

Product Costing and Pricing under Long-Term Capacity Commitment

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Abstract: We develop a model to analyze optimal product-costing and pricing decisions in a dynamic information environment under long-term-capacity commitment. The arrival of new information about demand and cost parameters each period makes the problem complex. The optimal prices and capacity choices in our model cannot be decoupled as in Banker and Hughes' (1994) single-period model.

The optimal prices are based on product costs that are adjusted each period to reflect changes in expected variable costs as well as utilization of fixed activity resources. The charge for each fixed resource is monotonically increasing in the expected demand for that resource in each state given the optimal capacity choice. The average optimal prices across periods and states are similar to Banker and Hughes' (1994) benchmark prices.

Finally, we investigate a two-period version of the model to explore the optimality of carrying idle capacity. The optimal product-cost charge for fixed capacity is strictly less in the first period than in the second period when the firm expects demand growth.

Keywords: product costing; fixed costs; pricing; capacity.

INTRODUCTION

This paper analyzes the issue of optimal product costing and pricing when a monopolist makes a long-term commitment to the capacity of activity resources. Both economic and relevant cost analyses in managerial accounting conclude that committed fixed costs such as capacity costs should not be charged to products because these sunk costs are irrelevant for decision making. However, most firms charge capacity costs to products and use the resulting costs for pricing decisions (Shim and Sudit 1995; Govindrajan and Anthony 1983). Activity-based costing advocates recommend charging committed costs to products based on cause-and-effect relationships (Cooper and Kaplan 1987). This practice of charging committed fixed costs to products and then using such product costs to price the products leads managerial accounting researchers to explore the economic rationale for such practices.

The managerial accounting literature offers several reasons for allocating fixed costs. These include opportunity costs (Kaplan and Thompson 1971; Kaplan and

Welam 1974; Miller and Buckman 1987; Balachandran and Srinidhi 1987; Banker et al. 1988), managerial incentives (Zimmerman 1979), and capacity choices (Balachandran et al. 1987; Whang 1989). Banker and Hughes (1994) study how the committed activity resource costs affect optimal capacity and pricing decisions, as well as what information decentralized decision makers need to implement those decisions in a single-period model. They show that the optimal pricing rule incorporates activity-based product costs that include charges for committed resources.

More recently, several papers have extended this line of research. Balakrishnan and Sivaramakrishnan (1996) and Balachandran et al. (1997) consider hard capacity constraints. Balakrishnan and Sivaramakrishnan (2001) and Banker and Hansen (2002) compare the performance of different heuristics for the joint capacity-planning and pricing problem. Göx (2001) evaluates the performance of different heuristic rules in a two-period planning model that considers price changes required to achieve better utilization of committed capacity levels.

Our paper is closely related to Banker and Hughes (1994), Balakrishnan and Sivaramakrishnan (2001), and Göx (2002). Balakrishnan and Sivaramakrishnan (2001) consider a single-period model with two products, two states (high and low demand), hard capacity constraints, and deterministic demand (demand is completely revealed after capacity decisions). In their model, the first product drives all demand for capacity because investing in capacity to produce the second product is unprofitable. Under these assumptions, they show that demand parameters must be known to design the optimal product-costing system and that activity-based costs are not sufficient to set prices as in Banker and Hughes' (1994) single-period model. Göx (2002) considers the capacity-planning and pricing problem under uncertain demand in a single-period model with both hard and soft capacity constraints. He finds that capacity costs are relevant only when the capacity-planning and pricing decisions are made under the same information about the state of nature. When the firm has more information at the time of the pricing decision, the firm uses marginal-cost pricing.

Our research extends and complements this earlier research by relaxing some of the assumptions and providing several new insights. We extend the Banker and Hughes (1994) framework and develop a general multiproduct, multiperiod model of long-term capacity commitment in a dynamic information environment where new information about demand, cost, and spot prices arrives each period. We analyze how optimal product costs differ across periods when the firm chooses capacity of different activity resources at the outset but new information about demand, costs, and spot prices becomes available for the pricing decision in each subsequent period. In our model, resource charges to determine optimal prices depend on the shortage probability and can be higher, lower, or equal to their activity-based full cost. Consequently, demand parameter and activity-based full-cost information together is not sufficient (Amershi et al. 1990) for the optimal pricing decision in our model. We show that the optimal charge for committed resources differs across periods by a factor that increases monotonically with the expected demand for the resource in each period. This feature distinguishes our results from those of Banker and Hughes (1994) in which price based on full cost is optimal.

In contrast, when the firm must commit to a fixed price for each product over the entire horizon, we obtain a benchmark solution similar to that for the Banker and Hughes (1994) single-period model. More importantly, the average of the expected optimal prices when prices fluctuate with new information equals the Banker and Hughes (1994) benchmark price based on full costs. This result may rationalize the empirical finding that firms use activity-based full costs in their pricing decisions. We speculate that rather than simply basing prices on full costs,

firms may first compute benchmark prices based on full costs, and then adjust these prices heuristically to reflect updated information on demand and cost conditions (see also, Atkinson et al. 1995, 342).

Our study adds to earlier research in three ways. First, we link short-term pricing and product-costing decisions in a multiperiod dynamic information environment to long-term benchmarks based on full costs. Second, we show that the optimal short-term charge for committed capacity resources increases with their expected demand. Finally, we identify when the firm should carry idle capacity and provide theoretical support for strategically charging less than the full cost of capacity to products when demand growth is expected.

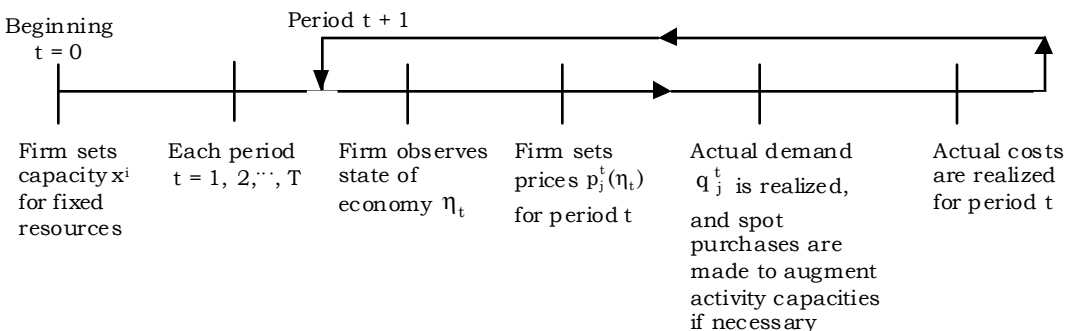
The remainder of this paper is organized as follows. The second section develops a multiproduct, multiperiod capacity-planning and pricing model. The third section presents the analytical results for optimal pricing and product-costing decisions in the multiperiod setting. A special case of the multiperiod model related to the optimality of idle capacity under demand growth expectations appears in the fourth section. The last section concludes the paper with a brief summary of the results.

LONG-TERM CAPACITY COMMITMENT MODEL

We consider a monopolist firm producing J products (indexed by $j = 1, 2, \dots, J$) in each of T periods (indexed by $t = 1, 2, \dots, T$). The firm has a Leontief production function that converts input resources in each period t into the output vector $q^t \equiv (q_1^t, q_2^t, \dots, q_J^t)$. There are two types of resources, corresponding to variable and fixed costs. The first type, such as some direct material, direct labor, and variable overhead, is acquired on an as-needed basis. The cost of these variable inputs is denoted by v_j^t for each unit of product j in period t . For the second type of resource, the firm must commit at the beginning of the planning horizon to levels of capacity, $x \equiv (x_1, x_2, \dots, x_I)$, that will be available throughout the planning horizon at a cost m_i^t per unit of capacity i , $i = 1, 2, \dots, I$, in each period t . These costs may fluctuate across period due to differences in maintenance, insurance, plant management, salaries, and other capacity-related activity costs. The firm can acquire an additional amount of any resource i on an as-needed basis at a premium θ_i^t on the spot market, resulting in a cost of $\theta_i^t m_i^t$ per unit of the resource. We require the *ex ante* expected value $E[\theta_i^t m_i^t]$ over all periods and states to be greater than $E[m_i^t]$, but unlike Banker and Hughes (1994), we allow θ_i^t to be less than 1 in some periods.

Figure 1 below shows the time line of events for our model. The firm faces uncertainty about the future state of the economy that may affect its cost and demand functions in each of the T periods. At the beginning of each period t , the

FIGURE 1
Time Line of Events



firm observes a signal η_t on the state of the economy, where $\eta_t \in \{1, 2, \dots, N_t\}$, and updates its initial beliefs about the distribution of the cost and demand function parameters. The probability of the signal being η_t in period t is given by a probability measure $r^t(\eta_t) : (t \times \eta_t) \rightarrow [0, 1]$. The firm sets a price, $p_j^t(\eta_t)$, based on the observed signal in each period. The actual demand q_j^t for each product j is realized in accordance with a linear demand function for the product with an additive stochastic component as in Banker and Hughes (1994):

$$q_j^t \equiv \alpha_j - \beta_j p_j^t + \varepsilon_j^t, \text{ for } j = 1, 2, \dots, J \text{ products, } t = 1, 2, \dots, T \text{ periods,} \quad (1)$$

where ε_j^t is distributed with mean zero and finite variance. Let $\bar{\varepsilon}_j^t(\eta_t)$ denote $E[\varepsilon_j^t | \eta_t]$, the revised expectation of the residual demand given the signal η_t such that conditional on the observation of η_t , ε_j^t is distributed as $\bar{\varepsilon}_j^t(\eta_t) + \varepsilon_j$ with $E[\varepsilon_j] = 0$ and ε_j has finite variance as in Banker and Hughes (1994). Therefore, the expected demand given that the firm observes a signal η_t and selects a price $p_j^t(\eta_t)$ is given by:

$$\bar{q}_j^t(\eta_t) \equiv E[q_j^t | \eta_t] = \alpha_j^t(\eta_t) - \beta_j p_j^t(\eta_t) \quad (2)$$

where $\alpha_j^t(\eta_t) = \alpha_j + \bar{\varepsilon}_j^t(\eta_t)$.

We use the following vector notation to simplify our presentation. The vectors $p^t(\eta_t) \equiv (p_1^t(\eta_t), p_2^t(\eta_t), \dots, p_J^t(\eta_t))$ and $\bar{q}^t(\eta_t) \equiv (\bar{q}_1^t(\eta_t), \bar{q}_2^t(\eta_t), \dots, \bar{q}_J^t(\eta_t))$ are $J \times 1$ vectors whose elements are the prices and expected demand quantities of J products given the state η_t in period t . The $\sum_{t=1}^T J \times N_t$ vectors $p \equiv (p^1, p^2, \dots, p^T)$ and

$q \equiv (q^1, q^2, \dots, q^T)$ include all possible prices and production quantities across all periods and all states.

The cost function $C(x, q)$ incorporates variable, fixed, and spot premium costs as in Banker and Hughes (1994). The expected cost given a signal η_t is:

$$C(x, q^t | \eta_t) = \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \left[\sum_{i=1}^I \bar{m}_i^t(\eta_t) x_i + \sum_{j=1}^J \bar{v}_j^t(\eta_t) \bar{q}_j^t(\eta_t) + E \left(\sum_{i=1}^I \xi_i(x_i, q^t) \mid \eta_t \right) \right] \quad (3)$$

where:

$$\bullet \quad \xi_i(x_i, q^t) = \begin{cases} \theta_i^t m_i^t \left(\sum_{j=1}^J \mu_{ij} q_j^t - x_i \right) & \text{if } \sum_{j=1}^J \mu_{ij} q_j^t > x_i \\ 0 & \text{otherwise} \end{cases} \quad \text{for } t = 1, 2, \dots, T,$$

- $x_i \equiv$ capacity implied by resources committed to activity i ,
- $\bar{v}_j^t(\eta_t) \equiv$ expected unit variable cost of product j given state η_t in period t ,
- $\bar{m}_i^t(\eta_t) \equiv$ expected unit cost for resource i given state η_t in period t ,
- $\bar{\theta}_i^t(\eta_t) \equiv$ expected spot market premium for resource i given state η_t in period t ,
- $\mu_{ij} \equiv$ units of activity i required to support one unit of product j , and
- $r^t(\eta_t) \equiv$ probability of state η_t occurring in period t .

The first term on the right-hand side of Equation (3) represents the expected fixed costs. These costs do not vary with actual production level q^t , but increase with the activity capacities x . The second term is the expected variable costs, which are directly proportional to production quantities. The third term reflects the expected spot premium to obtain additional capacity when the realized demand requires capacity resources exceeding the committed capacity.

We denote the expected total usage of activity i for the expected production

$\bar{q}^t(\eta_t)$ in period t given state η_t as $\bar{z}_i^t(\eta_t) \equiv \sum_{j=1}^J \mu_{ij} \bar{q}_j^t(\eta_t)$, and the actual total usage

of activity i as $z_i^t \equiv \sum_{j=1}^J \mu_{ij} q_j^t$. This allows us to simplify our analysis by separating

the additional demand due to additive stochastic component from the mean

average demand. In addition, we define $\psi_i^t = \sum_{j=1}^J \mu_{ij} \varepsilon_j^t$, $\bar{\psi}_i^t(\eta_t) = \sum_{j=1}^J \mu_{ij} \bar{\varepsilon}_j^t(\eta_t)$,

$y_i^t(\eta_t) = \psi_i^t - \bar{\psi}_i^t(\eta_t)$, and $\bar{s}_i^t(\eta_t) = x_i - \bar{z}_i^t(\eta_t)$ to simplify our exposition later. We denote the density function and the distribution function of $y_i^t(\eta_t)$ as $\phi_i(\cdot)$ and $\Phi_i(\cdot)$, respectively. Each $y_i^t(\eta_t)$ is distributed with finite variance as each ε_j^t has finite variance.

Using the above definition, we can rewrite the actual activity usage as $z_i^t = \bar{z}_i^t(\eta_t) + y_i^t(\eta_t)$. If the actual demand for the resource i exceeds its committed capacity, i.e., $z_i^t > x_i$, then the firm incurs an additional cost of $\theta_i^t m_i^t$ for each additional unit of the resource consumed in excess of the available capacity x_i . On the other hand, if $z_i^t < x_i$, there is excess capacity and the total expected cost

of activity i is $\sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{m}_i^t(\eta_t) x_i$. Using our definitions above, we obtain:

$$\xi_i(x_i, q^t) = \begin{cases} \theta_i^t(\eta_t) m_i^t(\eta_t) (\bar{z}_i^t(\eta_t) + y_i^t(\eta_t) - x_i) & \text{if } y_i^t(\eta_t) > x_i - \bar{z}_i^t(\eta_t) \\ 0 & \text{otherwise} \end{cases}$$

for $t = 1, 2, \dots, T$ and $\eta_t = 1, 2, \dots, N_t$.

The firm's expected profit is:

$$\begin{aligned} E[\pi(x, p)] = & - \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} \sum_{i=1}^I r^t(\eta_t) \bar{m}_i^t(\eta_t) x_i \\ & + \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \left\{ \sum_{j=1}^J (p_j^t(\eta_t) - \bar{v}_j^t(\eta_t)) (\alpha_j^t(\eta_t) - \beta_j p_j^t(\eta_t)) \right\} \\ & - \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} \sum_{i=1}^I r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) \int_{x_i - \bar{z}_i^t(\eta_t)}^{\infty} (\bar{z}_i^t(\eta_t) + y_i^t(\eta_t) - x_i) \phi_i(y_i^t(\eta_t)) dy_i^t(\eta_t). \quad (4) \end{aligned}$$

Given this expected profit function, we next analyze the *ex ante* optimal capacity and price choices of the firm.

ANALYSIS AND RESULTS

We begin our analysis with the optimal *ex ante* capacity and period-by-period pricing choices of the multiproduct monopolist. The firm first selects the capacity level x_i for each resource and subsequently chooses a different price for each period based on information available at that time. The following lemma characterizes the optimal choices of capacities and prices.

Lemma 1: The optimal choice of capacity levels x_i^* and prices $p_j^{t*}(\eta_t)$ solves the following set of equations.

$$\frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) [1 - \Phi_i(x_i^* - \bar{z}_i^{t*}(\eta_t))] = \bar{m}_i \quad \forall i \tag{5}$$

$$\begin{aligned} & \bar{q}_j^{t*}(\eta_t) - \beta_j(p_j^{t*}(\eta_t) - \bar{v}_j^t(\eta_t)) \\ & = \sum_{i=1}^I \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) \mu_{ij} \beta_j [1 - \Phi_i(x_i^* - \bar{z}_i^{t*}(\eta_t))] \quad \forall j, t, \text{ and } \eta_t \end{aligned} \tag{6}$$

where $\bar{m}_i = \frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{m}_i^t(\eta_t)$ and $\bar{z}_i^{t*}(\eta_t) = \sum_{j=1}^J \mu_{ij} (\alpha_j^t(\eta_t) - \beta_j p_j^{t*}(\eta_t))$.

Proof: All proofs are in the Appendix.

The firm solves a set of $I + \sum_{t=1}^T J \times N_t$ simultaneous equations to determine the optimal capacity levels for the I resources and the optimal prices for the J products in each state η_t in each of T periods. Clearly, this is a complex problem when the number of products or state realizations is large. The right-hand side of Equation (5) is the expected marginal cost of adding an extra unit of capacity averaged over all T periods and the left-hand side is equal to the average expected marginal cost of procuring an extra unit of capacity in the spot market, should the need arise. Given the optimum capacity, the firm is indifferent between these two options at the margin. Although complex, this expression is similar in spirit to the Banker and Hughes (1994) result. Rewriting Equation (5) yields:

$$\sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) \Phi_i(x_i^* - \bar{z}_i^{t*}(\eta_t)) = (\bar{w}_i - \bar{m}_i) T \tag{7}$$

where $\frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) = \bar{w}_i$.

Equation (7) indicates that the firm will invest in capacity only if the average expected spot premium exceeds the expected unit cost of installing an additional unit of capacity. This is analogous to Banker and Hughes' (1994) requirement that the spot premium factor be greater than 1. However, in our model the spot premium factor can be less than 1 in some states of the economy, e.g., in an economic downturn.

For the pricing decision the firm trades off the expected marginal loss in contribution margin on the left-hand side of Equation (6) against the expected marginal cost savings through reduction in spot premium for exceeding the optimal capacities

shown on the right-hand side of Equation (6). Rewriting the price equation in a form similar to Banker and Hughes (1994) yields:

$$p_j^{t*}(\eta_t) = \frac{\alpha_j^t(\eta_t)}{2\beta_j} + \frac{1}{2} \left(\bar{v}_j^t(\eta_t) + \sum_{i=1}^I \mu_{ij} \bar{m}_i \lambda_i^t(\eta_t) \right) \quad \forall j, t, \text{ and } \eta_t \quad (8)$$

where $\lambda_i^t(\eta_t) \equiv \frac{\bar{m}_i^t(\eta_t)}{\bar{m}_i} \bar{\theta}_i^t(\eta_t) [1 - \Phi_i(x_i^* - \bar{z}_i^{t*}(\eta_t))]$ is the fixed cost adjustment factor. The term $\frac{\bar{m}_i^t(\eta_t)}{\bar{m}_i}$ is the ratio of the expected marginal cost for the initially committed capacity resource given a certain state of nature to its average expected marginal cost. Also, $\bar{\theta}_i^t(\eta_t)$ is the expected spot premium and $[1 - \Phi_i(x_i^* - \bar{z}_i^{t*}(\eta_t))]$ is the probability of capacity shortage.

As in Banker and Hughes (1994), the optimal price in Equation (8) has two components, one related to the demand parameters $(\alpha_j^t(\eta_t), \beta_j)$, and the other related to production cost parameters based on the optimal capacity choice. However, unlike in Banker and Hughes (1994), the optimal prices depend on the optimal capacity choices, and the optimal capacity choices depend on the cost information $(\bar{v}_j^t(\eta_t), \bar{m}_i^t(\eta_t))$ indirectly through the expected demand $\bar{z}_i^{t*}(\eta_t)$, which depends on the optimal prices, $p_j^{t*}(\eta_t)$. Similarly, the optimal capacity charge used to determine prices deviates from the activity-based full cost obtained in the Banker and Hughes model.¹ This result that full-cost pricing is not optimal for short-term pricing parallels similar results obtained in Balakrishnan and Sivaramakrishnan (2001) and Göx (2002) under different sets of assumptions. In addition, we show later in Proposition 1 that the average of these short-term prices is related to the full-cost-based price in Banker and Hughes (1994).

Equation (8) shows that $c_j^t(\eta_t) = \bar{v}_j^t(\eta_t) + \sum_{i=1}^I \lambda_i^t(\eta_t) \bar{m}_i \mu_{ij}$ aggregates all of the detailed information about variable and committed activity resources that is required for the pricing decision in period t given the state η_t . This aggregate cost, $c_j^t(\eta_t)$, resembles the activity-based product cost in Banker and Hughes (1994)

$\left(c_j = v_j + \sum_{i=1}^I m_i \mu_{ij} \right)$ except that the charge per unit of the committed activities $i = 1,$

$2, \dots, I$ is the amount $\lambda_i^t(\eta_t) \bar{m}_i$ instead of simply the expected unit cost of the activity given by $\bar{m}_i^t(\eta_t)$. This implies that the knowledge of demand parameters and full-cost allocation based on activity-based costing is not adequate for the pricing decision.

¹ In addition, we also find that the capacity-planning and pricing decisions can no longer be decoupled as in Banker and Hughes (1994).

In the above analysis, we have allowed the firm to dynamically adjust the price of its products in the product market. Although this assumption reflects the case for many commodities and services, such as gasoline and air travel, firms often do not have this flexibility. This is especially true in long-term customer-supplier relationships that require upfront price commitments by the supplier, as is prevalent in the automobile components industry. Hence, we next explore the optimal prices when the firm must commit to a single price for each product through all periods.

Lemma 2: The optimal *ex ante* choice of capacity levels x_i^B and prices p_j^B under price commitment solves the following equations.

$$\frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) [1 - \Phi_i(x_i^B - \bar{z}_i^{tB}(\eta_t))] = \bar{m}_i \quad \forall i \tag{9}$$

$$p_j^B = \frac{\bar{\alpha}_j}{2\beta_j} + \frac{1}{2} \left(\bar{v}_j + \sum_{i=1}^I \bar{m}_i \mu_{ij} \right) \quad \forall j, t, \text{ and } \eta_t \tag{10}$$

where $\bar{\alpha}_j = \frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \alpha_j^t(\eta_t)$ and $\bar{v}_j = \frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{v}_j^t(\eta_t)$.

We call these restricted optimal choices the benchmark capacities and prices because they are similar in form to the Banker and Hughes (1994) single-period results that effectively decouple the pricing and capacity choices. Thus, when the firm must commit *ex ante* to a price, the charge for each activity is similar to that in the Banker and Hughes (1994) model.² Although these benchmark-capacity and price choices are interesting in their own right, Proposition 1 next explores how they relate to the unrestricted optimal choices in Lemma 1.

Proposition 1:

(i) The benchmark price is equal to the average expected optimal price \bar{p}_j^* of each product across all states and periods.³

$$p_j^B = \bar{p}_j^* \equiv \frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) p_j^{t*}(\eta_t)$$

(ii) The average expected demand for each resource over T periods is identical for the benchmark and optimal solutions.

$$\sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{z}_i^{tB}(\eta_t) = \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{z}_i^{t*}(\eta_t)$$

(iii) The expected shortage cost for each resource over T periods is identical for the benchmark and optimal solutions.

$$\sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) [1 - \Phi_i(x_i^B - \bar{z}_i^{tB}(\eta_t))]$$

² This result also corresponds to that in Göx (2002), demonstrating that full-cost-based pricing is optimal when the price and capacity decisions are based on identical information.

³ A similar result is obtained in a highly stylized setting by Balakrishnan and Sivaramakrishnan (2001). They note that it is due to demand stochasticity being captured by the intercept term in their model. However, we show that this result also holds in our more general setting.

$$= \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) \left[1 - \Phi_i(x_i^* - \bar{z}_i^{t*}(\eta_t)) \right]$$

The benchmark price is simply the average of the expected optimal prices over the T periods. Thus, when the firm must commit to a price for all T periods, it simply chooses the average of the expected optimal prices over the T periods. Parts (ii) and (iii) of Proposition 1 show that the expected demand for resources and the expected shortage cost are also identical over the T periods. However, firm profit with the unrestricted optimal prices must exceed the corresponding profit using the common benchmark price in all periods. The increase in expected profit comes from demand management. The firm stimulates demand by charging lower prices when demand is expected to be lower and dampens the demand by charging higher prices when demand is expected to be greater. Likewise, the opportunity cost of the available committed capacity is lower when the demand is low and higher when the demand is high. In turn, the firm adjusts the charge for the committed resources in different states based on the expected demand and cost parameters in each state.

The difference between the optimal price and the benchmark price $\Delta p_j^t(\eta_t) = p_j^{t*}(\eta_t) - p_j^B$ becomes:

$$\Delta p_j^{t*}(\eta_t) = \frac{\alpha_j^t(\eta_t) - \bar{\alpha}_j}{2\beta_j} + \frac{\bar{v}_j^t(\eta_t) - \bar{v}_j}{2} + \frac{1}{2} \sum_{i=1}^I \mu_{ij} \bar{m}_i \left[\lambda_i^t(\eta_t) - 1 \right].$$

This total difference includes three components: the base-level demand, the variable costs, and the charge for the committed resources. The first two components are exogenous to the firm, but the third results from the firm's accounting system adjusting the charge for the committed resource. Next, we characterize the fixed cost charge, $\lambda_i^t(\eta_t)$, in the optimal solution and compare it to the corresponding charge in the benchmark solution.

Proposition 2:

- (i) The fixed cost adjustment factor, $\lambda_i^t(\eta_t) \equiv \frac{\bar{m}_i^t(\eta_t)}{\bar{m}_i} \bar{\theta}_i^t(\eta_t) \left[1 - \Phi_i(x_i^* - \bar{z}_i^{t*}(\eta_t)) \right]$, i.e.,

the optimal charge for committed resource i is monotonically increasing in the expected demand $\bar{z}_i^{t*}(\eta_t)$ for that resource in each state, given the optimal capacity choice, x_i^* .

- (ii) Depending on the fixed cost adjustment factor (i.e., on whether $\lambda_i^t(\eta_t) > 1$, $\lambda_i^t(\eta_t) < 1$, or $\lambda_i^t(\eta_t) = 1$), the charge for the committed resource i is more than, less than, or equal to the corresponding full cost rate in the benchmark case.

Proposition 2 shows that the charge for a committed resource depends directly on the expected demand for that resource. The greater the demand, the smaller the slack capacity, and therefore the greater is the corresponding charge. Thus, when there is a high likelihood of idle capacity for a resource, the firm should reduce the charge for the resource, thereby reducing its price and stimulating demand.

OPTIMALITY OF IDLE CAPACITY

Firms acquire capacity based on long-term strategic plans that forecast future needs. The firm may acquire some capacity that is not fully utilized initially in order to have the capacity to meet future demand growth. For example, in the "Société Bonlieu" case (Shank 1996) the company invested 18 million francs in a

new kiln that had a capacity of 10.6 cubic meters per week but was operating at only 2.2 cubic meters a week. The firm chose the larger kiln capacity to meet anticipated growth in product demand. The main question in “Société Bonlieu” case is how the cost of the idle kiln capacity should be charged to production in the current period.

A second example is the Fab 30 plant built by AMD Saxony GmbH in Dresden, Germany (<http://www.amd.com/locations/aboutfab.html>). Fab 30 is a state-of-the-art manufacturing facility, costing \$1.9 billion. Production began in December 1998, and some initial capacity remained idle until AMD added new high-performance processors to the production schedule. As of December 2001, AMD expected that Fab would soon utilize all the initial capacity. The central question in both of these contexts is how the facility costs should be charged to products over the economic life of the facility. The opposite situation occurs when a firm is in a declining market for its product line.

Although the problem of product costing with strategic idle capacity is common, little theoretical guidance is available in the management accounting literature. To address this issue with a relatively simple model, we adapt our multiperiod capacity-planning and pricing model to a two-period setting where the firm expects future growth or decline of a product line. To focus on the capacity and pricing decisions for product growth or decline, we assume that there is no residual uncertainty in costs and that the spot premium factor, θ_j , is identical for both periods. As before, we represent demand as:

$$q_j^t \equiv \alpha_j^t - \beta_j p_j^t + \varepsilon_j \text{ for } j = 1, 2 \text{ products and } t = 1, 2 \text{ periods.}$$

Letting $\alpha_j^2 \geq \alpha_j^1$ for each product j , with a strict inequality for at least one product, say product 2, would represent expected demand growth. Conversely, $\alpha_1^2 \leq \alpha_1^1$, $\alpha_2^2 < \alpha_2^1$ reflects an anticipated decline in demand.

As before, the firm acquires capacity at the beginning of the first period. With $T = 2$ and $J = 2$, the cost function becomes:

$$C(x, q^t) = \sum_{i=1}^I 2m_i x_i + \sum_{t=1}^2 \sum_{j=1}^2 v_j q_j^t + \sum_{i=1}^I \xi_i(x_i, q^t) \tag{11}$$

$$\text{where } \xi_i(x_i, q^t) = \begin{cases} \theta_i m_i \left(\sum_{j=1}^2 \mu_{ij} q_j^t - x_i \right) & \text{if } \sum_{j=1}^2 \mu_{ij} q_j^t > x_i \\ 0 & \text{if } \sum_{j=1}^2 \mu_{ij} q_j^t \leq x_i \end{cases} \text{ for } t = 1, 2.$$

The first term on the right-hand side of Equation (11) is the committed resource capacity costs for the two-period model. The second term represents the variable costs for the two periods. The last term is the spot premium cost if the demand exceeds capacity in either period.

Using the notation defined in the second section, we can express the total usage of activity i to support actual production as:

$$z_i^t = \sum_{j=1}^2 \mu_{ij} q_j^t = \bar{z}_i^t + y_i^t \text{ for } t = 1, 2.$$

As in the original model, if the actual demand for resource i is greater than the initially committed resource capacity, i.e., $z_i^t > x_i$, the firm must pay $\theta_i m_i$ per additional unit of activity resource, and the expected profit function is:

$$\begin{aligned} E[\pi(x, p)] = & -\sum_{t=1}^2 \sum_{i=1}^I m_i x_i + \sum_{t=1}^2 \sum_{j=1}^2 (p_j^t - v_j) (\alpha_j^t - \beta_j p_j^t) \\ & - \sum_{t=1}^2 \sum_{i=1}^I \theta_i m_i \int_{x_i \bar{z}_i^t(\eta_t)}^{\infty} (\bar{z}_i^t + y_i^t - x_i) \phi_i(y_i^t) dy_i^t \end{aligned} \quad (12)$$

where $\bar{q}_j^t \equiv E[q_j^t | p_j^t] = \alpha_j^t - \beta_j p_j^t$, $\bar{z}_i^t = \sum_{j=1}^2 \mu_{ij} \bar{q}_j^t$, and $y_i^t = \sum_{j=1}^2 \mu_{ij} \epsilon_j^t$.

We assume that the error term in the demand function is independently and identically distributed across products and periods.

To analyze this model, we first characterize the optimal capacities, x_i^* , and prices, p_j^{t*} , as the solutions to the following set of equations:

$$\frac{\theta_i m_i}{2} \sum_{t=1}^2 [1 - \Phi_i(x_i^* - \bar{z}_i^{t*})] = m_i \quad \text{for } i = 1, 2, \dots, I \quad (13)$$

$$\bar{q}_j^{t*} - (p_j^{t*} - v_j) \beta_j = \sum_{i=1}^I \theta_i m_i \mu_{ij} \beta_j [1 - \Phi_i(x_i^* - \bar{z}_i^{t*})] = 0 \quad \text{for all } t \text{ and } j. \quad (14)$$

Equation (13) equates the average expected cost of spot premiums over two periods with the marginal cost of acquiring additional capacity in the first period. Equation (14) equates the expected marginal loss in contribution margin with the expected marginal cost savings in terms of reduction in spot premiums for exceeding the optimal capacities. Rewriting (14) yields:

$$p_j^{t*} = \frac{\alpha_j^t}{2\beta_j} + \frac{v_j}{2} + \sum_{i=1}^I \theta_i m_i \mu_{ij} [1 - \Phi_i(x_i^* - \bar{z}_i^{t*})] \quad \text{for all } t \text{ and } j. \quad (15)$$

The optimal price again has three components, the first related to demand and the other two related to production costs.

To analyze the capacity and pricing choices, define γ_i as the ratio of the shortage probabilities in periods 1 and 2:

$$\gamma_i = \frac{1 - \Phi_i(x_i - \bar{z}_i^1)}{1 - \Phi_i(x_i - \bar{z}_i^2)} = \frac{\Pr(y_i^1 \geq x_i - \bar{z}_i^1)}{\Pr(y_i^2 \geq x_i - \bar{z}_i^2)}. \quad (16)$$

Our assumption about demand growth then constrains the value of the optimal γ_i^* as follows:

Lemma 3:

- (i) When demand growth is expected in period 2 (i.e., $\alpha_1^2 \geq \alpha_1^1$ and $\alpha_2^2 > \alpha_2^1$), then $\gamma_i^* < 1$ for all i .
- (ii) When demand decline is expected in period 2 (i.e., $\alpha_1^2 \leq \alpha_1^1$ and $\alpha_2^2 < \alpha_2^1$), then $\gamma_i^* > 1$ for all i .

We can also rewrite the optimal capacities and prices as follows:

$$x_i^* = \bar{z}_i^{1*} + \Phi_i^{-1} \left(1 - \frac{2\gamma_i^*}{\theta_i(\gamma_i^* + 1)} \right) = \bar{z}_i^{2*} + \Phi_i^{-1} \left(1 - \frac{2}{\theta_i(\gamma_i^* + 1)} \right),$$

$$p_1^{1*} = \frac{\alpha_1^1}{2\beta_1} + \frac{v_1}{2} + \sum_{i=1}^I m_i \mu_{i1} \frac{2\gamma_i^*}{(\gamma_i^* + 1)}, \quad p_2^{1*} = \frac{\alpha_2^1}{2\beta_2} + \frac{v_2}{2} + \sum_{i=1}^I m_i \mu_{i2} \frac{2\gamma_i^*}{(\gamma_i^* + 1)},$$

$$p_1^{2*} = \frac{\alpha_1^2}{2\beta_1} + \frac{v_1}{2} + \sum_{i=1}^I m_i \mu_{i1} \frac{2}{(\gamma_i^* + 1)}, \quad \text{and} \quad p_2^{2*} = \frac{\alpha_2^2}{2\beta_2} + \frac{v_2}{2} + \sum_{i=1}^I m_i \mu_{i2} \frac{2}{(\gamma_i^* + 1)}.$$

Note that γ_i^* is a function of the optimal amount of committed resource i and the optimal expected demand for resource i in both periods. Given that the firm acquires capacity based on anticipated demand growth, Proposition 3 establishes when the firm should carry idle capacity.

Proposition 3: There exists a critical value of the spot premium factor $1 < \theta_i^c = 4\gamma_i^* / (\gamma_i^* + 1) < 2$ for each resource i , above which (i.e., for $\theta_i > \theta_i^c$), the firm should carry idle capacity for resource i in the first period when expecting demand growth in the second period.⁴

The expression in Proposition 2 (i) indicates that whether the firm chooses to carry idle capacity for future demand growth depends on the expected need for the future periods as well as the spot premium. In fact, as in Banker and Hughes (1994), if the spot premium factor for each resource is greater than or equal to 2 (even if the demand is invariant across the two periods), the firm expects to have idle capacity irrespective of the demand parameters. The critical value for the expected idle capacity in Proposition 3 is less than the critical value of 2 in the Banker and Hughes (1994) model because the firm anticipates future demand growth in the present model, whereas it does not in the Banker and Hughes (1994) model. The greater the difference between the first and second period expected demands, the smaller the critical value in Proposition 3. Thus, there are two reasons in our model for firms to carry idle capacity; first, to avoid paying the spot premium for capacity and, second, to accommodate anticipated demand growth.⁵

The final question we address in Proposition 4 is how the committed capacity cost should be charged to products to support optimal pricing decisions when the firm expects demand growth or demand decline.

Proposition 4:

- (i) When the firm expects demand growth in the second period ($\alpha_1^2 \geq \alpha_1^1$ and $\alpha_2^2 > \alpha_2^1$), the charge per unit of capacity resources in the first period is strictly less than the charge in the second period.

⁴ Balakrishnan and Sivaramakrishnan (1996) show with a numerical example in a two-period setting that a firm may carry idle capacity in anticipation of growth. In our model, demand growth alone does not imply that the firm should carry idle capacity. The spot premium must exceed a critical value, in addition to an anticipated growth in demand, to make it optimal for the firm to carry idle capacity.

⁵ Replacing the assumption that the spot premium is the same over both periods with the assumption that the premium is an increasing function of the aggregate demand for that resource

$$\left(\theta_i^t = \rho_i \sum_{j=1}^2 \mu_{ij} \alpha_j^t, \rho_i > 0 \right) \text{ yields expected idle capacity for } \theta_i^2 > \theta_i^c, \text{ where } \theta_i^c = \gamma_i^* (4 - \theta_i^1).$$

- (ii) The converse holds when the firm expects demand decline in the second period.

Proposition 4 supports the intuition that current products should not be charged the cost of idle capacity when the firm acquires capacity to meet expected demand in subsequent periods. Similarly, if the firm expects the demand for a particular product to decline, it should reduce the capacity charge in the second period to stimulate the demand.

CONCLUSION

This paper develops a product costing and pricing model under long-term capacity commitment to analyze the interaction between the initial capacity choice and subsequent product costing and pricing decisions in a dynamic information environment. The resulting optimal product costs differ from the activity-based costs in Banker and Hughes' (1994) single-period model except when the firm has to commit to a common price for all periods. Depending on the spot premium cost, the optimal charges for fixed resources could be greater than, less than, or equal to the benchmark charge that results under price commitment. In addition, the average expected optimal charge for an activity resource equals its expected full cost as in Banker and Hughes (1994). However, the optimal charge in each period fluctuates around this benchmark as the firm adjusts costing and pricing based on realized demand. Using our model in a two-period context demonstrates the optimality of carrying idle capacity. Further, when the firm expects demand growth in the second period, the charge per unit of capacity resources in the first period is strictly less than the charge in the second period. The opposite holds when the firm expects demand to decline in the second period.

As in Banker and Hughes (1994), we have made certain simplifying assumptions such as the Leontief production function. Future research may try to relax some of the restrictive assumptions, particularly the requirement that all products be produced in all periods. Other research avenues include adapting this model to different contexts, such as where the realized demand need not be met in all periods and states. Other extensions include the analysis of decisions related to the introduction of a new product or the dropping of an existing product. A promising approach involves the use of simple heuristics based on the complex expressions for optimal prices and capacity choices, such as those presented here. In support of these avenues for further research, the present study provides insights into how the optimal determination of product costs to facilitate pricing decisions in different periods interacts with expectations about the likelihood of committed capacity remaining idle in each period.

APPENDIX

Lemma 1: Expected profit function given the known parameters is:

$$E[\pi(x, p)] = - \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} \sum_{i=1}^I r^t(\eta_t) \bar{m}_i^t(\eta_t) x_i + \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \left\{ \sum_{j=1}^J (p_j^t(\eta_t) - \bar{v}_j^t(\eta_t)) \bar{q}_j^t(\eta_t) \right\} - \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} \sum_{i=1}^I r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) \int_{x_i - \bar{z}_i^t(\eta_t)}^{\infty} (\bar{z}_i^t(\eta_t) + y_i^t(\eta_t) - x_i) \phi_i(y_i^t(\eta_t)) dy_i^t(\eta_t)$$

where $\bar{q}_j^t(\eta_t) = \alpha_j^t(\eta_t) - \beta_j p_j^t(\eta_t)$. (A1)

The first-order condition (FOC) with respect to x_i gives:

$$\frac{\partial E[\pi(x, p)]}{\partial x_i} = - \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{m}_i^t(\eta_t) + \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) \int_{x_i - \bar{z}_i^t(\eta_t)}^{\infty} \phi_i(y_i^t(\eta_t)) dy_i^t(\eta_t) = 0 \quad \forall i.$$

$$- T \bar{m}_i + \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) [1 - \Phi_i(x_i - \bar{z}_i^t(\eta_t))] = 0$$

$$\frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) [1 - \Phi_i(x_i^* - \bar{z}_i^t(\eta_t))] = \bar{m}_i. \tag{A2}$$

Similarly, the FOC with respect to $p_j^t(\eta_t)$ gives:

$$\frac{\partial E[\pi(x, p)]}{\partial p_j^t(\eta_t)} = \bar{q}_j^t(\eta_t) - \beta_j (p_j^t(\eta_t) - \bar{v}_j^t(\eta_t)) + \beta_j \sum_{i=1}^I \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) \mu_{ij} \int_{x_i - \bar{z}_i^t(\eta_t)}^{\infty} \phi_i(y_i^t(\eta_t)) dy_i^t(\eta_t) = 0 \quad \forall j, t, \text{ and } \eta_t.$$

$$\bar{q}_j^{t*}(\eta_t) - \beta_j p_j^{t*}(\eta_t) = - \left(\beta_j \bar{v}_j^t(\eta_t) + \sum_{i=1}^I \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) \mu_{ij} \beta_j [1 - \Phi_i(x_i^* - \bar{z}_i^t(\eta_t))] \right)$$

$$p_j^{t*}(\eta_t) = \frac{\alpha_j^t(\eta_t)}{2\beta_j} + \frac{1}{2} \bar{v}_j^t(\eta_t) + \frac{1}{2} \sum_{i=1}^I \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) \mu_{ij} [1 - \Phi_i(x_i^* - \bar{z}_i^t(\eta_t))]. \tag{A3}$$

Lemma 2: The expected profit function under price commitment is:

$$E[\pi(x, p_j)] = - \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} \sum_{i=1}^I r^t(\eta_t) \bar{m}_i^t(\eta_t) x_i + \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \left\{ \sum_{j=1}^J (p_j - \bar{v}_j^t(\eta_t)) \bar{q}_j^{tB}(\eta_t) \right\} - \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} \sum_{i=1}^I r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) \int_{x_i - \bar{z}_i^{tB}(\eta_t)}^{\infty} (\bar{z}_i^{tB}(\eta_t) + y_i^t(\eta_t) - x_i) \phi_i(y_i^t(\eta_t)) dy_i^t(\eta_t)$$

(A4)

where $\bar{q}_j^{tB}(\eta_t) \equiv \alpha_j^t(\eta_t) - \beta_j p_j$ and $\bar{z}_i^{tB}(\eta_t) \equiv \sum_{j=1}^J \mu_{ij} \bar{q}_j^{tB}(\eta_t)$.

Equation (A4), the first-order condition (FOC) with respect to x_i , gives the optimal

$$x_i^B: \quad \frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) [1 - \Phi_i(x_i^B - \bar{z}_i^{tB}(\eta_t))] = \bar{m}_i \quad \forall i. \quad (\text{A5})$$

On the other hand, the FOC with respect to p_j gives:

$$\begin{aligned} \frac{\partial E[\pi(x, p_j)]}{\partial p_j} &= \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) (\alpha_j^t(\eta_t) - \beta_j p_j) - \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \beta_j (p_j - v_j^t(\eta_t)) \\ &\quad + \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} \sum_{i=1}^I r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) \mu_{ij} \beta_j \int_{x_i^B - \bar{z}_i^{tB}(\eta_t)}^{\infty} \phi_i(y_i^t(\eta_t)) dy_i^t(\eta_t) = 0. \end{aligned}$$

By denoting the optimal price as p_j^B , we have:

$$T\bar{\alpha}_j - 2T\beta_j p_j^B + T\beta_j \bar{v}_j + \sum_{i=1}^I \beta_j \mu_{ij} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) [1 - \Phi_i(x_i^B - \bar{z}_i^{tB}(\eta_t))] = 0,$$

that is: $T\bar{\alpha}_j - 2T\beta_j p_j^B + T\beta_j \bar{v}_j + T \sum_{i=1}^I \beta_j \mu_{ij} \bar{m}_i = 0$.

$$\text{Therefore, } p_j^B = \frac{\bar{\alpha}_j}{2\beta_j} + \frac{1}{2} \left(\bar{v}_j + \sum_{i=1}^I \mu_{ij} \bar{m}_i \right) \quad \forall j, t, \text{ and } \eta_t \quad (\text{A6})$$

where $\bar{\alpha}_j = \frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \alpha_j^t(\eta_t)$ and $\bar{v}_j = \frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) v_j^t(\eta_t)$.

Proposition 1:

(i) Averaging all the optimal prices in Equation (A3) gives:

$$\begin{aligned} \bar{p}_j^* &\equiv \frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) p_j^{t*}(\eta_t) \\ &= \frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \left[\frac{\alpha_j^t(\eta_t)}{2\beta_j} + \frac{1}{2} \left\{ v_j^t(\eta_t) + \sum_{i=1}^I \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) \mu_{ij} \left[1 - \Phi_i(x_i^* - \bar{z}_i^{t*}(\eta_t)) \right] \right\} \right] \\ &= \frac{\bar{\alpha}_j}{2\beta_j} + \frac{1}{2} \left(\bar{v}_j + \sum_{i=1}^I \mu_{ij} \bar{m}_i \right) \\ &= p_j^B. \end{aligned} \quad (\text{A7})$$

where:

$$\bar{\alpha}_j = \frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \alpha_j^t(\eta_t), \bar{v}_j = \frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) v_j^t(\eta_t) \text{ and}$$

$$\bar{m}_i = \frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) \left[1 - \Phi_i \left(x_i^* - \bar{z}_i^{t*}(\eta_t) \right) \right] \text{ from Equation (A2).}$$

(ii) $\bar{z}_i^{tB}(\eta_t) = \sum_{j=1}^J \mu_{ij} \bar{q}_j^{tB}(\eta_t) = \sum_{j=1}^J \mu_{ij} (\alpha_j^t(\eta_t) - \beta_j p_j^B)$

$$\bar{z}_i^{t*}(\eta_t) = \sum_{j=1}^J \mu_{ij} (\alpha_j^t(\eta_t) - \beta_j p_j^{t*}(\eta_t)).$$

Summing the above two demands for each resource gives:

$$\sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{z}_i^{tB}(\eta_t) = \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \sum_{j=1}^J \mu_{ij} (\alpha_j^t(\eta_t) - \beta_j p_j^B) = \sum_{j=1}^J \mu_{ij} (T \bar{\alpha}_j - T \beta_j p_j^B)$$

$$\sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{z}_i^{t*}(\eta_t) = \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \sum_{j=1}^J \mu_{ij} (\alpha_j^t(\eta_t) - \beta_j p_j^{t*}(\eta_t))$$

$$= \sum_{j=1}^J \mu_{ij} \left(T \bar{\alpha}_j - \beta_j \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) p_j^{t*}(\eta_t) \right)$$

$$= \sum_{j=1}^J \mu_{ij} (T \bar{\alpha}_j - T \beta_j p_j^B) \text{ from Equation (A7),}$$

that is, $\sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{z}_i^{tB}(\eta_t) = \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{z}_i^{t*}(\eta_t).$ (A8)

(iii) From Equations (A2) and (A5):

$$\frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) [1 - \Phi_i(x_i^* - \bar{z}_i^{t*}(\eta_t))] = \bar{m}_i \quad \forall i$$

$$\frac{1}{T} \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) [1 - \Phi_i(x_i^B - \bar{z}_i^{tB}(\eta_t))] = \bar{m}_i \quad \forall i$$

$$\sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) [1 - \Phi_i(x_i^* - \bar{z}_i^{t*}(\eta_t))]$$

$$= \sum_{t=1}^T \sum_{\eta_t=1}^{N_t} r^t(\eta_t) \bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) [1 - \Phi_i(x_i^B - \bar{z}_i^{tB}(\eta_t))] \tag{A9}$$

Proposition 2:

(i) We have $\lambda_i^t(\eta_t) \equiv \frac{\bar{m}_i^t(\eta_t)}{\bar{m}_i} \bar{\theta}_i^t(\eta_t) [1 - \Phi_i(x_i^* - \bar{z}_i^{t*}(\eta_t))]$, and the charge for the committed resource i is $\bar{\theta}_i^t(\eta_t) \bar{m}_i^t(\eta_t) [1 - \Phi_i(x_i^* - \bar{z}_i^{t*}(\eta_t))] = \bar{m}_i \lambda_i^t(\eta_t)$.

$$\frac{\partial \lambda_i^t(\eta_t)}{\partial \bar{z}_i^{t*}(\eta_t)} = \frac{\bar{m}_i^t(\eta_t)}{\bar{m}_i} \bar{\theta}_i^t(\eta_t) \phi_i(x_i^* - \bar{z}_i^{t*}(\eta_t)) > 0.$$

(ii) The required result follows directly from the above relation.

Lemma 3:

$$q_1^{1*} = \alpha_1^1 - \beta_1 p_1^1 = \frac{\alpha_1^1}{2} - \frac{\beta_1}{2} \left\{ v_1 + \sum_{i=1}^I m_i \mu_{i1} \frac{2\gamma_i^*}{(\gamma_i^* + 1)} \right\}. \quad (\text{A10})$$

$$q_2^{1*} = \alpha_2^1 - \beta_2 p_2^1 = \frac{\alpha_2^1}{2} - \frac{\beta_2}{2} \left\{ v_2 + \sum_{i=1}^I m_i \mu_{i2} \frac{2\gamma_i^*}{(\gamma_i^* + 1)} \right\}. \quad (\text{A11})$$

$$q_1^{2*} = \alpha_1^2 - \beta_1 p_1^2 = \frac{\alpha_1^2}{2} - \frac{\beta_1}{2} \left\{ v_1 + \sum_{i=1}^I m_i \mu_{i1} \frac{2}{(\gamma_i^* + 1)} \right\}. \quad (\text{A12})$$

$$q_2^{2*} = \alpha_2^2 - \beta_2 p_2^2 = \frac{\alpha_2^2}{2} - \frac{\beta_2}{2} \left\{ v_2 + \sum_{i=1}^I m_i \mu_{i2} \frac{2}{(\gamma_i^* + 1)} \right\}. \quad (\text{A13})$$

We provide a proof by contradiction. First, we consider the case when demand growth is expected in period 2 (i.e., $\alpha_1^2 \geq \alpha_1^1$ and $\alpha_2^2 > \alpha_2^1$). If $\bar{z}_i^{1*} \geq \bar{z}_i^{2*}$, then:

$$q_1^{2*} - q_1^{1*} = \frac{\alpha_1^2 - \alpha_1^1}{2} - \frac{\beta_1}{2} \left\{ \sum_{i=1}^I \theta_i m_i \mu_{i1} [-\Phi_i(x_i^* - \bar{z}_i^{2*}) + \Phi_i(x_i^* - \bar{z}_i^{1*})] \right\} > 0,$$

and

$$q_2^{2*} - q_2^{1*} = \frac{\alpha_2^2 - \alpha_2^1}{2} - \frac{\beta_2}{2} \left\{ \sum_{i=1}^I \theta_i m_i \mu_{i2} [-\Phi_i(x_i^* - \bar{z}_i^{2*}) + \Phi_i(x_i^* - \bar{z}_i^{1*})] \right\} > 0.$$

However, if $q_j^{2*} > q_j^{1*}$ for $j = 1, 2$, then $\bar{z}_i^{1*} = \sum_{j=1}^2 \mu_{ij} q_j^{1*} < \bar{z}_i^{2*} = \sum_{j=1}^2 \mu_{ij} q_j^{2*}$, which contradicts the assumption that $\bar{z}_i^{1*} \geq \bar{z}_i^{2*}$. Therefore, it must be the case that $\bar{z}_i^{1*} < \bar{z}_i^{2*}$.

Because $\Phi_i(\cdot)$ is an increasing function, $\gamma_i^* = \frac{1 - \Phi_i(x_i^* - \bar{z}_i^{1*})}{1 - \Phi_i(x_i^* - \bar{z}_i^{2*})} < 1$.

Next, consider the case when demand decline is expected in period 2 (i.e., $\alpha_1^2 \leq \alpha_1^1$ and $\alpha_2^2 < \alpha_2^1$). If $\bar{z}_i^{1*} \leq \bar{z}_i^{2*}$, then:

$$q_1^{2*} - q_1^{1*} = \frac{\alpha_1^2 - \alpha_1^1}{2} - \frac{\beta_1}{2} \left\{ \sum_{i=1}^I \theta_i m_i \mu_{i1} [-\Phi_i(x_i^* - \bar{z}_i^{2*}) + \Phi_i(x_i^* - \bar{z}_i^{1*})] \right\} < 0,$$

$$\text{and } q_2^{2*} - q_2^{1*} = \frac{\alpha_2^2 - \alpha_2^1}{2} - \frac{\beta_2}{2} \left\{ \sum_{i=1}^I \theta_i m_i \mu_{i2} \left[-\Phi_i(x_i^* - \bar{z}_i^{2*}) + \Phi_i(x_i^* - \bar{z}_i^{1*}) \right] \right\} < 0.$$

However, if $q_j^{1*} > q_j^{2*}$ for $j = 1, 2$, then $\bar{z}_i^{1*} > \bar{z}_i^{2*}$, which contradicts the assumption that $\bar{z}_i^{1*} \leq \bar{z}_i^{2*}$. Therefore, it must be that $\bar{z}_i^{1*} > \bar{z}_i^{2*}$. Because $\Phi_i(\cdot)$ is an increasing

$$\text{function, } \gamma_i^* = \frac{1 - \Phi_i(x_i^* - \bar{z}_i^{1*})}{1 - \Phi_i(x_i^* - \bar{z}_i^{2*})} > 1.$$

Proposition 3:

$$\text{Because } \gamma_i^* = \frac{1 - \Phi_i(x_i^* - \bar{z}_i^{1*})}{1 - \Phi_i(x_i^* - \bar{z}_i^{2*})} \text{ by definition, we have } x_i^* = \bar{z}_i^{1*} + \Phi_i^{-1} \left(1 - \frac{2\gamma_i^*}{\theta_i(\gamma_i^* + 1)} \right).$$

Idle capacity will be carried in the first period if and only if $\frac{1}{2} < 1 - \frac{2\gamma_i^*}{\theta_i(\gamma_i^* + 1)}$, or

equivalently for all $\theta_i > \theta_i^c$ where $\theta_i^c = \frac{4\gamma_i^*}{\gamma_i^* + 1}$. The fact that $1 < \theta_i^c$ follows because

otherwise it is cheaper to acquire the resources from the spot market rather than

contracting for it initially. The condition that $\theta_i^c = \frac{4\gamma_i^*}{\gamma_i^* + 1} < 2$ holds if $\gamma_i^* < 1$, which

follows from Lemma 3.

Proposition 4:

The optimal prices and quantities of the two products for the two periods are:

$$p_1^{1*} = \frac{\alpha_1^1}{2\beta_1} + \frac{v_1}{2} + \sum_{i=1}^I m_i \mu_{i1} \frac{2\gamma_i^*}{(\gamma_i^* + 1)}. \tag{A14}$$

$$p_2^{1*} = \frac{\alpha_2^1}{2\beta_2} + \frac{v_2}{2} + \sum_{i=1}^I m_i \mu_{i2} \frac{2\gamma_i^*}{(\gamma_i^* + 1)}. \tag{A15}$$

$$p_1^{2*} = \frac{\alpha_1^2}{2\beta_1} + \frac{v_1}{2} + \sum_{i=1}^I m_i \mu_{i1} \frac{2}{(\gamma_i^* + 1)}. \tag{A16}$$

$$p_2^{2*} = \frac{\alpha_2^2}{2\beta_2} + \frac{v_2}{2} + \sum_{i=1}^I m_i \mu_{i2} \frac{2}{(\gamma_i^* + 1)}. \tag{A17}$$

The charges per unit of capacity resources in period 1 and period 2 are $2\gamma_i^*/(\gamma_i^* + 1)$ and $2/(\gamma_i^* + 1)$, respectively. $2\gamma_i^*/(\gamma_i^* + 1)$ is less than $2/(\gamma_i^* + 1)$ if and only if $\gamma_i^* < 1$. From Lemma 3, when the firm expects demand growth in the second period, we have $\gamma_i^* < 1$, and when the firm expects demand decline in the second period, we have $\gamma_i^* > 1$. Hence, Proposition 4 follows directly from a comparison of charges by appealing to Lemma 3.

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