

The Adequacy of Full-Cost-Based Pricing Heuristics

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Abstract: We investigate the performance of a full-cost heuristic in a service setting. In our model, a service firm determines the amount of capacity, a price, and a price discount each period. Based upon the price, a stochastic number of customers will place service orders. If too many orders arrive in a period, the firm will offer a price discount to those customers willing to back order and accept service the next period. Even though the model is fairly simple, the optimal pricing, price discount, and capacity rules are complex and require extensive calculations. We examine how closely three distinct heuristics approximate the optimal performance. The best-performing heuristic is a full-cost pricing rule based upon a constrained version of the firm's optimization program. It consists of setting a price using full costs plus an adjustment based upon the nonlinear elasticity of demand. In 500 random simulations our full-cost heuristic obtains 99.5 percent of the optimal performance. Preliminary analysis suggests that a modified full-cost heuristic may continue to do well in settings where interim demand information arrives after the capacity choice, but before the pricing choice. However, a modified full-cost heuristic may not perform well when capacity lasts several periods.

INTRODUCTION

The argument over whether to include capacity (fixed cost) charges into product costs is one of the longest running debates in management accounting (Church 1911; Williams et al. ([1921] 1990); Vatter 1945; Zimmerman 1979; Shim and Sudit 1995). The classic exchange of views (Church 1915; Gantt 1915) is that if managers set prices using only variable costs, a proxy for marginal costs, then they will generate low prices and high sales. However, if the price is too low, then the firm will not recover its fixed costs and it will lose money in the long run. Conversely, if the managers set prices using full costs (variable plus unit fixed costs), then every order will cover its portion of fixed costs. However, the higher prices may reduce orders to the point where the firm again loses money in the long run. This discussion suggests that a manager would be able to choose between direct and full-costing methods if she understood how product prices interact with customer demand and the firm's capacity. A more detailed business model that captures the marketing and capacity trade-offs would show which alternative was preferred.

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However, with a sophisticated model of the firm's operations, the manager could directly choose the optimal prices and capacity. Given this complete model and analysis, the choice of full or variable costing becomes irrelevant because the cost system does not affect any of the manager's decisions.

Although this is a telling point, managers can rarely generate robust, solvable models of their markets and operations. Pricing and capacity decisions are very complex and determining the optimal solution may be prohibitively expensive. An alternative is to identify simple rules (heuristics) that generate close to the optimal solution (Banker et al. 2002; Göx 2002; Hansen and Magee 1993). Prior literature on the efficacy of heuristics (reviewed in Balakrishnan and Sivaramakrishnan [2002]) has concentrated on full-cost heuristics because full-cost prices are commonly used by managers (Shim and Sudit 1995; Govindarajan and Anthony 1983).

We investigate the performance of three simple heuristics, including one full-cost heuristic, in a model where costs, prices, and capacity choices are intertwined. We model a service company that determines both the amount of capacity and the price each period. Based upon the price, a stochastic number of customers will place service orders. If too many orders arrive in a period, the manager will offer a price discount, set at the start of the period, to those customers willing to back order and accept service in the next period. Even though the model is fairly simple, the optimal pricing, discount, and capacity rules are determined simultaneously via a system of nonlinear equations and require extensive calculations.

Three heuristics naturally emerge from the limiting behavior of the model. They incorporate either the limiting equation for the optimal price (full-cost heuristic), the limiting equation for the optimal capacity (balanced-capacity heuristic), or both limiting equations (basic heuristic). All three heuristics can be implemented recursively by solving comparatively simple equations for the price, discount, and capacity.

We evaluate the performance of these three heuristics relative to the optimal solution and determine the size of the expected loss for each. The results of this simulation horse race are striking. The full-cost-based heuristic wins and achieves a median performance of 99.5 percent of optimal profits in a simulation with 500 observations vs. 94.0 percent (94.1 percent) for the balanced-capacity heuristic (basic heuristic). Our results provide evidence in support of reporting unit costs for pricing decisions.

Our paper is distinguished from prior work in several ways. First, we investigate a service setting, which complements the prior work that focuses exclusively on manufacturing. Second, we allow the manager to select a price discount, a method of shifting future capacity into the present period. As section six demonstrates, this discount is related to the concept of soft capacity in manufacturing, additional capacity purchased at a premium to augment existing capacity (e.g., overtime). Depending on the parameters, the price discount captures the cases of no soft capacity allowed, unlimited soft capacity permitted, and a new case in which the amount of soft capacity is endogenously determined by the manager. Finally, unlike prior simulations, our analysis does not restrict the parameters to fall into one of the two polar soft capacity cases (no soft capacity allowed, unlimited soft capacity permitted). Our simulations cut across several cases and, therefore, our full-cost heuristic is robust to the form of capacity.

Once we complete our basic analysis, we provide preliminary evidence about the robustness of our results. Our preliminary analysis suggests that a full-cost heuristic may continue to do well in a setting where interim-demand information arrives after the capacity choice, but before the pricing decision. However, our

preliminary evidence also suggests that a full-cost heuristic does not perform well in situations where capacity lasts several periods.

The organization of this paper is as follows. The second section presents the basic model, and the third section presents the complex, optimal solution. Section four provides the limiting solution as the number of customers increases. The fifth section creates heuristics using this benchmark, and section six presents simulation results. We recast our model into a manufacturing setting in section seven. Section eight discusses several extensions, and section nine presents our conclusions.

THE BASIC MODEL

We model a generic service company in which the manager determines the service capacity at the start of each period.¹ The manager then creates and publicizes a price list that describes the service and its price. Customers read the price list and queue up to purchase the service. If too many customers arrive during the period, the manager offers customers a predetermined price discount if they are willing to wait and accept service the next period.

For example, our service company could be a barbershop. The manager's capacity decision is the number of barbers to schedule during a day. The price of a haircut is posted in the window. If too many customers arrive at one time, the manager may offer the customers a preprinted discount coupon if they will come back for a haircut the next day. Returning customers are always served first.

Figure 1 shows the timeline for our model. The model reflects a representative period in an infinite horizon game with no discounting and, hence, the time subscript is suppressed. At the start of the period the manager chooses the service capacity level, y . Capacity consists of two parts: the amount committed to fill the prior period's backorders, b_{-1} , and the amount that can be devoted to new orders in the current period, x . The manager selects the capacity level before knowing the current period demand for his product. The cost of capacity is linear with parameter k , so that the total cost of capacity is ky . Next, the manager sets the current period price, $p > 0$ and the price discount, $d \geq 0$, to induce customers to back order when all service capacity is committed. Both the price and price discount are "sticky" and, once set, cannot be changed until the start of the next period (Blinder et al. 1998; Wolman 2000).² To simplify the analysis, all customers order one unit of service that requires one unit of capacity and generates variable costs, $v > 0$.³ Performing the service requires a short amount of time and all sales begun in a period are completed in that period.

¹ For simplicity, we assume that there is no upper bound on the amount of potential service capacity.

² If the manager were allowed to adjust the optimal price throughout the period, then the optimal price would vary with the number of customers who have already ordered, the number of potential customers remaining who could order, and the amount of remaining current capacity available. We conjecture that the price would vary for every customer until capacity was filled. However, once current capacity is assigned, we believe that the manager would offer the same price discount to all remaining customers. The difference is driven by the distinction between constrained current capacity and unconstrained future capacity. The manager can always purchase enough capacity in the following period to fill back orders. Therefore, the manager sets the price discount by making the same trade-off between price and probability of ordering for every customer.

³ Our model assumes that all capacity costs are proportional to the service-level capacity. Adding in fixed capacity costs that would not vary with the service capacity or the actual service level is straightforward. We would just subtract a constant from the objective functions throughout the paper. This additional, constant, term would not affect any of the manager's choices or the performance of any heuristic. However, we would have to add the additional condition that the firm would shut down if its profits were insufficient to cover their fixed capacity costs.

annoyance at not being served. To increase the likelihood a customer will back order, the manager may offer a predetermined price discount, $d \geq 0$, if the customer is willing to back order. This price discount is determined *ex ante*, before the number of over-capacity orders is known. Combining these effects, customer i will back order if and only if $u_i - (a - d) \geq p$, otherwise she will not order. Because only customers who tried to order are offered the discount, the manager never offers a discount greater than the annoyance factor, $a \geq d \geq 0$. Offering a larger discount would lower the amount received from the sale without affecting the probability the customer would return the next day.

The percentage of customers offered the price discount who are willing to wait is given by:

$$w(p,d) = \frac{(1 - F(p - d + a))}{(1 - F(p))},$$

where $1 \geq w(d,p) \geq 0$. If the manager offers a discount equal to the customer's disutility from back ordering, $d = a$, then all customers who cannot immediately fill their orders would be willing to wait, and $w(p,d) = 1$. If the disutility of a back order is very large, $a \rightarrow \infty$, then no customers would be willing to wait, and $w(p,d) = 0$.

Each of the m potential customers has an identical *ex ante* probability of placing or not placing an order. With this structure, the *ex ante* distribution of sales orders, s , is binomial. The binomial distribution is discrete and cannot be analyzed by the standard calculus techniques. Therefore we use continuous normal approximations to the binomial to perform the analysis. In our model, the density of current customer sales, s , is $f(s;m,p)$, which is approximately normally distributed with mean $m(1 - F(p))$ and variance $mF(p)(1 - F(p))$. Given that the current sales, s , are greater than the capacity, x , available for current sales, $s - x > 0$, customer back orders, b , then have a density $f(b;w(p,d),s - x)$, which is also approximately normally distributed with mean $w(p,d)[s - x]$ and variance $w(p,d)[1 - w(p,d)][s - x]$.

The manager selects the current capacity, x , the price, p , and the price discount, d , to maximize expected profits:

$$\begin{aligned} \max_{x,p,d} & \int_{-\infty}^x (p - v)sf(s; m, p)ds + \int_x^{\infty} (p - v)xf(s; m, p)ds \\ & + \int_x^{\infty} (p - d - v - k) \left\{ \int bf(b; w(p,d), s - x)db \right\} f(s,m, p)ds - kx. \end{aligned}$$

We will suppress the waiting percentage's dependence upon the price and discount in order to simplify the notation.

Integrating out the back order density and rearranging terms provides the following mathematical program.

The Manager's Program

$$\max_{x,p,d} \int_{-\infty}^x (p - v)sf(sm,p)ds + \int_x^{\infty} \{ (p - v)x + w(p - d - v - k)(s - x) \} f(sm,p)ds - kx,$$

where $x, p \geq 0, \quad a \geq d \geq 0.$

The Manager's Program contains two integrals. The first integrates the marginal profits when current orders are less than the capacity devoted to current period orders. The second reflects the marginal profits when current orders are greater than capacity for current orders. This second integral contains two terms.

The $(p - v)x$ term captures profits for those orders satisfied by current capacity, while the $w(p - d - v - k)(s - x)$ term contains the additional profits from those customers willing to wait.

We now analyze the manager's optimal choices.

THE OPTIMAL SOLUTION

We begin with the manager's capacity choice.

The Manager's Capacity Choice

Differentiating The Manager's Program with respect to the current capacity amount, x , generates the capacity first-order condition:

$$1 - \int_{-\infty}^x f(s|m,p) ds = \frac{k}{[(1-w)(p-v) + w(d+k)]} \quad (1)$$

The left-hand side of Equation (1) is the expected fraction of customers not served by the current capacity. The right-hand side is the ratio of the cost of purchasing one more unit of capacity to the opportunity cost of lost customers. Equation (1) states that the fraction of customers who are not served depends on the ratio of the cost of excess capacity to the cost of insufficient capacity.

The capacity choice in Equation (1) is similar to the inventory choice in the classic news-vendor problem. The solution requires selecting capacity so that the fractile of customers serviced equates the expected costs of over- and under-capacity.⁷ Although the results are similar, our model is more general because the characteristics of the manager's demand function are partially under the control of the manager, while in the news-vendor problem the price and the stochastic demand function are exogenously fixed. When the manager selects a product price, he adjusts the mean and the variance of the number of customer orders. In this way, the manager endogenously selects the stochastic demand distribution.⁸

As with all first-order conditions, Equation (1) shows the manager's optimal capacity choice given any price, not just the optimal one. As such, Equation (1) suggests how the manager should react to a heuristically selected price. This observation serves as the basis for the certainty price heuristic described later in the paper.

Using standard properties of the normal distribution, we can rewrite Equation (1) as follows in Proposition 1.

Proposition 1: The manager's optimal current capacity choice solves:

$$x = m(1 - F(p)) + \tau(p)[mF(p)\{1 - F(p)\}]^{1/2}, \quad (2)$$

where $\tau(p) \equiv \Phi^{-1}\left(1 - \frac{k}{[(1-w)(p-v) + w(d+k)]}\right)$ and $\Phi(\cdot)$ is the cumulative unit normal distribution.

Proposition 1 shows that the manager chooses capacity for current sales equal to the expected demand plus a multiple of the standard deviation of sales. The multiplier, $\tau(p)$, depends upon the ratio of the cost of excess capacity to the cost of

⁷ For example, Equation (1) is similar to the news-vendor-capacity Equation (10.6) (Silver et al. 1998, 388).

⁸ In the Operations Research literature this problem is termed the inverse news-vendor problem (Carr and Lovejoy 1999; Lau and Lau 1988).

insufficient capacity. Because the multiplier can be negative, the manager may choose to serve less than 50 percent of the customers. If no customers are willing to wait, $w = 0$, the multiplier simplifies to $\tau(p) = \Phi^{-1}\left(\frac{p-v-k}{p-v}\right)$, reflecting the ratio of the gross profit to the contribution margin. If all customers are willing to wait, $w = 1$, the multiplier becomes $\tau(p) = \Phi^{-1}\left(\frac{d}{d+k}\right)$, reflecting the relative cost of the price discount to the incremental cost of a waiting customer.

Managers often attempt to “balance” the production facility by purchasing capacity equal to expected demand. The following corollary describes a specialized setting where the stochastic capacity rule (Equation (2)) reduces to this balanced approach.

Corollary 1.1: The manager will purchase current capacity equal to expected current period demand, $x = m(1 - F(p))$, if and only if the ratio of excess to insufficient capacity costs is one-half:

$$\frac{k}{[(1-w)(p-v) + w(d+k)]} = \frac{1}{2}. \quad (3)$$

Proof: Equation (3) implies $\tau(p) = 0$ in Equation (2).

The intuition behind Corollary 1.1 is straightforward. Equation (3) implicitly equates the expected marginal costs of over- and under-capacity. When the manager purchases a unit of capacity for current sales he pays k for certain. When the manager sets capacity equal to expected demand,⁹ the probability that capacity is too low is 50 percent, yielding an expected cost of $(.50)[1-w](p-v) + w(d+k) = (.50)(2k) = k$.

Rewriting Equation (3) yields a form that highlights its connection to the pricing heuristics to be introduced in Section Five, specifically:

$$p = v + k + \frac{k-wd}{(1-w)}. \quad (4)$$

When the optimal capacity equals expected demand, the optimal price is full cost (variable plus unit fixed cost) plus a markup where the markup reflects the difference between paying for capacity up front, k , and the expected cost of paying the price discount, wd .

Now that we have described the manager’s capacity choice, we turn to examining the manager’s selection of the back order discount.

The Manager’s Back Order Discount Decision

Differentiating The Manager’s Program with respect to the price discount, d , provides the following Proposition.

Proposition 2: The optimal backorder price discount involves three cases:

Case 1. When the gross profit from back-ordered customers is relatively small,

$$\frac{(1-F(p+a))}{f(p+a)} > (p-v-k), \text{ the manager does not offer a price discount, } d = 0, \text{ and the percentage of waiting customers is } w(p,0) = \frac{(1-F(p+a))}{(1-F(p))}.$$

⁹ Our model uses a symmetric ordering distribution and therefore the mean equals the median.

Case 2. When the gross profit from back-ordered customers is intermediate, $\frac{(1 - F(p))}{f(p)} \geq (p - v - k - a)$ and $(p - v - k) \geq \frac{(1 - F(p + a))}{f(p + a)}$, the manager offers a price discount based on the first-order condition:

$$p - v - k - d = \frac{(1 - F(p - d + a))}{f(p - d + a)}, \quad (5)$$

and the percentage of waiting customers is $w(p, d) = \frac{(1 - F(p - d + a))}{(1 - F(p))}$.

Case 3. When the gross profit from back-ordered customers is relatively large, $(p - v - k - a) \geq \frac{(1 - F(p))}{f(p)}$, the manager offers the maximum price discount, $d = a$, and the percentage of waiting customers is $w(p, d) = 1$.

The proof is given in the Appendix.

Proposition 2 shows that there are three qualitatively distinct discount settings. In Case 1 a price discount does not generate a sufficient increase in waiting customers to be cost effective and no discount is given. In Case 2 the manager can increase expected profits by offering a price discount. The optimal discount (Equation (5)) trades off the decrease in the unit gross profit from the discount against the increase in the expected number of customer's orders, captured by the inverse hazard rate. In Case 3 every over-capacity customer order is valuable and the manager reimburses all over-capacity customers for the annoyance of returning the next period. Every over-capacity customer is induced to return the following period.

The simple form of the interior solution (Equation (5)) comes from the decision to fill back orders being unconstrained by capacity considerations. The manager can always purchase enough additional capacity in the next period to produce back orders. In contrast, the pricing decision is more complex and must allow for the presence of binding capacity constraints. The next subsection considers the manager's optimal pricing decision.

The Manager's Pricing Choice

Because no information is revealed between the choice of capacity and the choice of price we can treat them as simultaneous choices. Using the capacity first-order condition (Equation (1)) and following the sequence of steps in the Appendix, the manager's pricing program can be written as follows.

The Manager's Pricing Program:

$$\max_p (p - v - k)m(1 - F(p)) - [(1 - w)(p - v) + w(d + k)][mF(p)(1 - F(p))]^{1/2} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\tau(p)^2}{2}\right),$$

such that $p > 0$.

The Manager's Pricing Program begins with the certainty profits and subtracts off a penalty that captures the additional costs due to stochastic orders. The penalty has two elements. The first element reflects the marginal impact of not having a unit of capacity for current sales. The manager loses the contribution margin from the customers who are unwilling to wait, $(1 - w)(p - v)$, and also incurs additional costs for those customers who do wait, $w(d + k)$. The second

element in the penalty, involves the exponential term, $\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\tau(p)^2}{2}\right)$, the unit normal density in $\tau(p)$.

The behavior of the penalty for stochastic orders can be seen by rewriting capacity Equation (1) as:

$$\tau(p) = \frac{x - m(1 - F(p))}{[mF(p)(1 - F(p))]^{1/2}}. \quad (6)$$

This exponential penalty term is symmetric about the mean demand and is greatest when capacity for current sales equals demand. Therefore, when capacity is balanced with the expected demand, the manager has the greatest costs due to stochastic orders.

The following proposition presents the manager's pricing decision.

Proposition 3: The manager's optimal service price solves:

$$\begin{aligned} p - \frac{(1 - F(p))}{f(p)} = v + k & \quad (7) \\ + \frac{k\tau(p)[mF(p)(1 - F(p))]^{1/2}}{[(1 - w)(p - v) + w(d + k)]mf(p)} \left[(1 - w) - \frac{(p - v - k)[wf(p) - f(p - d + a)]}{(1 - F(p))} \right] \\ - \frac{[mF(p)(1 - F(p))]^{1/2}}{\sqrt{2\pi}} \exp\left(-\frac{\tau(p)^2}{2}\right) \\ \bullet \left[\frac{(1 - w)}{mf(p)} - \frac{(p - v - k)[wf(p) - f(p - d + a)]}{mf(p)(1 - F(p))} + \frac{[(1 - w)(p - v) + w(d + k)](1 - 2F(p))}{2[mF(p)(1 - F(p))]} \right]. \end{aligned}$$

See the Appendix for the proof.

The pricing rule (Equation (7)) is difficult to calculate, though straightforward to describe. The pricing rule depends upon the inverse hazard rate, $[(1 - F(p))/f(p)]$, which captures the change in the number of potential orders as the price rises, the variable costs of production $[v]$, the marginal costs of capacity $[k]$, a term that captures the increase in the per unit opportunity loss when the manager raises

prices, $\left[\frac{k\tau(p)[mF(p)\dots]}{[(1 - w)(p - v)]\dots} \right]$, and a term that captures the shift in the probability that the capacity for current sales will be insufficient (over-sufficient) to satisfy expected demand, $\left[\frac{[mF(p)(1 - F(p))]^{1/2}}{\sqrt{2\pi}} \dots \right]$.

We have now completed the characterization of the optimal capacity, price discount, and price. Determining the optimal solution involves solving the complex, nonlinear, simultaneous system of equations inherent in Propositions 1, 2, and 3. We will examine whether simpler, easier to calculate heuristics can approach the optimal profits. The next section provides a logical source for these simple heuristic rules.

THE CERTAINTY BENCHMARK

Our benchmark solution replaces the random-order distribution generated by stochastic reservation prices with the same deterministic demand function, $m(1 - F(p))$, in every period.

Given a deterministic demand, the manager will never discount the price and will never generate back orders, as shown in the Appendix. Therefore, in the certainty setting the manager always purchases just enough capacity to satisfy demand:

$$s = x = m(1 - F(p)).$$

The manager sets each period's price by solving the following program.

The Certainty Price Program

$$\max_p (p - v)s - kx \quad (8)$$

$$\text{s.t. } s = x = m(1 - F(p)).$$

The Certainty Price Program is very simple. The manager's profits consist of the gross profit margin, $(p - v - k)$, times demand, $m(1 - F(p))$. Since the manager does not allow back orders, the probability of waiting customers and the costs of servicing waiting customers do not influence the manager's decisions.

The manager's pricing first-order condition is:

$$p - \frac{(1 - F(p))}{f(p)} = v + k. \quad (9)$$

If we define the function $g(p) \equiv p - \frac{(1 - F(p))}{f(p)}$, then Equation (9) can be rewritten as:

$$p^* \equiv g^{-1}(v + k), \quad (10)$$

an expression which highlights the certainty pricing rule's dependence upon full costs.

The certainty pricing rule (Equation (9)) is relatively simple. The price equals full costs, $v + k$, plus an adjustment based upon the inverse hazard rate, $(1 - F(p))/f(p)$. The presence of the inverse hazard rate illustrates the distinction between our model and that in Hansen and Magee (1993), where the manager produces for other divisions of the firm. In their setting, because the firm captures the benefits from all final product sales, the manager simply charges other divisions the full cost of production. In contrast, in our model the manager sells products to external customers, and therefore he must set prices above full cost to generate profits. The inverse hazard rate trades off the increased profits due to higher prices with the reduction in the number of customers whose reservation prices are above the higher price threshold.

Equation (9) provides a second connection to prior work. Rewriting Equation (9) yields the formulation:

$$p \left[1 + \frac{1}{\text{elasticity of demand}} \right] = \text{full cost},$$

the classic monopolist's first-order condition. To generate the standard sign on the elasticity of demand, we consider only reservation price distributions with non-negative inverse hazard rates, that is, for all p .¹⁰

Capacity Equation (8) and Pricing Equation (9) jointly comprise the certainty benchmark.

Besides the simple decision rules, the certainty benchmark also has the following useful property.

¹⁰ This assumption is satisfied by many standard distributions, among them are the normal, log-normal, exponential, and gamma.

Proposition 4: As the number of potential customers increases, the profits from using the certainty benchmark solution converge to the profits of the optimal solution. See the Appendix for the proof.

The next section uses the certainty benchmark to construct the heuristics.

THE HEURISTICS

Our heuristics are based upon a constrained optimization approach. We restrict the price (capacity) choice to follow a simple rule and then allow the firm to select the optimal capacity (price) choice given these simple rules. Because the optimal discount already follows a simple form we will not construct a heuristic for this choice.

The source of our simple decision rules is the certainty benchmark setting. Table 1 summarizes the heuristics. We focus on two decisions: price and capacity, and two potential approaches for each decision: use a simple heuristic rule or optimize against the other choices. The *basic certainty heuristic* uses the benchmark solution to generate heuristics for both choices. The *certainty price heuristic* uses the certainty price equation and generates the optimal capacity choice given this price. The *certainty capacity heuristic* is the converse of the certainty price heuristic. Now the capacity choice uses a heuristic, while the price is optimal given the capacity choice. Finally, in the optimal solution the manager chooses the optimal capacity given the price and the optimal price given the capacity.

Because the basic certainty heuristic uses heuristics for both decisions, we anticipate that it will not perform as well as the certainty price and the certainty capacity heuristics.

By comparing the performance in the off-diagonal heuristics in Table 1, we will be able to determine the relative importance of the pricing and capacity decisions for the firm. For instance, if the certainty price heuristic performs better than the certainty capacity heuristic, then optimizing over the capacity would be more important than optimizing over the price.

Our heuristics have two additional attractive features. First, they correspond to practice in that the two heuristics employing the limiting pricing equation rely upon full-cost information. Second, and more importantly, all three heuristics enable decentralized decision making, in contrast to the optimal solution that requires a central authority to have access to all information and, implicitly, to solve a system of simultaneous equations.

The Basic Certainty Heuristic

The basic certainty heuristic uses both the benchmark price and capacity choices as simple decision rules in the stochastic setting.

The basic certainty heuristic decomposes the firm’s pricing and capacity decisions into four parts. First, the accounting group calculates the variable and fixed

TABLE 1
The Decision Rules for the Heuristics

	Heuristic Price	Optimal Price given Capacity Choice
Heuristic Capacity	Basic Certainty	Certainty Capacity
Optimal Capacity given Price Choice	Certainty Price	Optimal Solution

cost parameters, $v + k$. Accounting passes these charges to the marketing department, which forecasts the elasticity of demand (the inverse hazard rate), and sets the price using the benchmark rule:

$$p - \frac{(1 - F(p))}{f(p)} = v + k. \quad (9)$$

Given this price, the marketing group determines the price discount using Proposition 2.

The marketing group then sends the price and price discount to the production group. The production group uses the benchmark capacity rule and purchases capacity equal to expected demand:

$$x = m(1 - F(p)). \quad (8)$$

The certainty price rule is related to standard accounting procedures for several special cases.

Proposition 5:

- (1) If the reservation prices have a pareto density $f(p) = cp^{-(c+1)}$, $p \geq 1$; then the certainty price heuristic is a constant markup over full costs:

$$p = (v + k) * \frac{c}{(c-1)} = (\text{full costs})(\text{mean reservation price}) > (\text{full costs}).$$

- (2) If the reservation prices have an exponential density, $f(p) = \frac{1}{\lambda} \exp(-\frac{p}{\lambda})$, then the certainty price heuristic adds a fixed amount of profits to full costs:

$$p = v + k + \lambda = \text{full costs} + \text{mean reservation price}. \quad (11)$$

Proposition 5 shows that for very specific distributions the certainty price rule takes a very simple form. The pricing rule is a positive markup over full costs. Proposition 5 allows us to relate the price and the distribution of underlying consumer preferences. Both pricing rules in Proposition 5 imply that the manager sells only to customers with reservation prices above the population mean. In other words, the firm sells only to the upper tail of the customer price distribution.

The basic certainty heuristic assumes that the marketing and operations groups use heuristics, that is, simple “rules of thumb” to determine their actions. The second and third heuristics relax this assumption by allowing one group to use a heuristic rule, while the other group optimizes against that heuristic.

The Certainty Price Heuristic

The certainty price heuristic follows a similar sequence as the basic certainty heuristic. First, the accounting group calculates the unit variable and fixed cost parameters, $v + k$. Accounting passes these charges to the marketing department, which forecasts the elasticity of demand (the inverse hazard rate), and sets the price using the benchmark rule:

$$p - \frac{(1 - F(p))}{f(p)} = v + k. \quad (9)$$

Using this price, the marketing group determines the optimal price discount using Proposition 2. The marketing group sends their price and price discount to the production group. The production group uses marketing’s price and accounting’s cost estimates to determine the optimal capacity choice given this price. Production selects capacity for new sales, x , to solve:

$$x = m(1 - F(p)) + \tau(p)[mF(p)(1 - F(p))]^{1/2}, \quad (2)$$

where $\tau(p) \equiv \Phi^{-1}\left(1 - \frac{k}{[(1-w)(p-v) + aw]}\right)$ and $\Phi(\cdot)$ is the cumulative unit normal distribution.

The certainty price heuristic assumes that the marketing group uses a simple pricing rule, while the operations group is sophisticated and selects the optimal capacity given marketing's simplistic action.

The Certainty Capacity Heuristic

The certainty capacity heuristic reverses the sophisticated and unsophisticated roles from the certainty price heuristic. Now the operations group uses a simple capacity rule while the marketing group is sophisticated and selects the optimal price and discount given operation's simplistic capacity choice.

The certainty capacity heuristic follows a more complex procedure than the other heuristics. As before, the accounting group calculates the unit variable and fixed cost charges and passes these charges to the marketing department. The marketing department knows that the production department is going to use their price to choose capacity equal to expected demand. Given this restriction, the Appendix shows that the optimal pricing rule is:

$$\begin{aligned}
 p - \frac{(1 - F(p))}{f(p)} = v + k & \tag{12} \\
 - \frac{[mF(p)(1 - F(p))]^{1/2}}{\sqrt{2\pi}} \left(\frac{(1 - w)}{mf(p)} + \frac{[(1 - w)(p - v) + w(d + k)](1 - 2F(p))}{2[mF(p)(1 - F(p))]} \right) \\
 - \frac{(p - v - k)[wf(p) - f(p - d + a)]}{mf(p)(1 - F(p))}.
 \end{aligned}$$

Because the optimal pricing rule depends upon the optimal price discount (through the waiting percentage's [w's] dependence on p and d), the marketing department must simultaneously solve for the price and discount. The marketing department then passes the price and discount to production. In the final step, production uses the simplified capacity rule:

$$x = m(1 - F(p)). \tag{8}$$

The next section uses a sequence of simulations to run a horse race to investigate the circumstances where each heuristic performs best.

THE SIMULATON STUDY

To run the simulation horse race, we must parameterize two remaining functions.

For the potential customers' reservation price density, f(p), we use the Weibull density:

$$f(p) = \frac{c}{\alpha} \left(\frac{p}{\alpha} \right)^{c-1} \exp[-\{p/\alpha\}^c], \quad \text{where } p, c, \alpha > 0;$$

which displays a wide variety of shapes. By varying the parameter c from 1 to 3.6, the Weibull density moves from an exponential shape to approximately log-normal, gamma, and normal (Johnson et al. 1994, 630-635). As c increases, the distribution evolves from highly asymmetric to completely symmetric. The Weibull distribution

has an inverse hazard rate with the simple form, $\frac{\alpha}{c} \left(\frac{\alpha}{p} \right)^{c-1}$, which increases in the scale parameter, a, and decreases in the shape parameter, c.

The final functional choice is how to generate the variable production costs. Because we focus on how the firm uses heuristics to make pricing and capacity decisions, the variable costs are interesting only relative to capacity costs. The simulations determine the variable costs by first randomly drawing a value of the unit capacity costs, k , then randomly drawing a multiplier, t . The variable cost parameter is then calculated as $v = tk$. The regressions described later in the paper use the value of the unit capacity costs, k , and the multiplier, t , in the analysis. This approach allows us to determine how the mix of variable vs. capacity costs affects the results.

Using the functional choices, we created a program to calculate the optimal solution and the heuristics. The flow of the program logic is as follows. First, we draw the parameters for an example and calculate the basic certainty heuristic solution. If the basic heuristic solution generates negative profits for these parameters, we then redraw the parameters.¹¹ Once we have a set of parameters that generate positive profits, we use these basic certainty numbers as the starting values for calculating the optimal solution. We then solve for the optimal solution initially assuming that the solution falls into Case 1 (no price discount), the simplest case of Proposition 2. If the Case 1 condition is not satisfied, we next assumed that the solution fell into Case 2 and generated a second preliminary solution. We found that every candidate Case 2 solution satisfied the Case 2 conditions. As a result, Case 3, the maximum price discount, never emerged in our simulation. After generating the optimal Case 1 or Case 2 solution the program calculates the certainty price and certainty capacity heuristics. We programmed the simulations in LISP and ran them using Mathematica on an HP Vectra IBM clone with a 66 Mhz Pentium Chip and 32 MB RAM.

Table 2 presents the range of the various parameter choices that we selected to meet four criteria. First, the parameters span a broad range of values to generate different performances across the heuristics. The *median* profit performance for the different heuristics ranged from 94 percent to 99.5 percent of the optimal solution.¹² Second, the gains from using a news-vendor optimization approach should be roughly in line with the gains reported in prior studies. Our simulations exhibit a 5.9 percent increase in profits over the basic certainty case from the use of the optimal news-vendor-type approach, similar to the 4.7 percent found in Lieberman (1989).¹³ Third, the capacity cost parameters vary by a factor of 233 percent, reflecting a wide range of capacity cost values. Finally, the optimal capacity should, on average, be close to expected demand, providing the most favorable environment for the certainty capacity heuristic. The average $\tau(p)$ is $-.4777$, which means that the optimal capacity was less than half a standard deviation below expected demand.

The average run time per simulation was 413 seconds (6.9 minutes). Even for our simple one-product setting, the firm faces a substantial calculation burden. Finding the optimal solution consumed 98 percent of the total calculation time, while the three heuristics together required only two percent of the total calculation

¹¹ In the entire set of 500 simulations, the program redrew the parameters 242 times. We performed an analysis of the rejected parameters and found that the rejected parameters were more likely to have: (1) low numbers of potential customers, (2) high unit capacity costs, and (3) high distribution-shape parameters.

¹² The *average* profit performance ranged from 88 percent to 99 percent.

¹³ Lieberman (1989) investigates the determinants of industry capacity utilization in 40 chemical-process industries over a period of roughly two decades. The gain due to the news vendor model is not explicitly calculated in the paper but can be easily constructed using the data in his Table 3.

TABLE 2
The Simulation Parameters

$m \in [10, 40]$
$k \in [.30, .70]$
$t \in [.50, .65]$
$a \in [.20, .80]$
$c \in [1, 4]$
$\alpha \in [.70, .80]$

Variable Definitions:

- m = number of potential customers;
 - k = cost of capacity;
 - t = variable cost multiplier, variable cost = t (capacity cost);
 - a = customer's disutility from back ordering;
 - c = shape parameter, customers' Weibull reservation price density; and
 - α = scale parameter, customers' Weibull reservation price density.
-

time.¹⁴ Given the substantial calculation time, we investigated which parameters cause the problems. Table 3 shows the results of regressing the optimal solution calculation time on the model parameters for each case with a dummy when the optimal price discount rate is 0.¹⁵ We find that the calculation time increases in the unit-capacity costs and distribution-shape parameter, but decreases in the number of potential customers, the customer-annoyance factor, and if the solution has a 0 price discount. These results are intuitive. As the capacity costs become more important, the nonlinear penalty for stochastic orders increases in importance and increases the program run time. For larger shape parameter values, the inverse hazard rate drops, and the additional nonlinear terms in the program become relatively more important. As the customer annoyance factor increases, fewer customers wish to back order, the nonlinear penalty for stochastic orders declines in importance, and run time drops. Finally, as the number of customers increases, the solution converges to the simpler certainty benchmark, decreasing the calculation time.

The global pattern for all 500 simulations is very strong. As anticipated, the certainty capacity and certainty price heuristics outperformed the basic certainty heuristic. In every case, the basic certainty heuristic lost out to one of the other heuristics.¹⁶ The median (mean) profit for the basic certainty heuristic was 94.10 percent (86.05 percent) of the optimal solution, while for the certainty capacity it was 94.03 percent (86.34 percent), and for the certainty price it was 99.47 percent (98.83 percent). However, the certainty capacity heuristic resulted in a statistically insignificant decline in performance relative to the basic certainty (94.03 percent vs. 94.10 percent), while the certainty price generated a statistically

¹⁴ On average, calculating the certainty price heuristic took only 1.23 percent (5.34 sec), as long as finding the optimal solution. The basic certainty case took 0.58 percent (2.51 sec) of the optimal solution time, while the certainty capacity heuristic took 0.68 percent (2.95 sec) of the time.

¹⁵ Running the time regression separately for each case yields similar results.

¹⁶ The certainty price heuristic performed best in 96.00 percent of the examples, while the certainty capacity heuristic won in the other 4.00 percent. When the certainty price heuristic won, it won by a large margin, an average of 13.01 percent of the optimal profits. When the certainty price heuristic lost, it came in a close second, and lost by an average of 0.0628 percent of the optimal profits. As expected, the certainty capacity heuristic won only when the optimal solution was very close to balanced. When the certainty capacity won, the average $\tau(p)$ was -0.024 and the range was $[-.1314, .046]$. The entire set of 500 simulations has an average $\tau(p)$ of -0.4775 with a range of $[-1.025, .2201]$.

TABLE 3
Regression Analysis of Optimal Solution Calculation Time
on the Model Parameters and Case Dummy

$$\text{Solution Time} = \beta_0 + \beta_1 m + \beta_2 k + \beta_3 t + \beta_4 a + \beta_5 c + \beta_6 \alpha + \beta_7 \text{Case 1}$$

<u>Variable</u>	<u>Estimate</u> <u>(p-value)</u>
Intercept	257.54*** (0.0001)
m	-0.76*** (.0029)
k	539.10*** (.0001)
t	-41.72 (.4024)
a	-42.51*** (.0007)
c	21.65*** (.0001)
α	-47.48 (.5235)
Case 1	-228.32*** (.0001)
Adj. R ²	0.7254
n	500

*** Significant at the 1 percent level.

Variable Definitions:

m = number of potential customers;

k = cost of capacity;

t = variable cost multiplier, variable cost = t (capacity cost);

a = customer's disutility from back ordering;

c = shape parameter, customers' Weibull reservation price density;

α = scale parameter, customers' Weibull reservation price density; and

Case 1 = dummy variable, 1 if optimal discount is 0.

significant improvement over the basic certainty heuristic (99.47 percent vs. 94.10 percent).¹⁷ The global results demonstrate that a sophisticated capacity rule (certainty price heuristic) outperforms a sophisticated pricing rule (certainty capacity heuristic). In summary, the capacity decision is more crucial than the pricing decision.

We report one additional analysis for the certainty price heuristic. Table 4 reports the results of regressing the certainty price heuristic's percent of optimal profits on

¹⁷ A t-test fails to reject the hypothesis that the difference between the certainty capacity and basic certainty heuristics is 0 ($t = .2781$). A t-test does reject the hypothesis that the difference between the certainty price and basic certainty heuristics is 0 ($t = .0001$). Similar results are obtained with the nonparametric Mann-Whitney (or Wilcoxon rank) test.

the model parameters.¹⁸ The results support and clarify our previous observations. The performance of the certainty price heuristic increases in the number of potential customers and the distribution scale parameter (α), but decreases in the capacity costs and distribution-shape parameter. The intuition is identical to that associated with the increase in program run time.

Using a price heuristic is less costly than using a capacity heuristic, making the certainty price heuristic the clear winner in our simulations. In other words, optimizing over capacity dominates optimizing over price in our model.

To relate our results to prior research, the next section recasts our model into a manufacturing setting.

¹⁸ Table 4 shows that regressions for the basic certainty and certainty capacity heuristics display similar patterns. Performance increases in the number of potential customers and decreases in the unit capacity costs and the distribution-shape parameter. The basic certainty regression, but not the certainty capacity regression, shows that the performance increases in the distribution scale (α) parameter.

TABLE 4
Regression Analysis of the Heuristics' Simulation
Performance on the Model Parameters

% optimal profits = $\beta_0 + \beta_1 m + \beta_2 k + \beta_3 t + \beta_4 a + \beta_5 c + \beta_6 \alpha$
(Coefficient Estimates with p-values in parentheses)

Variable	Basic Certainty	Certainty Price	Certainty Capacity
Intercept	1.3425*** (0.0001)	1.0180*** (0.0001)	1.3418*** (0.0001)
m	0.00295*** (0.0001)	0.00062*** (0.0001)	0.00259*** (.0005)
k	-1.3825*** (0.0001)	-0.10592*** (0.0001)	-1.2862*** (.0001)
t	-0.01814 (0.9051)	-0.00545 (0.6964)	-0.01008 (0.9451)
a	0.05542 (0.1303)	-0.00287 (0.3922)	0.06747* (0.0555)
c	-0.11311*** (0.0001)	-0.00854*** (0.0001)	-0.10513*** (0.0001)
α	0.40658* (0.0742)	0.03469* (0.0968)	0.32800 (0.1338)
Adj. R ²	0.4575	0.3966	0.4421
n	500	500	500

*, ***, Significant at the 10 percent and 1 percent levels, respectively.

Variable Definitions:

m = number of potential customers;

k = cost of capacity;

t = variable cost multiplier, variable cost = t(capacity cost);

a = customer's disutility from back ordering;

c = shape parameter, customers' Weibull reservation price density; and

α = scale parameter, customers' Weibull reservation price density.

A MANUFACTURING INTERPRETATION

Our manufacturing firm model has a structure similar to our service firm model. At the start of each period the manager determines the capacity, price, and a price premium (described below), and m potential customers arrive sequentially. The only significant change is that when a customer orders when all current capacity has been allocated, the manager can pay a soft capacity surcharge, $\hat{d} \geq 0$, to purchase an additional capacity unit. The manager can also charge a price premium, $\hat{a} \geq 0$, in order to fill the over-capacity order.¹⁹

In this manufacturing firm model, a price premium replaces the customer-annoyance term, and the manager’s soft capacity surcharge replaces the customer’s price discount. This reinterpretation generates a few subtle changes in the derivation and analysis.

First, a necessary condition for the manager to purchase additional soft capacity is that the profit from an over-capacity sale be positive, $p + \hat{a} > \hat{d} + v + k$. Similarly, a customer will pay the over-capacity price premium if her utility from the purchase is greater than the price plus the premium, $u_i \geq p + \hat{a}$. The percentage of over-capacity customers willing to pay the premium is:

$$\hat{w}(p, \hat{a}) = \frac{(1 - F(p + \hat{a}))}{(1 - F(p))}$$

With no price premium, $\hat{a} = 0$, all customers stay, $w(p, \hat{a}) = 1$, and all over-capacity orders are filled. If the price premium is very large, $\hat{a} \rightarrow \infty$, all customers leave, $w(p, \hat{a}) = 0$, and no over-capacity orders are placed.

We obtain the following maximization program by following a similar process to that for the service firm.

The Manager’s Program in the Manufacturing Setting

$$\max_{x, p, \hat{a}} \int_{-\infty}^x (p - v)sf(s; m, p)ds + \int_x^{\infty} \{(p - v)x + \hat{w}(p, \hat{a})(p + \hat{a} - \hat{d} - v - k)(s - x)\}f(s; m, p)ds - kx,$$

where $x, p, \hat{a} \geq 0$.

The following proposition is the manufacturing setting equivalent of Proposition 2.

Proposition 6: In a manufacturing setting where the manager can select a price premium, \hat{a} , and can pay a soft capacity surcharge, \hat{d} , there are three possible solutions:

Case 1. When the soft capacity surcharge is relatively high, $\hat{d} + v + k \geq p$, the manager never purchases soft capacity; the effective price premium is infinite, $\hat{a} = \infty$, and no over-capacity customers order, $w(p, \hat{a}) = 0$.

Case 2. When the soft capacity surcharge is intermediate, $p > \hat{d} + v + k$, and

$$p - \hat{d} - v - k > \frac{(1 - F(p))}{f(p)},$$

the manager charges over-capacity customers a price premium, \hat{a} that solves:

$$(p + \hat{a}) - (\hat{d} - v - k) = \frac{(1 - F(p + \hat{a}))}{f(p + \hat{a})},$$

¹⁹ We thank Ramji Balakrishnan for suggesting this interpretation of our model.

and the percentage of over-capacity customers who order is:

$$1 > w(p, \hat{a}) = \frac{(1 - F(p + \hat{a}))}{(1 - F(p))} > 0.$$

Case 3. When the soft capacity surcharge is relatively low, $p > \hat{d} + v + k$ and

$$\frac{(1 - F(p))}{f(p)} \geq p - \hat{d} - v - k, \text{ the manager never charges a price premium,}$$

$\hat{a} = 0$, and always purchases enough soft capacity to satisfy all over-capacity orders. All over-capacity customers order, $w(p, \hat{a}) = 1$.

See the Appendix for the proof.

The three cases in Proposition 6 capture the three distinct settings from the full-cost heuristic literature. In the first case the cost of purchasing soft capacity is prohibitive, yielding a “hard” capacity constraint. Hansen and Magee (1993) have shown that full-cost heuristics perform well with hard capacity constraints. However, Balakrishnan and Sivaramakrishnan (2001) reach the opposite conclusion in a more complex environment.

In the third case, the cost of soft capacity is very low, while the value of the over-capacity orders is high, so the manager purchases soft capacity to fill all orders. To ensure that all customers buy from the firm, he does not charge over-capacity customers a price premium. Banker and Hughes (1994) show that in this setting using a full-cost decision rule is not just a good heuristic, but is actually the economically optimal decision.

Although prior work has addressed the two polar cases of completely soft and totally hard capacity, the hybrid second case of Proposition 6 is new to the literature. In this hybrid setting the manager selects the amount of soft capacity by trading off the increase in profit from the price premium against the decrease in the probability that an over-capacity customer will order.

A second distinguishing feature of our analysis is that we allow all three possible soft capacity conditions in our simulations. As mentioned in Section Six, we never drew a set of parameters where the manager used unlimited soft capacity (Case 3). Because Case 3 is the setting where a full-cost heuristic is actually the optimal decision rule, our simulations are biased against a full-cost heuristic performing well. In spite of this handicap, our full-cost heuristic still emerges as our best heuristic.

SEVERAL EXTENSIONS

Our model is an interesting first pass at examining full costs in a service setting. There are three potential extensions that prior analysis suggests could lead to different results (Balakrishnan and Sivaramakrishnan 2002).

In our model, the manager selects the amount of hard capacity every period. The first extension would have the hard capacity last multiple periods. With longer-lived hard capacity, shifting customers into the next period reduces the next period’s potential sales, and may make the price discount/soft capacity less valuable. To explore the potential effect of multiperiod capacity on our results, we constructed a simple two-period model and slightly modified the heuristics in order to run a trial simulation. In the modified heuristic, at the start of the first period the manager chooses the amount of the two-period capacity, a common price that holds for both periods, and a price discount. In this trial two-period horse race with 100 simulations, the balanced-capacity heuristic won 42 percent

of the time, the full-cost heuristic 39 percent, and the basic heuristic 19 percent. Contrary to the one-period setting, the full-cost heuristic no longer performed the best, but it still achieved 90.5 percent of the optimal profits. We conjecture that lengthening the hard capacity's life beyond two periods would lead to further erosion in the performance of the full-cost heuristic.

In our model no new information arrives between the capacity decision and the pricing choice. The second extension would allow information about the product demand to arrive after the capacity decision, but before the pricing decision. We modified our model to incorporate such interim demand information. In this modified setting, the demand information was whether the shift parameter in the Weibull distribution was high or low.²⁰ We modified the full-cost heuristic to use the updated information to set the price, and ran 50 simulations. The full-cost heuristic still achieved a respectable 97.4 percent of the optimal profits. The multiperiod analysis in Banker et al. (2002) suggests how a modified full-cost heuristic can adjust for new demand information to provide further improvements in performance.

Our model currently has one product and one capacity resource. The third extension would expand our setting to multiple products and multiple resources. Given the complexity of a model with multiple products and resources and our limited computing capacity, we decided to focus on the effect of two products in a one-resource model. We modified the heuristics to set one common capacity level, two service prices, and two service-price discounts, and found that calculating even one example in this setting proved beyond the power of our computer. Hence, the efficacy of our full-cost heuristic in this expanded setting is an open question.²¹

CONCLUSIONS

The debate over the proper role of fixed-capacity charges in product costs is a classic management accounting issue. Our paper examines how well a full-cost-based pricing heuristic performs in solving the capacity and pricing decisions for a service company. We set up a basic model of capacity and pricing choices and derive three heuristics from the limiting behavior of the optimal solution. We then run a horse race between three contenders: a full-cost-based pricing heuristic, a balanced-capacity heuristic, and a heuristic based solely on the model's limiting behavior. We find that the full-cost-based pricing heuristic yields a superior performance of 99.5 percent of the optimal profits in 500 random simulations vs. 94 percent for the other two heuristics.

The full-cost-based heuristic's performance yields an interesting corollary: accuracy is more important for the capacity decision than the pricing decision in our setting. When the firm takes a sophisticated approach to selecting capacity, a simple nonlinear pricing rule generates close to optimal profits.

We describe our model as reflecting a service, rather than a manufacturing setting for several reasons. First, services have a short production cycle and completing all services inside a period is a reasonable assumption. Second, services cannot be stored, so omitting inventory is more reasonable than it would be

²⁰ This modification generated a substantial dispersion in the prices across the high- and low-demand situations. The price across states varied by an average of 11 percent, while the profits across states varied by an average of 40 percent.

²¹ The two-product model's objective function involves seven nonlinear double integrals, while our one-product objective involves two nonlinear single integrals. We abandoned the simulation after 58 hours, when the time required for the first two-product example exceeded the time for of the 500 one-product examples discussed in the text.

in a manufacturing setting. Finally, price discounts are an accepted practice in service firms.²²

Introducing a price discount is our major modeling innovation. The price discount allows the manager to serve more customers today by shifting over-capacity customers into the next period. Our analysis shows that price discounts behave in a similar fashion as soft capacity; expensive capacity purchased at a premium to augment existing capacity (e.g., overtime). Both price discounts and soft capacity allow the manager to serve additional customers once the initial capacity has been exhausted. Because the manager can choose the amount of the price discount, we can generate a new soft capacity case where the manager uses the price discount to serve some, but not all, of the over-capacity customers.

Our second innovation is our novel method of performing the simulations. Rather than restricting the simulations to selecting parameters with a specific type of hard or soft capacity, as in prior work, we randomly select the parameters and let the cases fall where they will. Our approach generates a heuristic that is robust to the type of capacity.

In the extensions section of our paper, we examine the bounds of our results. Preliminary evidence suggests that a full-cost heuristic may continue to work well when interim demand information arrives after the capacity choice has been made, but before the pricing decision is completed. However, our evidence also suggests that a full-cost heuristic performs poorly in settings where capacity lasts several periods.

In summary, this paper advances the growing body of work in management accounting that recognizes that many relatively simple accounting practices may not provide information that leads to optimal decisions. However, these simple practices are often easier to implement in a decentralized environment and involve considerably less computational complexity. These potential benefits justify the practice when the expected loss due to deviation from optimal decisions is small enough. We evaluate a simple pricing rule based on the sum of variable and unit capacity cost. Extensive simulations reveal that this heuristic requires only 1 percent of the computational time required for the optimal decision, at the cost of only a 0.5 percent decline in the median expected profit relative to the optimum. Although this evidence supports the cost accounting practice of reporting unit costs for pricing decisions, preliminary analysis indicates that our results may extend to some, but not all environments.

²² For instance, a muffler shop in Los Angeles hands out 5 percent discount coupons to every customer turned away more than one-half hour before closing time.

APPENDIX

Proof of Proposition 2

Differentiating The Manager’s Program with respect to d provides:

$$\left[\frac{f(p-d+a)}{(1-F(p))} (p-d-v-k) - \frac{(1-F(p-d+a))}{(1-F(p))} \right]. \tag{A1}$$

If the second-order condition holds, then the expression (A1) is concave and has a positive, then a negative slope. The cases in the proposition are derived from the following observations. If (A1) is negative at the lower bound of $d = 0$, then the first-order condition cannot increase for higher values and the manager prefers to not offer a price discount (Case 1). Setting (A1) equal to 0 provides the interior solution (Case 2). If (A1) is positive at the upper bound of $d = a$, then the first-order condition must be smaller for lower values and the manager prefers to offer the maximum price discount (Case 3).

Derivation of The Pricing Program

The objective in The Manager’s Program can be rewritten as:

$$\begin{aligned} & [(1-w)(p-v) + w(d+k)]x \int_x^\infty f(s; m, p) ds + w(p-d-v-k) \int_x^\infty sf(s; m, p) ds \\ & + (p-v) \int_{-\infty}^x sf(s; m, p) ds - kx . \end{aligned}$$

Inserting in the formula:

$$\int_x^\infty sf(s; m, p) ds = m(1-F(p)) - \int_{-\infty}^x sf(s; m, p) ds ,$$

and simplifying provides:

$$\begin{aligned} & w(p-d-v-k)m(1-F(p)) + [(1-w)(p-v) + w(d+k)]x \int_x^\infty f(s; m, p) ds \\ & + [(1-w)(p-v) + w(d+k)] \int_{-\infty}^x sf(s; m, p) ds - kx . \end{aligned} \tag{A2}$$

The next step uses a minor variant of the formula for the mean of a truncated normal distribution (Johnson et al. 1994, 156, Equation 13.134):

$$\int_{-\infty}^x sf(s; m, p) ds = m(1-F(p)) \int_{-\infty}^x f(s; m, p) ds - f(x; m, p)[m(1-F(p))F(p)], \tag{A3}$$

Inserting (A3) into (A2) and rearranging terms yields:

$$\begin{aligned} & w(p-d-v-k)m(1-F(p)) + [(1-w)(p-v) + w(d+k)]x \int_x^\infty f(s; m, p) ds \\ & + [(1-w)(p-v) + w(d+k)]m(1-F(p)) \int_{-\infty}^x f(s; m, p) ds \\ & - [(1-w)(p-v) + w(d+k)][m(1-F(p))F(p)]f(x; m, p) - kx . \end{aligned} \tag{A4}$$

Inserting in:

$$\int_{-\infty}^x f(s; m, p) ds = 1 - \int_x^{\infty} f(s; m, p) ds$$

allows us to rewrite (A4) as:

$$(p - v)m(1 - F(p)) + [(1 - w)(p - v) + w(d + k)][x - m(1 - F(p))] \int_x^{\infty} f(s; m, p) ds - [(1 - w)(p - v) + w(d + k)][m(1 - F(p))F(p)]f(x; m, p) - kx. \quad (A5)$$

The next step inserts the rewritten capacity first-order condition:

$$[(1 - w)(p - v) + aw] \int_x^{\infty} f(s; m, p) ds = k,$$

into (A5) to provide:

$$(p - v)m(1 - F(p)) + [x - m(1 - F(p))]k - [(1 - w)(p - v) + w(d + k)][m(1 - F(p))F(p)]f(x; m, p) - kx. \quad (A6)$$

Canceling out the kx terms and rearranging terms provides:

$$(p - v - k)m(1 - F(p)) - [(1 - w)(p - v) + w(d + k)][mF(p)(1 - F(p))]f(x; m, p) \quad (A7)$$

Inserting Equation (6) into the last term of (A7) provides:

$$- [(1 - w)(p - v) + w(d + k)][mF(p)(1 - F(p))]f(x; m, p) = - [(1 - w)(p - v) + w(d + k)] \frac{[mF(p)(1 - F(p))]}{[mF(p)(1 - F(p))]^{1/2} \sqrt{2\pi}} \exp \left[\frac{-1}{2} \left(\frac{x - m(1 - F(p))}{[mF(p)(1 - F(p))]^{1/2}} \right)^2 \right]. \quad (A8)$$

Using Equation (6), expression (A8) becomes:

$$- [(1 - w)(p - v) + w(d + k)][mF(p)(1 - F(p))]^{1/2} \frac{1}{\sqrt{2\pi}} \exp \left(- \frac{\tau(p)^2}{2} \right),$$

and the pricing program now can be written in the form given in the text.

Proof of Proposition 3

Differentiating The Manager's Pricing Program with respect to p provides the first-order condition:

$$m[(1 - F(p)) - (p - v - k)f(p)] - \frac{[mF(p)(1 - F(p))]^{1/2}}{\sqrt{2\pi}} \exp \left(- \frac{\tau(p)^2}{2} \right) \left\{ (1 - w) - \frac{dw(p, d)}{dp} (p - v - k) + \frac{[(1 - w)(p - v) + aw]mf(p)(1 - 2F(p))}{2[mF(p)(1 - F(p))]} - [(1 - w)(p - v) + aw]\tau(p) \frac{d\tau(p)}{dp} \right\} = 0. \quad (A9)$$

The two missing pieces in (A9) are $\frac{dw(p, d)}{dp}$ and $\frac{d\tau(p)}{dp}$. Differentiating the definition of $w(p, d)$ with respect to p provides:

$$\begin{aligned} \frac{dw(p, d)}{dp} &= \frac{(1 - F(p + a))f(p) - (1 - F(p))f(p + a)}{(1 - F(p))^2} \\ &= \frac{w(p, d)f(p) - f(p + a)}{(1 - F(p))}. \end{aligned} \quad (A10)$$

The definition of $\tau(p)$ in Equation (6) can be rewritten as:

$$\int_{-\infty}^{\tau(p)} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) dz = 1 - \frac{k}{[(1-w)(p-v) + w(d+k)]}. \quad (\text{A11})$$

Differentiating both sides of (A11) with respect to p and rearranging terms generates:

$$\frac{d\tau(p)}{dp} = \frac{(1-w)k - \frac{[wf(p) - f(p+a)]}{(1-F(p))}(p-v-k)}{[(1-w)(p-v) + w(d+k)]^2 \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\tau(p)^2}{2}\right)}. \quad (\text{A12})$$

Inserting (A10), (A12) into (A9) and canceling out common terms leads to Equation (7) in the text.

Proof That the Firm Does Not Use Back Orders in the Certainty Benchmark

At the first stage the firm selects capacity, x , by solving the following program:

$$\begin{aligned} \max_x \quad & (p-v-k)x + w(p-d-v-k)[m(1-F(p)) - x] \\ \text{such that} \quad & m(1-F(p)) \geq x \geq 0. \end{aligned}$$

This program has a first-order condition which never holds, namely:

$$(1-w)(p-v-k) + wd \gg 0. \quad (\text{A13})$$

Therefore, in the first period the firm chooses a capacity amount at one of the corners. Since (A13) holds, the objective function is strictly increasing in x , and the firm selects the corner $x = m(1-F(p))$. At this corner, capacity equals demand and the firm never allows back ordering.

Proof of Proposition 4

The objective function in The Manager's Pricing Program:

$$(p-v-k)m(1-F(p)) - [(1-w)(p-v) + w(d+k)][mF(p)(1-F(p))]^{1/2} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\tau(p)^2}{2}\right)$$

is composed of two pieces. The first term is the objective function for the certainty benchmark, $(p-v-k)m(1-F(p))$. The second term is the penalty for stochastic

ordering, $[(1-w)(p-v) + w(d+k)][mF(p)(1-F(p))]^{1/2} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\tau(p)^2}{2}\right)$. We will show that

as the number of customers rises, $m \rightarrow \infty$, the second term becomes small relative

to the first term, $\lim_{m \rightarrow \infty} \frac{\text{term 2}}{\text{term 1}} = 0$. Since the second term has declining influence on

the firm's profits as the number of customers rises, the relative loss from using a solution based only on the first term (the certainty benchmark solution) goes to 0.

The proof that the second term becomes relatively small consists of (1) constructing an upper bound for the absolute value of the second term, and then (2) showing that the ratio of the second term's upper bound to the first term goes to 0 as $m \rightarrow \infty$.

The upper bound is constructed using the following two inequalities. First, the firm's unit profit from a sale before capacity binds is greater than the profits from a sale after capacity binds:

$$(p-v-k) \geq (1-w)(p-v) + w(k+d). \quad (\text{A14})$$

Second, the normal density is less than one for all values:

$$1 \geq \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\tau(p)^2}{2}\right) \forall \tau(p). \quad (\text{A15})$$

Using (A14) and (A15) in sequence yields:

$$\begin{aligned} & (p - v - k)[mF(p)(1 - F(p))]^{1/2} \\ & \geq [(1 - w)(p - v) + w(d + k)][mF(p)(1 - F(p))]^{1/2} \\ & \geq [(1 - w)(p - v) + w(d + k)][mF(p)(1 - F(p))]^{1/2} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\tau(p)^2}{2}\right) > 0. \end{aligned}$$

The limit of the ratio of the second term's upper bound to the first term is:

$$\lim_{m \rightarrow \infty} \frac{(p - v - k)[mF(p)(1 - F(p))]^{1/2}}{(p - v - k)m(1 - F(p))} = \frac{F(p)^{1/2}}{(m(1 - F(p)))^{1/2}} = 0.$$

The Derivation of the Pricing Rule Used in the Certainty Capacity Heuristic

The derivation follows the same steps used earlier to obtain expression (A5). Inserting $x = m(1 - F(p))$ into (A5) generates:

$$\begin{aligned} & (p - v - k)m(1 - F(p)) - [(1 - w)(p - v) + w(d + k)][m(1 - F(p))F(p)]f(x; m, p) \quad (\text{A15}) \\ & = (p - v - k)m(1 - F(p)) - \frac{[(1 - w)(p - v) + w(d + k)][m(1 - F(p))F(p)]^{1/2}}{\sqrt{2\pi}}. \end{aligned}$$

Differentiating (A15) with respect to p and rearranging terms provides the expression in the text.

Proof of Proposition 6

Proof for Case 1

If $\hat{d} + v + k \geq p$, then every over-capacity order is unprofitable and the manager prefers to not fill any over-capacity order.

Proof for Cases 2 and 3

Differentiating The Manager's Program in the Manufacturing Setting with respect to \hat{a} provides:

$$\left[-\frac{f(p + \hat{a})}{(1 - F(p))} (p + \hat{a} - \hat{d} - v - k) - \frac{(1 - F(p + \hat{a}))}{(1 - F(p))} \right]. \quad (\text{A16})$$

If the second-order condition holds, then the expression (A16) is concave and has a positive, then a negative slope. The proofs for the second and third cases in the proposition are derived from the following observations. If (A16) is negative at the lower bound of $\hat{a} = 0$, then the first-order condition does not rise for higher values and the firm prefers to not offer a price premium (Case 3). Setting (A16) equal to 0 provides the interior solution (Case 2).

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