This equilibrium solution also achieves ex ante full information efficiency. The proof is omitted as it is analogous to the proof in Groves (1973). The intuition is that the reward function for each division manager is independent of his or her message choice, and the incentives are such that the consequent utility maximization by each manager corresponds exactly to the headquarters' full information maximization problem specified in (2''). Therefore, for each realization of  $\underline{\theta}$ , the proposed scheme allocates resources and induces action choices that maximize total revenues less total costs (including rewards to managers).

Similar modifications to the Groves scheme have been discussed before in the resource allocation and coordination literature. See for instance, Ronen and McKinney (1970), Banker (1981), Conn (1982), and Cohen and Loeb (1984). Cohen and Loeb consider a situation similar to the one considered here and claim (p. 20) that "a (reinterpreted) Groves scheme handles the problem of moral hazard" considered by HKR. They do not, however, explicitly recognize the precontract or postcontract nature of private information in their model.

Although in the no-collusion case, the modified Groves-Vickrey-Clark scheme achieves a first best allocation when the private information is obtained by the agent postcontract (and the agent commits to remain with the firm), this scheme may not be optimal if, as in HKR, the private information is received by the agent prior to accepting the contract. We demonstrate the nonoptimality of the scheme in the case of precontract private information via a counterexample discussed in Appendix A.

A second major assumption in implementing the modified Groves mechanism is the assumption of risk neutrality of the agents and the separability of the utility function in wealth and effort. The Groves mechanism exploits risk neutrality because each agent reveals his or her  $\theta_i$  via a choice among lotteries. Risk is imposed because the agent's compensation depends not only on the particular realization of his or her own  $\theta_i$  but also on the productivity parameter realizations of the other agents. This scheme may not work for risk-averse agents because the agents must then be additionally compensated for the additional risk they are required to bear.

Finally, we focus our attention on another major assumption that underlies the modified Groves scheme. This is the assumption that collusion is exogenously ruled out. HKR maintained a similar assumption. In using the "revelation principle," HKR (1982, p. 609) derive their results within a framework that rules out collusion.

In the next section, we show that the modified Groves mechanism discussed in this section is susceptible to collusion by divisional managers coordinating their message strategies. Collusion is worthwhile even in the absence of transfer payments between division managers and the first best allocation mechanism is not implementable. We formally show that this is *always* the case for the HKR-type linear technology with postcontract perfect information, which is a special case of the general model discussed in this section. We further show that in the postcontract private information case with HKR-type linear technology there exists a transfer price mechanism that is not susceptible to any of the forms of collusion discussed above and yet achieves full information efficiency.

### Linear technology

In this section, we consider a special case of the more general model considered in the second section. Specifically, we examine the model considered by HKR except that we consider the case of postcontract information rather than precontract information. Our general model is easily specialized to yield the HKR model. In particular, we let

$$c(\underline{x}, a_0, \theta_0) = \left( \sum_{i=1}^N x_i - a_0 \right) / \theta_0 \text{ where } \underline{x} = (x_1, \dots, x_N) \quad (7)$$

and

$$y_i = t_i(x_i, a_i, \theta_i) = a_i + \theta_i x_i \text{ for all } i = 1, \dots, N. \quad (8)$$

We continue to assume a more general probability distribution for  $\theta_i$ ,  $i = 0, 1, \dots, N$ , than that in HKR. As in HKR (1982, p. 608), we impose one more set of simplifying assumptions on the feasible allocations:

$$x_i \leq y_i/\beta_i \quad \forall i = 0, 1 \dots N \tag{9}$$

where  $\beta_i$  is the highest possible realization of  $\theta_i$ ,  $i = 0, 1 \dots N$ . The constraints in (9) ensure that no division can be allocated enough resource (or capital) to produce its required output without effort, even if that division has the highest possible productivity parameter value  $\beta_i$ . These assumptions enable us to rewrite (7) and (8) as

$$a_i = y_i - \theta_i x_i, \quad \forall i = 0, \dots, N. \tag{10}$$

The assumption in (9) forces a minimum amount of  $y_0 = \sum_{i=1}^N x_i$  to be produced by division 0 since it implies  $\sum_{i=1}^N x_i(\underline{\theta}) = y_0(\underline{\theta}) > \beta_0 x_0(\underline{\theta})$  for all realizations of  $\underline{\theta}$ .

In the general problem we considered in the second section, changes in the productivities ( $\theta_i$ ) affect marginal costs and thereby influence the profit-maximizing levels of outputs  $\underline{y}$ . Maximizing expected profits will be consistent with minimizing expected costs for a given amount of  $\underline{y}$  either if the demand for  $\underline{y}$  is fixed or if the marginal costs of producing  $\underline{y}$  are independent of  $\underline{\theta}$  for all realizations of  $\underline{\theta}$ . Assumption (9) makes the marginal cost of  $\underline{y}$  independent of  $\underline{\theta}$  and enables us to rewrite the expected profit maximization problem of Section 2 as an expected cost minimization problem for a given  $\underline{y}$ . We therefore suppress the revenue or sales-producing division (division  $(N + 1)$ ) of our general model. We assume, without loss of generality, that the output sold is given by  $y = \min_i \{y_i | i = 1, \dots, N\}$  so that the optimal  $y_i$  to be produced are  $y_i = y$  for all  $i = 1, \dots, N$ .

Since the model described above is a special case of the model described in the previous section, the modified Groves scheme will achieve full information efficiency if collusion is ruled out. Under the scheme,

$$\begin{aligned} \Phi_i^*(\cdot) &= -x_0^*(\underline{m}) - \sum_{\substack{j=0 \\ j \neq i}}^N a_j^*(\underline{m}) + E(c^*) \quad \forall i = 1, \dots, N \\ &= -(N - 1)y + \sum_{\substack{j=0 \\ j \neq i}}^N (m_j - 1)x_j^*(\underline{m}) - x_0^*(\underline{m}) + E(c^*) \end{aligned}$$

and

$$\begin{aligned} \Phi_0^*(\cdot) &= - \sum_{j=1}^N a_j^*(\underline{m}) + E(c^*) - x_0(y_0^*, a_0, \theta_0) \\ &= -Ny + \sum_{j=1}^N m_j x_j^*(\underline{m}) + E(c^*) - x_0(y_0^*, a_0, \theta_0) \end{aligned}$$

where

$$c^* = x_0^* + \sum_{j=0}^N a_j^*,$$

the full information efficient solution is obtained by solving the following problem:

$$\text{Minimize}_{\underline{x}, \underline{\Phi}} E \left[ x_0(\underline{\theta}) + \sum_{i=0}^N \Phi_i(\cdot) \right] \tag{11a}$$

subject to

$$E[\Phi_i(\cdot) - a_i(\underline{\theta})] \geq 0 \quad \forall i = 0, \dots, N. \tag{11b}$$

As discussed in the previous section, constraint (11b) is binding since perfect private information is obtained postcontract, and the full information efficient solution solves the following unconstrained problem:

$$\text{Minimize}_{\underline{x}} E \left[ x_0(\underline{\theta}) + \sum_{i=0}^N a_i(\underline{\theta}) \right]. \tag{12}$$

Substituting  $a_i(\underline{\theta}) = y - \theta_i x_i(\underline{\theta})$ ,  $i = 1, \dots, N$ , and  $y_0(\underline{\theta}) = \sum_{i=1}^N x_i(\underline{\theta})$  in (12), we have

$$\text{Minimize}_{\underline{x}} E \left[ Ny - \sum_{i=0}^N \theta_i x_i(\underline{\theta}) + \sum_{i=0}^N x_i(\underline{\theta}) \right]. \tag{13}$$

Since information about  $\underline{\theta}$  is recorded before the allocations are made, (13) is minimized by

$$\text{Minimize}_{\underline{x}} Ny - \sum_{i=0}^N \theta_i x_i(\underline{\theta}) + \sum_{i=0}^N x_i(\underline{\theta}) \tag{14}$$

at each  $\underline{\theta}$ . Solving (14) directly yields result 1.

*Result 1*

The optimal allocations are given by

$$x_i^* = \begin{cases} x_i^{\max} & \text{if } \theta_i > 1 \\ x_i^{\min} & \text{if } \theta_i < 1 \end{cases} \quad \text{for all } i = 0, 1, \dots, N.$$

where  $x_i^{\max}$  and  $x_i^{\min}$  refer to the maximum and minimum amount of resources that can be allocated to division  $i$ .

*Proof:* See Appendix B.

Proposition 1 indicates that the optimal allocation is a “bang-bang”-type scheme. For values of  $\theta_i > 1$ , division  $i$  is allocated as much of the common

resource (or capital) as possible. For values of  $\theta_i < 1$ , division  $i$  is allocated the minimum amount of the common resource (or capital). Each division manager is then paid  $\Phi_i$  to compensate for the effort  $a_i$  required to produce the required level of outputs,  $y_0 = \sum_{i=1}^N x_i$  for division 0 and  $y$  for divisions  $1, \dots, N$ .

It is interesting that the optimal scheme in the HKR model is also of the "bang-bang" variety in which the optimal  $x$  is at a maximum or minimum, depending on whether  $\theta_i$  is greater than or less than a certain cut-off value that exceeds the full information marginal cost of the common resource to reduce the incentive for underreporting productivity. However, since we are able to implement the full information (first best) efficient solution in the case of postcontract private information, the maximum amount of the common resource (or capital) is allocated whenever  $\theta_i > 1$ , the marginal cost of capital. When agents possess precontract private information as in the HKR model, full information efficiency cannot be attained, and first best allocations must be traded off with the cost of providing incentives to report productivity information.

We next provide a counterexample to show that division managers could benefit by coordinating their messages and deviating from truth-telling strategies. In fact, such collusive behavior can benefit both division managers without any transfer payments being made. For clarity in exposition, we consider the case of  $N = 2$ . Recall that  $y = a_1 + \theta_1 x_1(\underline{\theta}) = a_2 + \theta_2 x_2(\underline{\theta})$  and  $y_0 = x_1(\underline{\theta}) + x_2(\underline{\theta}) = a_0 + \theta_0 x_0(\underline{\theta})$ . Under the modified Groves scheme,

$$\begin{aligned}\Phi_1^*(\cdot) &= -x_0^*(\underline{m}) - a_0^*(\underline{m}) - a_2^*(\underline{m}) + E(c^*) \\ &= (m_0 - 1)x_0^*(\underline{m}) + (m_2 - 1)x_2^*(\underline{m}) - y - x_1^*(\underline{m}) + E(c^*).\end{aligned}$$

Similarly,

$$\Phi_2^*(\cdot) = (m_0 - 1)x_0^*(\underline{m}) + (m_1 - 1)x_1^*(\underline{m}) - y - x_2^*(\underline{m}) + E(c^*).$$

We consider how division manager 1 benefits if division 2 reports  $m_2$  instead of the observed  $\theta_2$  where  $m_2$  is chosen such that<sup>6</sup>

$$\theta_2 < m_2 \leq \theta_2^{\max} \quad \text{if } \theta_2 > 1$$

and

$$\theta_2 < m_2 < 1 \quad \text{if } \theta_2 < 1.$$

First we observe that the allocations  $x_1^*(\underline{\theta})$ ,  $x_2^*(\underline{\theta})$  do not change with the particular choice of  $m_2$  since the allocations, because of their "bang-bang" nature, do not depend on the particular values of  $m_i$  but only on whether the reported values are greater than or less than 1. We choose  $m_2$  such that when  $\theta_2 > 1$ ,  $m_2$  is greater than 1, and when  $\theta_2 < 1$ ,  $m_2$  is less than one. Furthermore, by choosing  $m_2$  to be greater than  $\theta_2$ , the reward  $\Phi_1$  to division manager 1 increases (since the second term increases and the other terms are unchanged).

6 The variable  $\theta_2$  is assumed to be continuous so that an  $m_2$  always exists. Even if  $\theta_2$  were discrete, such an  $m_2$  will generally exist except for some specific values of  $\theta_2$ .

The payment to the manager of division 2 is unaffected by reporting a higher  $m_2$  than the observed  $\theta_2$ . In an analogous manner, the manager of division 2 can be made better off if the manager of division 1 reports  $m_1$  such that

$$\theta_1 < m_1 \leq \theta_1^{\max} \quad \text{if } \theta_1 > 1$$

and

$$\theta_1 < m_1 < 1 \quad \text{if } \theta_1 < 1.$$

Our counterexample demonstrates that the modified Groves scheme is susceptible to collusion (even if transfer payments are precluded) and cannot be implemented unless collusion can be exogenously ruled out. The special nature of the HKR technology, however, permits us to construct an alternative mechanism that is immune to any type of collusion discussed earlier and achieves the first best efficient solution.

The mechanism is a transfer price-type one-round communication scheme rather than the two-round communication scheme in the modified Groves mechanism. Division managers are not required to reveal the values of their privately observed parameters, which constitute the basis for their allocations in accordance with the decision rules specified by the headquarters in the first round of communication. Instead, the headquarters (principal) announces a price of  $-1$  for each unit of  $x_i$  demanded by the division. After receiving private information about  $\theta_i$ , each division manager  $i$  chooses the levels of  $x_i$  and  $a_i$  that maximize his or her utility. These choices yield the full information efficient solutions, as formalized in the following proposition.

#### *Result 2*

A one-round communication mechanism in which each division manager  $i = 1, \dots, N$ , is compensated via  $\Phi_i = E[(1 - \theta_i)x_i^*(\theta_i)] + y - x_i$  is collusion free and achieves the full information efficient solution.

*Proof:* See Appendix B.

Linearity of the technology is not the key factor in achieving a collusion-free mechanism yielding the full information efficient solution. The key factor is that the optimal allocation  $x_i^*$  depends only on the realized value of  $\theta_i$ , not other  $\theta_j$ ,  $j \neq i$ . Such decision separability does not imply that there is no value for the firm to comprise several divisions or that it is optimal to have each division manager operate as a separate entity. This is because the common resource  $y_0$  is being shared by all divisions. We formalize and discuss this in more detail in the next section.

### **Optimal transfer prices**

In this section we consider a more general technology than the linear technology considered in the previous section. We continue to focus on the cost minimization problem for a given level of output  $y$  for each division. The technology is given by  $y_i = a_i + t_i(x_i, \theta_i)$  rather than the linear technology employed in the previous

section. To simplify the exposition, however, we consider only two intermediate divisions using resources of  $x_1$  and  $x_2$ . In contrast to the model in the earlier section, we make the simplifying assumption that there is no uncertainty in the cost of producing the resource vector  $(x_1, x_2)$  and assume that the resources are provided directly by headquarters. We consider a general cost function for the shared resource given by  $c(x_1, x_2)$  in contrast to the linear cost structure assumed in the previous section. The rest of our model is as described in the third section. Our model enables us to emphasize the important characteristics of transfer pricing for a shared resource. As indicated in our discussion in the previous section, expected cost minimization is equivalent to expected profit maximization under our particular technology assumptions.

The full information solution here is characterized by the following problem:

$$\text{Minimize } E[c(x_1, x_2) + \Phi_1(\cdot) + \Phi_2(\cdot)] \tag{15a}$$

subject to

$$E[\Phi_i(\cdot) - a_i] \geq 0, \quad i = 1, 2 \tag{15b}$$

As discussed in the second section, the individual rationality constraint (15b) is binding. Substituting  $E[\Phi_i(\cdot)] = E(a_i)$ ,  $i = 1, 2$ , in (15a), the full information problem may be restated as

$$\text{Minimize } E[c(x_1, x_2) + a_1 + a_2] \tag{16a}$$

We next examine the properties of the first best solution<sup>7</sup> to gain insights as to how the allocations might change with changes in the productivity parameters. Full information efficiency will be attained by minimizing  $c(x_1, x_2) + a_1 + a_2$  for every possible realization of  $\theta = (\theta_1, \theta_2)$ . The production technology here specifies  $a_i = y_i - t_i(x_i, \theta_i)$  so that the first best<sup>8</sup> allocation is achieved by

$$\text{Minimize}_{x_1, x_2} c(x_1, x_2) + 2y - \sum_{i=1}^2 t_i(x_i, \theta_i). \tag{17}$$

for a fixed  $y$ , the optimal  $x_i^*$  is obtained from solving

$$\pi(\theta_1, \theta_2, x_1, x_2) = \text{Maximize}_{x_1, x_2} \sum_{i=1}^2 t_i(x_i, \theta_i) - c(x_1, x_2) \text{ for each } \theta = (\theta_1, \theta_2).$$

Assuming the first best solution  $x_1^*$  and  $x_2^*$  satisfy Lagrange's first- and second-order conditions for a maximum, we have at the point  $(x_1^*, x_2^*)$

7 The model described here is a special case of the model discussed in the second section. Therefore, assuming no collusion, the modified Groves scheme can be employed to implement a first best allocation.

8 Extensions to the case of profit maximization rather than cost minimization discussed above is direct.

$$1 \quad \frac{\partial \pi}{\partial x_i} = \frac{\partial t_i}{\partial x_i} - \frac{\partial}{\partial x_i} c(x_1, x_2) = 0 \quad \text{for } i = 1, 2 \quad (18a)$$

$$2 \quad D = \begin{vmatrix} \frac{\partial^2 \pi}{\partial x_1^2} & \frac{\partial^2 \pi}{\partial x_1 \partial x_2} \\ \frac{\partial^2 \pi}{\partial x_1 \partial x_2} & \frac{\partial^2 \pi}{\partial x_2^2} \end{vmatrix} > 0 \quad (18b)$$

$$3 \quad \frac{\partial^2 \pi}{\partial x_i^2} < 0 \quad \text{for } i = 1, 2 \quad (18c)$$

Before stating and proving proposition 1, we provide some additional notation. We denote

$$c_{11} = \frac{\partial^2 c}{\partial x_1^2}, \quad c_{22} = \frac{\partial^2 c}{\partial x_2^2}, \quad c_{12} = \frac{\partial^2 c}{\partial x_1 \partial x_2}$$

$$t_{ix} = \frac{\partial t_i}{\partial x_i}, \quad t_{i\theta} = \frac{\partial t_i}{\partial \theta_i}, \quad i = 1, 2$$

$$t_{ixx} = \frac{\partial^2 t_i}{\partial x_i^2}, \quad t_{ix\theta} = \frac{\partial^2 t_i}{\partial x_i \partial \theta_i}, \quad i = 1, 2$$

$$z_{ij} = \frac{\partial x_i^*}{\partial \theta_j} \quad \text{for } i, j = 1, 2$$

$$\alpha_{ij} = \frac{\partial a_i^*}{\partial \theta_j} \quad \text{for } i, j = 1, 2$$

*Proposition 1*

- 1 If  $c_{12} > 0$ , then  $\begin{cases} z_{ij} > 0, \alpha_{ij} < 0 & \text{if } j = i \\ z_{ij} < 0, \alpha_{ij} > 0 & \text{if } j \neq i \end{cases}$
- 2 If  $c_{12} = 0$ , then  $\begin{cases} z_{ij} > 0, \alpha_{ij} < 0 & \text{if } j = i \\ z_{ij} = 0, \alpha_{ij} = 0 & \text{if } j \neq i \end{cases}$
- 3 If  $c_{12} < 0$ , then  $\begin{cases} z_{ij} > 0, \alpha_{ij} < 0 & \text{if } j = i \\ z_{ij} > 0, \alpha_{ij} < 0 & \text{if } j \neq i \end{cases}$

*Proof:* See Appendix B.

The intuition behind proposition 1 is as follows. As productivity of a division (say division 1) increases, its marginal cost decreases. Cost minimization requires that larger allocations of  $x_1$  be made. This is true for all values of  $c_{12}$ . The cross partial, however, plays an important role in determining how the allocations to division 2 are affected by changes in the productivity of division 1. If  $c_{12} = 0$ , the marginal cost of  $x_2$  is independent of increases in  $x_1$  and

hence the allocation  $x_2^*$  to division 2 is unaffected. If  $c_{12} > 0$ , the marginal cost of  $x_2$  increases (with the increase in the  $x_1$  allocation as a result of the higher productivity in division 1) and, hence, the optimal allocation of  $x_2^*$  decreases. If  $c_{12} < 0$ , the marginal cost of  $x_2$  decreases with higher allocations of  $x_1$  (resulting from a higher productivity in division 1) and the  $x_2^*$  allocation increases.

The  $\alpha_{ij}$  terms that represent changes in the optimal effort choices of the agents with changes in the productivity parameters always have signs that are the opposite of the  $z_{ij}$  terms. Thus, for instance, as the productivity of division 1 increases, the allocation  $x_1^*$  to division 1 increases and the optimal effort level  $a_1^*$  decreases. Furthermore, if  $c_{12} > 0$ , the allocation  $x_2^*$  decreases and the effort  $a_2^*$  required of division manager 2 increases. This follows directly from the fact that as allocations increase (decrease), lower (higher) levels of effort are required to produce the specified quantity of output  $y$  since effort and resources are substitutable inputs in producing  $y$ .

We use the properties of the first best allocation described above to establish the (necessary and) sufficient conditions to implement a collusion-free first best allocation. Recall that we were able to achieve this in the HKR type of setting in the fourth section since the allocations were independent of the messages and action choices of the other agents. The interesting question here is whether in the more general case considered here, a collusion-free transfer pricing scheme can be implemented that achieves the first best allocation. This is answered in the following proposition.

### *Proposition 2*

A necessary and sufficient condition for a (collusion-free) full information efficient solution to be implementable is that  $c(x_1, x_2) = \beta_0 + \beta_1(x_1) + \beta_2(x_2)$  where  $\beta_0$  is an arbitrary constant, and  $\beta_1(\cdot)$  and  $\beta_2(\cdot)$  are arbitrary functions of  $x_1$  and  $x_2$ , respectively.

*Proof:* See Appendix B.

Proposition 2 indicates that if each division  $i$ ,  $i = 1, 2$ , is charged for the resource  $x_i$  on the basis of the transfer price schedule  $\beta_i(x_i)$ , then a collusion-free first best allocation can be implemented. Note that if there exist *identifiable* fixed costs associated with either of the divisions, such costs may be charged as part of the transfer price schedule  $\beta_i(x_i)$ . Also note that even though the marginal costs are separable, the total costs are not because of the fixed costs that cannot be identified exclusively with division 1 or division 2.<sup>9</sup> Consequently, the asymmetric information problem cannot be eliminated simply by having each

9 Such joint fixed costs are zero only if the resource demands of *both* division 1 and division 2 are zero. We believe that a demand for the *allocation* of such joint fixed costs can also arise endogenously in a model in which the first stage considers the decision to have a common facility and the second stage considers the optimal levels of utilization of the common resource as in this paper, after division managers acquire their additional private information. For a formal development of this setting, see Banker (1988).

division manager  $i$  operate independently with his or her own facility producing  $x_i$ , in addition to the production of  $y_i$  with the technology  $t_i$ .

The driving force behind proposition 2 is the independence of the full information optimal allocations  $x_i^*$  from the productivities  $\theta_j$ ,  $j \neq i$  of the other divisions. Essentially, we exploit the property that when  $c_{ij} = 0$ , the allocation to division  $i$  is independent of the particular realizations of  $\theta_j$ ,  $j \neq i$ . The rationale for transfer pricing exists since divisional managers have private information about their productivities. The appropriate choice of a transfer price provides divisional managers with incentives that induce them to take decisions that would have been taken by headquarters if it had full information.

Thus, the demand for transfer pricing and resource allocation in our model arises endogenously. In the presence of asymmetric information and moral hazard, transfer prices serve as a mechanism that forces managers acting in their own interest to make choices that achieve a first best, full information efficient allocation. Furthermore, the allocation mechanism described in proposition 2 is decentralized in the sense that headquarters plays no active role in the allocation process after announcing the transfer price schedules. Each individual division observes its private information and decides on the amount of resource it should acquire at the specified transfer price.

### Conclusion

The earlier literature argues that the modified Groves scheme handles the problem of moral hazard in the presence of asymmetric information. We demonstrate that a budget-based modified Groves mechanism achieves full information efficiency if (1) agents are risk neutral, (2) asymmetry in information is postcontract, and (3) collusion is precluded. This scheme can be interpreted as setting rewards based on budgets and endogenously computed accounting variances. The Groves-type mechanism, however, is not optimal for the case of precontract private information.

The modified Groves mechanism assumes that collusion among agents is precluded. In searching for collusion-free mechanisms, we show that a (one-round communication) marginal cost pricing scheme is collusion free, meets the endogenous demand for transfer pricing, and achieves full information efficiency.

A simple marginal cost pricing rule is optimal if (1) agents are risk neutral, (2) the cost function is separable but not necessarily linear, and (3) asymmetry in information is postcontract. Given risk-neutral agents and postcontract information asymmetry, we also show that separability of the cost function is a necessary condition for the optimality of marginal cost pricing.

Several authors, beginning with Hirschleifer (1956), have argued in favor of marginal cost transfer pricing. It must be emphasized, however, that these earlier models did not explicitly consider transfer pricing issues within an environment in which a demand for transfer pricing arose endogenously. The need for transfer pricing was assumed by these models, and the marginal pricing scheme was suggested as one that would be consistent with headquarters' preferences. Marginal

cost transfer pricing arises endogenously in our model. Our results are in contrast to other papers (Harris, Kriebel, and Raviv, 1982; and Antle and Eppen, 1985) in which marginal cost transfer pricing is not optimal because agents are assumed to receive precontract private information.

### Appendix A

We consider an example similar to the HKR setting with two divisions, division 0 and division 1, producing an output  $y_1$ . The principal's problem is to implement a resource allocation mechanism that will minimize the expected cost to produce a specified level of output. Division 0 uses effort  $a_0$  and inputs  $x_0$  to produce an intermediate product  $x_1 = a_0 + \theta_0 x_0$  where  $\theta_0$  is the productivity of division 0. The intermediate product  $x_1$  is used by division 1 to produce an output  $y_1$  with a technology given by  $y_1 = a_1 + \theta_1 x_1$  where  $a_1$  is the effort provided by the manager of division 1 and  $\theta_1$  is the productivity of division 1. The productivity  $\theta_i$  is known to division manager  $i$ ,  $i = 0, 1$ , whereas headquarters and the other division manager  $j$ ,  $j \neq i$ , share a common probability distribution about  $\theta_i$ , known also to division  $i$ . The parameter  $\theta_i$ ,  $i = 0, 1$  can take on values of 0.5 and 1.2 with equal probability. The two stochastic variables  $\theta_0$  and  $\theta_1$  are independent. The choice of  $a_i$ ,  $i = 0, 1$ , is also unobservable to headquarters. The problem is to minimize expected costs equal to  $E[x_0 + \Phi_0 + \Phi_1]$  to produce 3.0 units of output  $y_1$ , where  $\Phi_0$  and  $\Phi_1$  are the rewards paid to division managers 0 and 1, respectively. As in HKR, two restrictions are imposed on  $x_0$  and  $x_1$ :  $x_1^{\max} = 2.0$ ,  $x_1^{\min} = 0$ ,  $x_0^{\max} = 1.0$ , and  $x_0^{\min} = 0$ . Since we are considering the precontract asymmetric information case, agents are guaranteed a minimum expected utility level of 0 for all  $\theta_i$ . Since output is prespecified, the principal's objective function and the agents' individual rationality constraints can be specified as follows:

$$\underset{a, x}{\text{Maximize}} E[x_0 + \Phi_0 + \Phi_1] \quad (\text{A1})$$

$$E[\Phi_i(\cdot) - a_i | \theta_i] \geq 0, \forall \theta_i, i = 0, 1. \quad (\text{A2})$$

For a two-round communication mechanism, there will be additional constraints for the decision rules for resource allocation as in (2c) and the agents' self-selection constraints as in (2d) and (2e). In addition, of course, we must consider the technology constraints specified earlier. Note that the agents' individual rationality constraints in (A2) must be satisfied for each  $\theta_i$ . This is in contrast to the postcontract case (equation 2b), where the constraint considers only the overall expected value.

Under the modified Groves scheme, rewards to division managers are computed as follows:

$$\Phi_0 = -x_0 - a_1 + k_{-0}$$

$$\Phi_1 = -x_0 - a_0 + k_{-1}$$

where the constants  $k_{-0}$  and  $k_{-1}$  are chosen to ensure that the minimum utility constraint is satisfied for all possible realizations of  $\theta_0$  and  $\theta_1$ , since otherwise the agent will not accept the contract. The optimal allocations  $x_0^*$  and  $x_1^*$  and the corresponding choices of  $a_0^*$  and  $a_1^*$  under various combinations of  $\theta_0$  and  $\theta_1$  using the modified Groves schemes described above are listed in Table 1. Note that  $\Phi_i^* - k_{-i}^* = -x_0^* - a_j^*$  where  $j \neq i$ . Therefore,

$$k_{-0}^* = \max\{-(-3.0 - 2.6)/2, -(-3.0 - 2.4)/2\} = 2.8$$

$$k_{-1}^* = \max\{-(-3.0 - 3.0)/2, -(-2.6 - 2.4)/2\} = 3.0$$

We briefly describe the computations underlying the figures in Table 1. The choice of parameters  $\theta_0$  and  $\theta_1$  ensures that the first best allocations  $x_0^*$  and  $x_1^*$  achieved under the modified Groves scheme will be such that

$$x_i^* = \begin{cases} x_i^{\min} & \text{if } \theta_i = 0.5 \\ x_i^{\max} & \text{if } \theta_i = 1.2 \end{cases} \quad \forall i = 0, 1.$$

A formal proof of this is provided in the fourth section. The effort levels  $a_i$  are chosen to achieve these allocations. For instance, when  $\theta_0 = 0.5$  and  $\theta_1 = 1.2$ ,  $x_0^* = 0$  and  $x_1^* = 2.0$ . To produce  $x_1^* = 2.0$  with  $x_0^* = 0$ , we must have  $a_0^* = 2.0$ . Then to get  $y_1 = 3$ ,  $a_1 = 3 - (2.0)(1.2) = 0.6$ . Columns (7) and (8) indicate the utilities of the managers of divisions 0 and 1, respectively, under each possible combination of  $\theta_0$  and  $\theta_1$  before adding the constants  $k_{-0}^*$  and  $k_{-1}^*$ . The constant  $k_{-0}^*$ , for instance, is chosen to ensure that the expected utility of the manager of division 0 is at least 0 whether  $\theta_0 = 0.5$  or  $\theta_0 = 1.2$ . When  $\theta_0 = 0.5$ ,  $\theta_1$  may take values 0.5 or 1.2 resulting in an expected utility before adjusting for  $k_{-0}^*$  of  $\frac{-3.0 - 2.6}{2} = -2.8$  for the manager of division 0. Similarly, if  $\theta_0 = 1.2$ ,

this manager's expected utility before adjusting for  $k_{-0}^*$  is  $\frac{-3.0 - 2.4}{2} = -2.7$ .

Since he or she must get an expected utility greater than or equal to zero whether  $\theta_0 = 0.5$  or  $\theta_0 = 1.2$ ,  $k_{-0}^* = \max\{-(-2.8), -(-2.7)\} = 2.8$ . Columns (9) and (10) compute  $\Phi_0^* = k_{-0}^* - x_0^* - a_1^*$  and  $\Phi_1^* = k_{-1}^* - x_0^* - a_0^*$ . Our computations indicate that this manager derives expected rent equal to 0.1 when  $\theta_0 = 1.2$  since  $E(\Phi_0^* - a_0^*) = \frac{1}{2}(-0.2.0) + \frac{1}{2}(1.2 - 0.8) = 0.1 > 0$  and no rent when  $\theta_0 = 0.5$ . This is a well-known result when managers possess precontract private information. A similar computation yields  $k_{-1}^* = 3.0$  and results in expected rent of 0.5 to manager 1 when  $\theta_1 = 1.2$  and no rent when  $\theta_1 = 0.5$ . Column (11) indicates the costs (for producing  $y_1 = 3.0$ ) associated with each of the possible combinations of  $\theta_0$  and  $\theta_1$ . Since each of these alternatives is equally likely, the modified Groves scheme results in an expected cost of  $(0.25)(2.8 + 3.2 + 2.8 + 3.4) = 3.05$ .

Note that in our example if the private information about the particular realizations of  $\theta_0$  and  $\theta_1$  were acquired by the division managers postcontract, the expected cost of implementing the first best allocation is 2.75. This is

achieved because  $k_{-0}^*$  and  $k_{-1}^*$  can be chosen such that the minimum expected utility constraint is satisfied only in expected terms, when expectation is taken over all realizations of  $\theta_0$  and  $\theta_1$ , and hence no rents accrue to the agents. Table 2 indicates the computations of expected costs under the first best allocation.

The computations in Table 2 are very similar to those in Table 1. Indeed, the numbers in columns (1) through (8) are identical to those in the corresponding columns in Table 1. The expected cost is calculated in Column (11) as  $(0.25) \cdot (2.5 + 2.9 + 2.5 + 3.1) = 2.75$ . It is easy to verify that for the full information case also the expected cost will be 2.75. Note that the first best resource allocations  $x_0^*$ ,  $x_1^*$  and action choices  $a_0^*$ ,  $a_1^*$  were achieved by the modified Groves schemes even in the precontract information case; however, because rents accrue to the agents in this case, the expected cost of implementing it is higher ( $= 3.05$ ). It may be possible, therefore, to find some other mechanism, which does not induce the first best allocations and action choices, but is preferred in the precontract information case because the rents accruing to the agents are lower. This possibility is explored next.

We demonstrate that in the case of precontract private information the modified Groves scheme may not even be optimal in the sense that an alternative scheme may exist that produces  $y_1 = 3$  units at a lower expected cost than the expected cost of 3.05 under the modified Groves scheme. Consider the following mechanism. For all pairs of realizations of  $(\theta_i, \theta_j)$ ,  $i, j = 0, 1$ , choose  $x_0^* = 0$  and  $x_1^* = 0$ . In all states  $(\theta_i, \theta_j)$ ,  $i, j = 0, 1$ , the manager of division 0 is paid  $\Phi_0^* = 0$  and is not required to produce any  $x_1$ . The manager of division 1 is required to produce three units of  $y_1$  by providing an effort level of  $a_1 = 3$  units (since  $x_1$  is set equal to 0) for which he or she is compensated with  $\Phi_1 = 3$ . This ensures his or her minimum utility level of 0. The expected cost to implement this plan is 3.0. Indeed, the cost of 3.0 is independent of the particular realizations of  $\theta_0$  and  $\theta_1$ . This cost is less than the expected cost of 3.05 to implement the modified Groves scheme, which proves the nonoptimality of the latter mechanism in the case of precontract private information.

In summary, three points need to be noted.

- 1 When precontract private information exists, the first best allocation may not be achieved. This is because rents may accrue to the agent since the minimum utility constraint must be satisfied for all realizations of  $\theta_i$  (Demski and Sappington, 1984). In our example, the first best cost is 2.75 whereas the expected cost with precontract private information is higher.
- 2 We have shown that in the precontract private information case the modified Groves scheme is not optimal. Our counterexample illustrates that in this case an alternative scheme reduces expected cost to 3.0 compared to the expected cost of 3.05 under the modified Groves scheme.
- 3 Our results do not preclude the possibility of the existence of other modifications of Groves-type schemes under which first best allocations can be

TABLE 1

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
$\theta_0$	$\theta_1$	$a_0^*$	$a_1^*$	$x_0^*$	$x_1^*$	$\phi_0^* - k_{-0}^* - a_0^*$	$\phi_1^* - k_{-1}^* - a_1^*$	$\phi_0^*$	$\phi_1^*$	$x_0^* + \phi_0^* + \phi_1^*$
0.5	0.5	0	3.0	0	0	-3.0	-3.0	-0.2	3.0	2.8
0.5	1.2	2.0	0.6	0	2.0	-2.6	-2.6	2.2	1.0	3.2
1.2	0.5	0	3.0	0	0	-3.0	-3.0	-0.2	3.0	2.8
1.2	1.2	0.8	0.6	1.0	2.0	-2.4	-2.4	1.2	1.2	3.4

Expected cost = 3.05

TABLE 2

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
$\theta_0$	$\theta_1$	$a_0^*$	$a_1^*$	$x_0^*$	$x_1^*$	$\phi_0^* - k_{-0}^* - a_0^*$	$\phi_1^* - k_{-1}^* - a_1^*$	$\phi_0^*$	$\phi_1^*$	$x_0^* + \phi_0^* + \phi_1^*$
0.5	0.5	0	3.0	0	0	-3.0	-3.0	-0.25	2.75	2.5
0.5	1.2	2.0	0.6	0	2.0	-2.6	-2.6	2.15	0.75	2.9
1.2	0.5	0	3.0	0	0	-3.0	-3.0	-0.25	2.75	2.5
1.2	1.2	0.8	0.6	1.0	2.0	-2.4	-2.4	1.15	0.95	3.1

Expected cost = 2.75

achieved as a dominant strategy equilibrium even in the case of precontract private information.

## Appendix B

### *Proof of result 1*

Differentiating (14) with respect to  $x_i$ , we obtain

$$\frac{\partial C}{\partial x_i} = 1 - \theta_i$$

where

$$C = Ny - \sum_{i=0}^N \theta_i x_i + \sum_{i=0}^N x_i$$

is the objective function in (14). We seek to minimize  $C$  for each realization of  $\underline{\theta}$  in the full information case. It is evident that if  $\theta_i > 1$  then  $x_i^* = x_i^{\max}$  and if  $\theta_i < 1$ , then  $x_i^* = x_i^{\min}$ .

$$\Phi_i^* - k_{-i}^* = -x_0^* - a_j^* \text{ where } j \neq i.$$

### *Proof of result 2*

Each agent chooses  $a_i$  on observing  $\theta_i$  to maximize his or her utility

$$\Phi_i^* - a_i = (k_i^* + y - x_i) - (y - \theta_i x_i)$$

where

$$k_i^* = E[(1 - \theta_i)x_i^*(\theta_i)]$$

is a constant.

Therefore, each agent chooses  $x_i$  to maximize  $-(1 - \theta_i)x_i$ . Therefore, for  $\theta_i > 1$ , he or she chooses  $x_i^{\min}$ , which is exactly consistent with the full information allocations. It is also easy to verify that the individual rationality constraint is satisfied, and the agent is held to his or her minimum expected utility. Further, since each agent's utility ( $\Phi_i^* - a_i$ ) is independent of the actions of the other agents, the scheme is collusion free.  $\square \square \square$

### *Proof of proposition 1*

The full information allocation problem is characterized by the following cost minimization problem given  $y$ .

$$\text{Minimize } c(x_1, x_2) + a_1 + a_2 \tag{A.3.1}$$

subject to

$$y_i = a_i + t_i(x_i, \theta_i), \quad i = 1, 2 \tag{A.3.2}$$

Substituting (A.3.2) into (A.3.1) we have

$$\text{Minimize } c(x_1, x_2) + \sum_{i=1}^2 (y_i - t_i(x_i, \theta_i)) \tag{A.3.3}$$

Therefore, the full information efficient solution is characterized by the following first- and second-order conditions:

$$c_i - t_{ix}(\cdot) = 0, \quad i = 1, 2 \tag{A.3.4}$$

$$c_{ii} - t_{ixx}(\cdot) > 0, \quad i = 1, 2 \tag{A.3.5}$$

$$D = (c_{11} - t_{1xx})(c_{22} - t_{2xx}) - c_{12}^2 > 0 \tag{A.3.6}$$

Differentiating (A.3.4) with respect to  $\theta_1$  for each  $i = 1, 2$  yields

$$(c_{11} - t_{1xx})z_{11} + c_{12}z_{21} - t_{1x\theta} = 0 \tag{A.3.7}$$

$$(c_{22} - t_{2xx})z_{21} + c_{12}z_{11} = 0 \tag{A.3.8}$$

Therefore, substituting (A.3.8) into (A.3.7), we obtain

$$z_{21} = -c_{12}t_{1x\theta}/D \tag{A.3.9}$$

which has the opposite sign as  $c_{12}$  since  $t_{1x\theta}/D > 0$ . Next, differentiating (A.3.2) with respect to  $\theta$  for each  $i = 1, 2$  yields  $\alpha_{11} = -t_{1\theta} - t_{1x}z_{11} < 0$  since  $z_{11}, t_{1\theta}, t_{1x} > 0$ , and  $\alpha_{21} = -t_{2x}z_{21}$ , which has the opposite sign as  $z_{21}$  since  $t_{2x} > 0$ .

□ □ □

*Proof of proposition 2*

If  $c(x_1, x_2) = \beta_0 + \beta_1(x_1) + \beta_2(x_2)$ , the full information efficient allocations  $x_i^*$  for division  $i, i = 1, 2$ , solve the problem in (A.3.3) and satisfy the conditions in (A.3.4), (A.3.5), and (A.3.6). Note that in a decentralized setup, each agent will choose  $x_i^*, a_i^*$  to maximize his or her own utility. Agent  $i$ 's problem is  $\max \Phi_i^*(x) - a_i$  corresponding to any  $\theta_i$  observed by him or her. Therefore, if we set  $\Phi_i^*(x) = k_i^* - \beta_i(x_i)$  where  $k_i^*$  is a constant chosen to satisfy the individual rationality constraint, then each agent's optimization corresponds exactly to the first best allocations characterized by (A.3.4), (A.3.5), and (A.3.6).

To prove the necessity of the condition on the form of  $c(x_1, x_2)$ , we note that each agent's optimization implies  $x_i^* \in \text{argmax } E_{x_j} \{ \Phi_i^*(x) - y_i + t_i(x_i, \theta_i) \}$  where  $E_{x_j}$  is the expectation operator over  $x_j$  since each agent has to choose his or her actions before the outcomes of the other agent are realized and he or she does not know the other agent's outcomes. Each agent's first-order condition is  $\int \Phi_{ix_i}(\cdot) p(x_j) dx_j - \frac{dt_i}{dx_i}(x_i, \theta_i) = 0$ , which is independent of  $x_j$ . The first-

order condition for the first best solution is  $\frac{\partial t_i(\cdot)}{\partial x_i} - \frac{\partial}{\partial x_i} c(x_i, x_j) = 0$  (from 18a).

If the two conditions are to hold simultaneously, it implies  $c_{12} = 0$ ; that is,  $c(x_1, x_2) = \beta_0 + \beta_1(x_1) + \beta_2(x_2)$ . Therefore, the full information efficient allocations  $x^*$  are implementable only if  $c_{12} = 0$ .

□ □ □

## References

- Amershi, A. and P. Cheng, "Intrafirm Resource Allocation: The Economics of Transfer Pricing and Cost Allocation in Accounting," *Contemporary Accounting Research* (Fall 1990) pp. 61-99.
- Antle, R. and D. Eppen, "Capital Rationing and Organizational Slack in Capital Budgeting," *Management Science* (February 1985) pp. 163-174.
- Baiman, S., "Agency Research in Managerial Accounting: A Survey," *Journal of Accounting Literature* (Spring 1982) pp. 154-213.
- and J.S. Demski, "Economically Optimal Performance Evaluation and Control Systems," *Journal of Accounting Research* (Supplement 1980) pp. 184-220.
- Banker, R.D., "Differential Information, Coordination, and Efficiency of Decentralized Decisions," Working Paper, Carnegie Mellon University, American Accounting Association Annual Meeting (1981).
- , "Value of Cost Allocations: Elicitation of Information," Working Paper, Carnegie Mellon University (1988).
- , S.M. Datar, and A. Maindiratta, "Unobservable Outcomes and Multiattribute Preferences in the Evaluation of Managerial Performance," *Contemporary Accounting Research* (Fall 1988) pp. 96-124.
- Banker, R.D. and A. Maindiratta, "Principal Agent Models with Generalized Multiattribute Preferences," Working Paper, Carnegie Mellon University (1986).
- Clarke, E., "Multipart Pricing of Public Goods," *Public Choice* (Fall 1971) pp. 17-33.
- Cohen, S.I. and M. Loeb, "The Groves Scheme, Profit Sharing and Moral Hazard," *Management Science* (January 1984) pp. 20-24.
- Conn, D., "Effort, Efficiency and Incentives in Economic Organizations," *Journal of Comparative Economics* (September 1982) pp. 223-234.
- Demski, J.S. and D. Sappington, "Optimal Incentive Contracts with Multiple Agents," *Journal of Economic Theory* (1984) pp. 152-171.
- Groves, T., "Incentives in Teams," *Econometrica* (July 1973) pp. 617-631.
- and M. Loeb, "Incentives in a Decentralized Firm," *Management Science* (March 1979) pp. 221-230.
- Harris, M. and R. Townsend, "Resource Allocation under Asymmetric Information," *Econometrica* (1981) pp. 33-64.
- Harris, M., C.H. Kriebel, and A. Raviv, "Asymmetric Information, Incentives and Intrafirm Resource Allocation," *Management Science* (1982) pp. 604-620.
- Harris, M. and A. Raviv, "Optimal Incentive Contracts with Imperfect Information," *Journal of Economic Theory* (1979) pp. 231-259.
- Hirschleifer, J., "On the Economics of Transfer Pricing," *Journal of Business* (July 1956) pp. 172-184.
- Myerson, R., "Incentive Compatibility and the Bargaining Problem," *Econometrica* (1979) pp. 61-73.
- , "Bayesian Equilibrium and Incentive Compatibility: An Introduction," in L. Hurwicz, D. Schmeidler, and M. Sonnenschein (eds.) *Social Goals and Social Organizations* (Cambridge: Cambridge University Press, 1985).
- Ronen, J. and G. McKinney, "Transfer Pricing for Divisional Autonomy," *Journal of Accounting Research* (Spring 1970) pp. 99-113.
- Vickrey, W., "Counterspeculation, Auctions, and Competitive Sealed Tenders," *Journal of Finance* (May 1961) pp. 8-37.

